100 Years of Quantum Mechanics

From Quantum Mechanics to Modern Particle Theory

Thomas Mannel, University of Siegen

color meets flavor

CPPS Center for Particle Physics Siegen

TP1 Theoretical Particle Physics

P H

University of Vienna Physics Colloquium, Dec. 1st, 2025

Menue

- The Beginning of Quantum Mechanics
- Early Contributors
- Milestone experiments: Estabslishing the quantum world
- Quantum technologies
- From Relativistic Quantum Mechanics and Quantum Field Theory
- Quantum Theory in Particle Physics
- Quantum Electrodynamics: The Blueprint for modern Particle Theories
- Renormalization and Renormalization Group
- The Standard Model of Particle Physics
- Outlook: What could come next?

The Beginning of Quantum Mechanics

- At the end of the 19th century:
  - Classical Mechanics
  - Waves and Fields: Classical Electrodynamics
- Some problems remained
  - Blackbody radiation
  - Spectra of atoms
- Development of empirical formulae and simple models

- Inherent paradigm shift ...

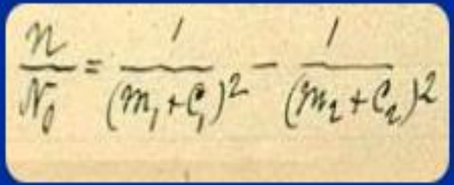
Quantum History Wall

(from quantum-history.de)

1884 1890 1900

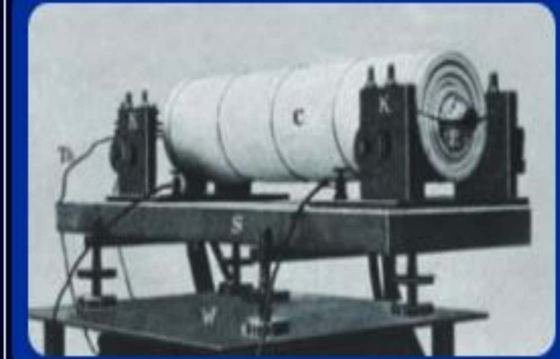
1885

Hydrogen Spectral Lines in (Architectural) Perspective



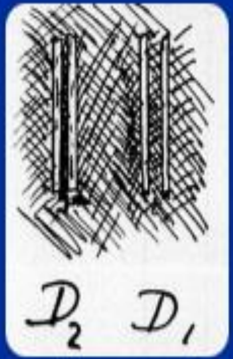
1888

Wavelengths of Spectral Series Lines



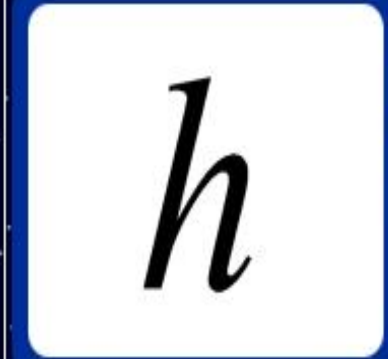
1895

A Black Body Explaining the Colors of Light



1896

What Spectral Lines Do When In a Magnetic Field

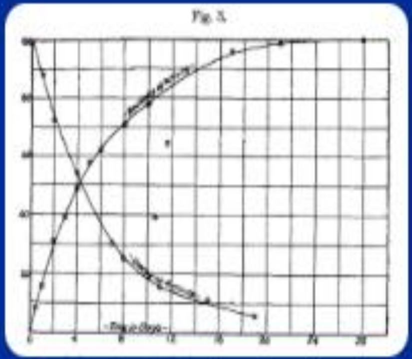


December 1900

A New Universal Constant With a Meaning

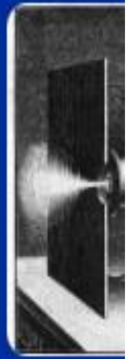
1902

Frequency, Not Intensity Reigns the Photo Effect



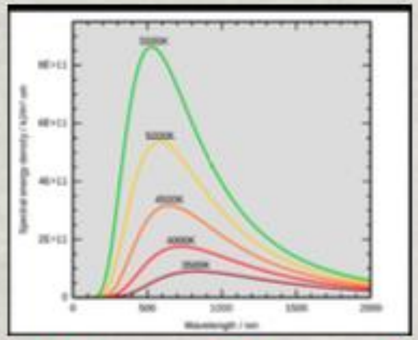
1902

The Law of Radiactive Decay

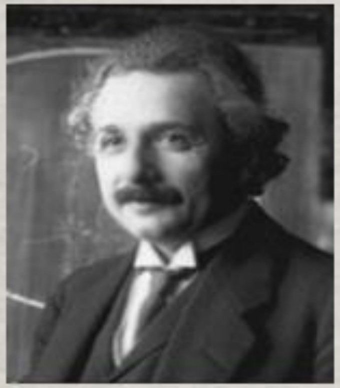


Early Contributors

Max Planck: Planck constant: black-body radiation 1900 (1918)



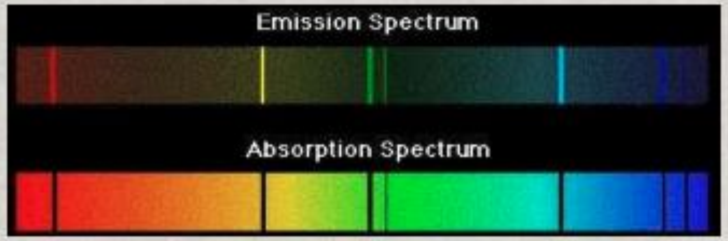
$$I(\nu,T)d\nu = \left(\frac{2h\nu^3}{c^2}\right)\frac{1}{e^{\frac{h\nu}{kT}}-1}\,d\nu$$



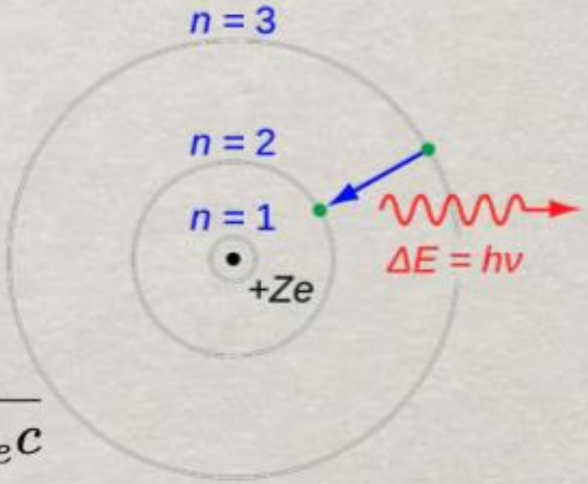
Albert Einstein: Photo-electric effect 1905 (1921)

$$E_{\max} = h\nu - W_0$$

Neils Bohr: Atomic model 1913 (1922)



$$a_0 = \frac{\hbar}{\alpha m_e c}$$



Towards the theoretical Desription

- Particle-Wave Dualism
- Matrix Mechanics
- Wave Mechanics

Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.

Von **W. Heisenberg** in Göttingen.

(Eingegangen am 29. Juli 1925.)


Heisenberg: Vacation on Helgoland 1923

The “umdeutung paper” & the Matrix Mechanics

Heisenberg 1925

- Follow the “correspondence principle”: classical – quantum correspondence in large quantum numbers
- Give up the unobservable orbits (x) and momentum (p)
- Focus on observable spectral lines, intensities …
- Establish selection rules & time-evolution

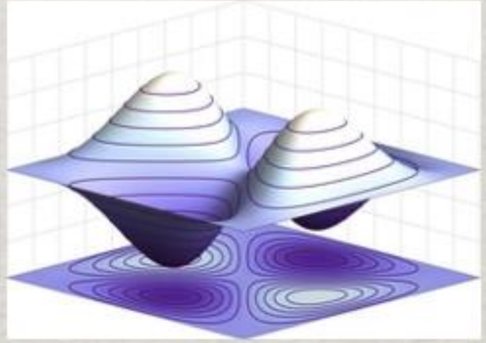
Heisenberg-Born-Jordan 1926

- Established non-communitive relations

$$[\hat{x}(t), \hat{p}(t)] = i\hbar$$

Erwin Schrödinger: Schrödinger Equation 1926

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$



The Papers laying out the Theory Foundations

Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.

Von **W. Heisenberg** in Göttingen.

(Eingegangen am 29. Juli 1925.)

Zur Quantenmechanik.

Von **M. Born** und **P. Jordan** in Göttingen.

(Eingegangen am 27. September 1925.)

Zur Quantenmechanik. II.

Von **M. Born, W. Heisenberg** und **P. Jordan** in Göttingen.

(Eingegangen am 16. November 1925.)

Second Series *December, 1926* *Vol. 28, No. 6*

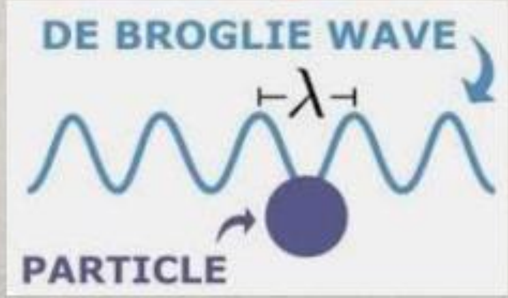
THE

PHYSICAL REVIEW

AN UNDULATORY THEORY OF THE MECHANICS OF ATOMS AND MOLECULES

By E. Schrödinger

Louis de Broglie: Matter wave 1924



$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

(1929)

Werner Heisenberg:

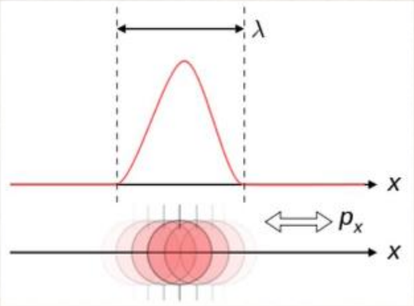
Matrix mechanics 1925
(Born & Jordan, 1926)

$$\mathbf{PQ} - \mathbf{QP} = \frac{\hbar}{i}\mathbf{I} \qquad \frac{\partial \hat{O}(t)}{\partial t} = \frac{1}{i\hbar}[\hat{O}(t), \hat{H}]$$

Uncertainty principle 1927

$$\Delta x \Delta p \geq \hbar/2 \quad \Delta E \Delta t \geq \hbar/2$$

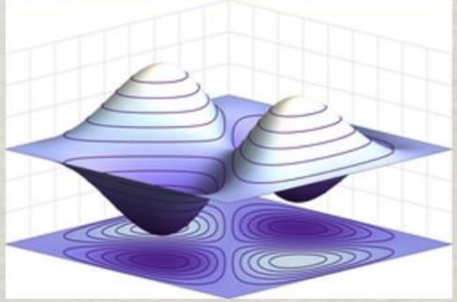
... ...



(1932)

Erwin Schrödinger: Schrödinger Equation 1926

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

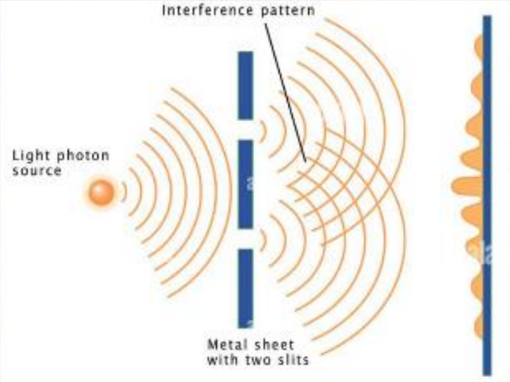


Spin and Relativistic Equations

- Interpretation of the Wave Function
- Development of Special Relativity
  - Klein Gordon Equation
  - **Dirac Equation**

- Intrinsic Properties: Spin of particles
- Exclusion Principle, Spin-Statistics Theorem

Max Born: Wave function statistical interpretation 1926



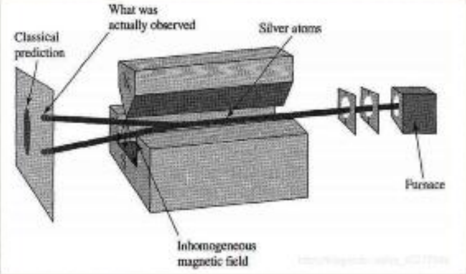
$$P = |\Psi|^2$$

(1954)

Wolfgang Pauli: Electron spin & exclusion principle 1925, Spin–statistics theorem 1940

(1945)

$$\psi(\mathbf{r}_a, \mathbf{r}_b) = [\psi_1(\mathbf{r}_a)\psi_2(\mathbf{r}_b) \pm \psi_1(\mathbf{r}_b)\psi_2(\mathbf{r}_a)]/\sqrt{2}$$



Paul Dirac:

$$\{F, G\} \Rightarrow \frac{1}{i\hbar}[\hat{F}, \hat{G}]$$

(1965)

Dirac relativistic Equation 1928

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \left(\frac{\hbar c}{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta mc^2\right)\psi(\mathbf{x},t)$$

(Early) Milestone Experiments

1. Blackbody Radiation (Planck, 1900)

- **Problem:** Classical physics couldn't explain why the radiation emitted by a blackbody diverged at short wavelengths (the "ultraviolet catastrophe").
- **Insight:** Max Planck proposed that energy is quantized and emitted in discrete packets (quanta) introducing the idea of **energy quanta** $E = h\nu$.

2. Photoelectric Effect (Einstein, 1905)

- **Problem:** Classical wave theory couldn't explain why light below a certain frequency doesn't eject electrons from a metal surface.
- **Insight:** Albert Einstein proposed that light is made of **photons** with quantized energy, supporting the **particle nature of light** and confirming Planck's quantum hypothesis.

3. Franck–Hertz Experiment (1914)

- **Problem:** Needed to test the quantized nature of atomic energy levels.
- **Insight:** Electrons gained/expended energy in discrete amounts, providing direct evidence for **quantized energy levels in atoms** (especially mercury atoms).

4. Compton Scattering (1923)

- **Problem:** Classical wave theory couldn't account for the scattering of X-rays by electrons.
- **Insight:** Arthur Compton demonstrated that photons carry momentum, confirming the **particle-like behavior of light** and supporting Einstein's photon model.

5. Stern–Gerlach Experiment (1922)

- **Problem:** Classical physics predicted a continuous range of outcomes for magnetic moment directions.
- **Insight:** Only discrete values of angular momentum (spin) were observed, showing **quantum spin quantization**.

6. Davisson–Germer Experiment (1927)

- **Problem:** Could electrons exhibit wave-like behavior?
- **Insight:** Electrons diffracted off a crystal lattice just like waves, confirming **de Broglie's hypothesis** that particles have **wave-particle duality**.

7. Double-Slit Experiment with Electrons (1927, further by Tonomura in 1980s)

- **Problem:** Could single particles interfere with themselves?
- **Insight:** Electrons sent one at a time through two slits still formed an interference pattern, showing **wave-particle duality** and **quantum superposition**.

8. Zeeman Effect (1896)

- **Problem:** Why do spectral lines split in a magnetic field?
- **Insight:** The splitting of spectral lines in a magnetic field hinted at **quantized angular momentum** and **magnetic properties of electrons**, helping develop quantum models of the atom.

Implications of QM: Entanglement

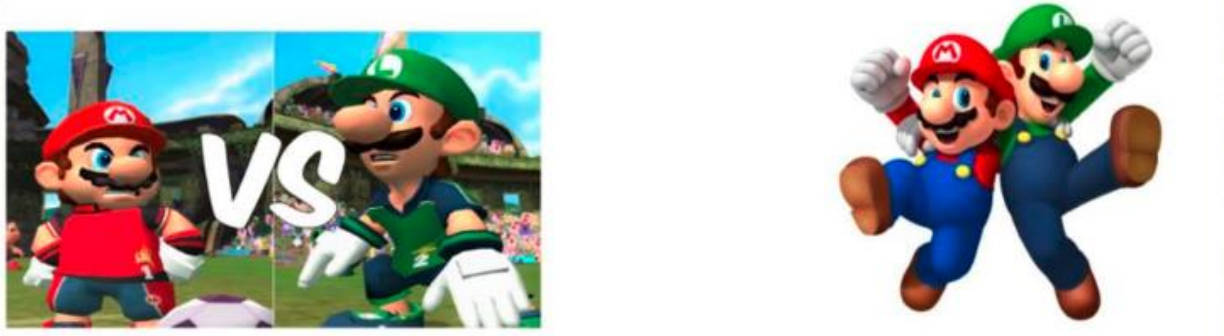
For a bipartite system, *i.e.*, $\frac{1}{2}\otimes\frac{1}{2} = 1\oplus 0$:

Singlet: $|0,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

Triplet:

$$
\begin{aligned}
|1,1\rangle &= \uparrow\uparrow \\
|1,0\rangle &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\
|1,-1\rangle &= \downarrow\downarrow
\end{aligned}
$$

Entangled



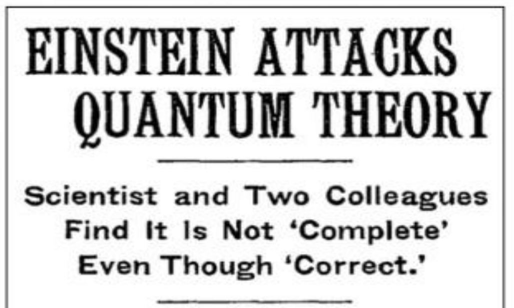
Separable Non-Separable

Quantum entanglement
→ sub-systems inseparable

Einstein-Podolsky-Rosen Paradox (Phys. Rev. 1935)

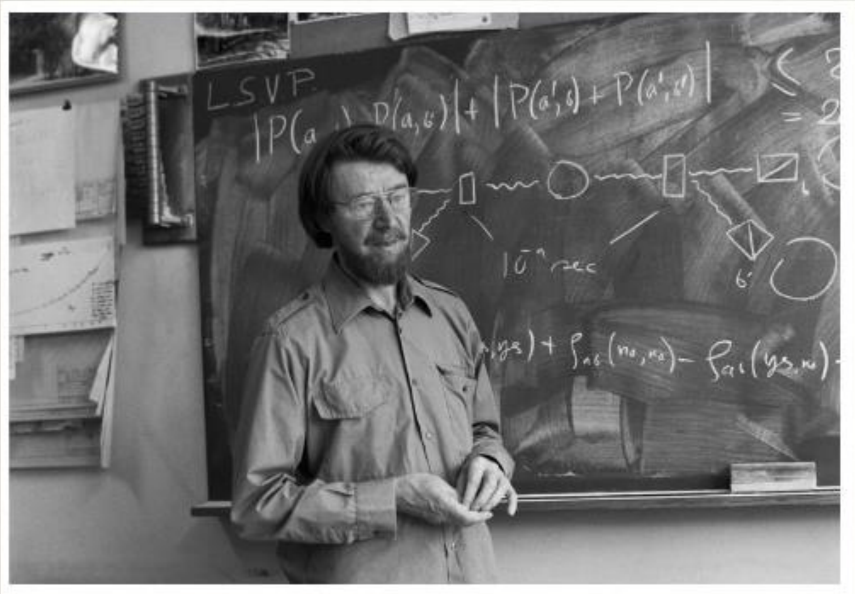
"Can quantum-mechanical description of physical reality be considered complete?"

"God doesn't play dice with the universe" – A. Einstein

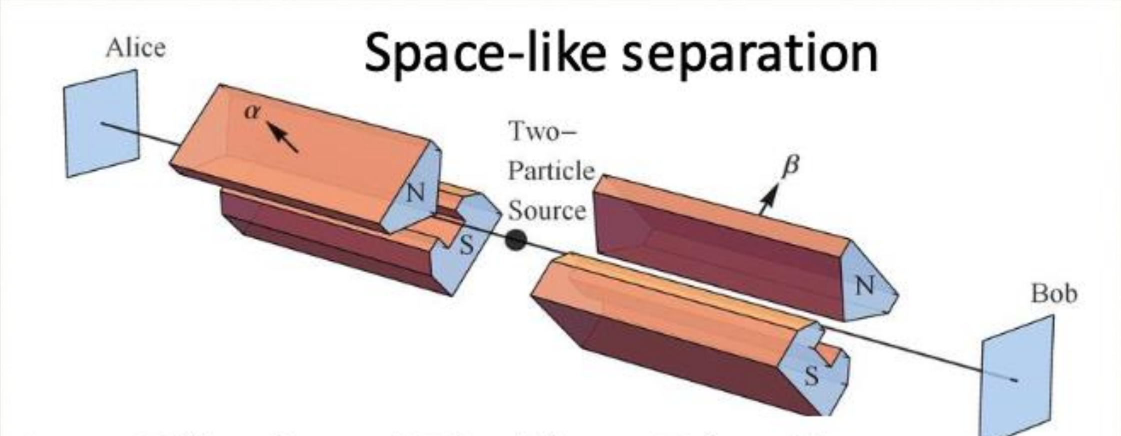

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

John S. Bell´s Inequality



“On the Einstein-Podolsky-Rosen paradox” (1964)



$$1 + P(\mathbf{b},\mathbf{c}) \geq |P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})|.$$

QM Non-Communitivity is the key:

Bell’s Inequality CAN BE violated by QM measurements;

but NOT by an EPR’s Local Hidden Variable Theory .

→ **“Quantum Information”**

NOBELPRISET I FYSIK 2022

THE NOBEL PRIZE IN PHYSICS 2022

KUNGL. VETENSKAPS-AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES


Photo: Royal Society

Alain Aspect
Université Paris-Saclay & École Polytechnique, France

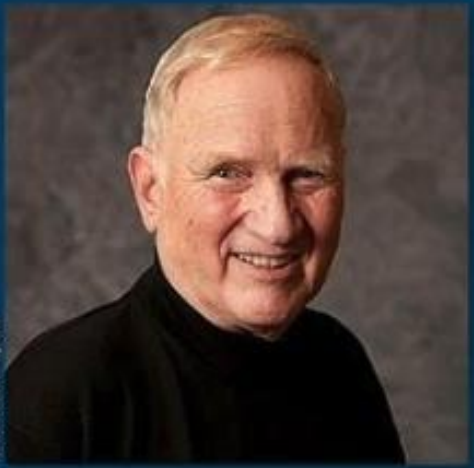


Photo: Peter Lyons

John F. Clauser
J.F. Clauser & Assoc., USA


Photo: Sepp Dreissinger

Anton Zeilinger
University of Vienna, Austria

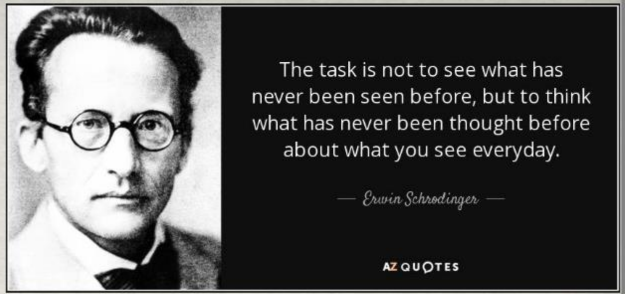
”för experiment med sammanflätade fotoner som påvisat brott mot Bell-olikheter och banat väg för kvantinformationsvetenskap”

“for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science”

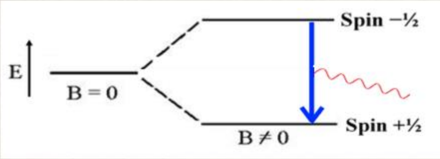
#nobelprize

THE NOBEL PRIZE

Quantum Technology in the real world



Spin $\frac{1}{2}$ system in magnetic field:



$\Delta E = \gamma \hbar B$

gyromagnetic ratio

$h\nu = \Delta E$

Magnetic Resonance Imaging (MRI):

Applying position dependent $B(x)$

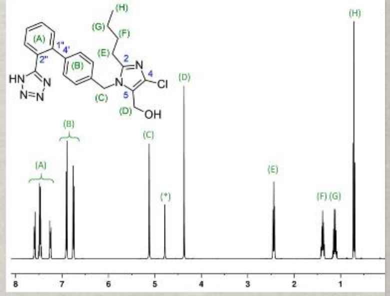
⇒ location of hydrogen atoms


MRI scan of brain | MRI scan of knee

Nuclear Magnetic Resonance (NMR):

Measuring the frequency of radiation

⇒ characteristic types of atoms



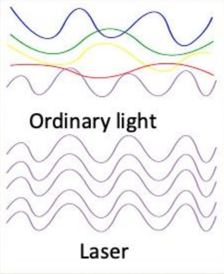
- Numerous technological applications of quantum effects

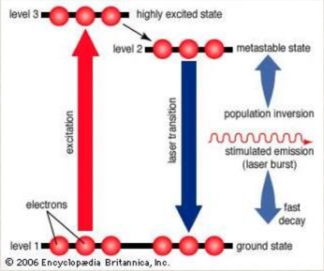
Laser:

Monochromatic

Coherence

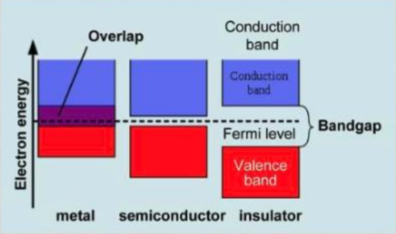
Directionality





$h\nu = E_2 - E_1$

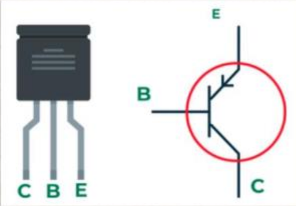
Semiconductor / Transistor:

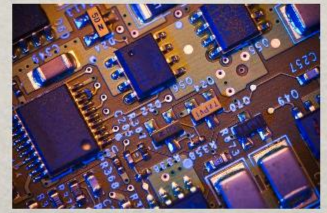


intermediate energy band:

- switch between conductor and insulator
- ideal for controlling current!

EVERYWHERE!





Smartphones and Laptops

Power Control, Displays

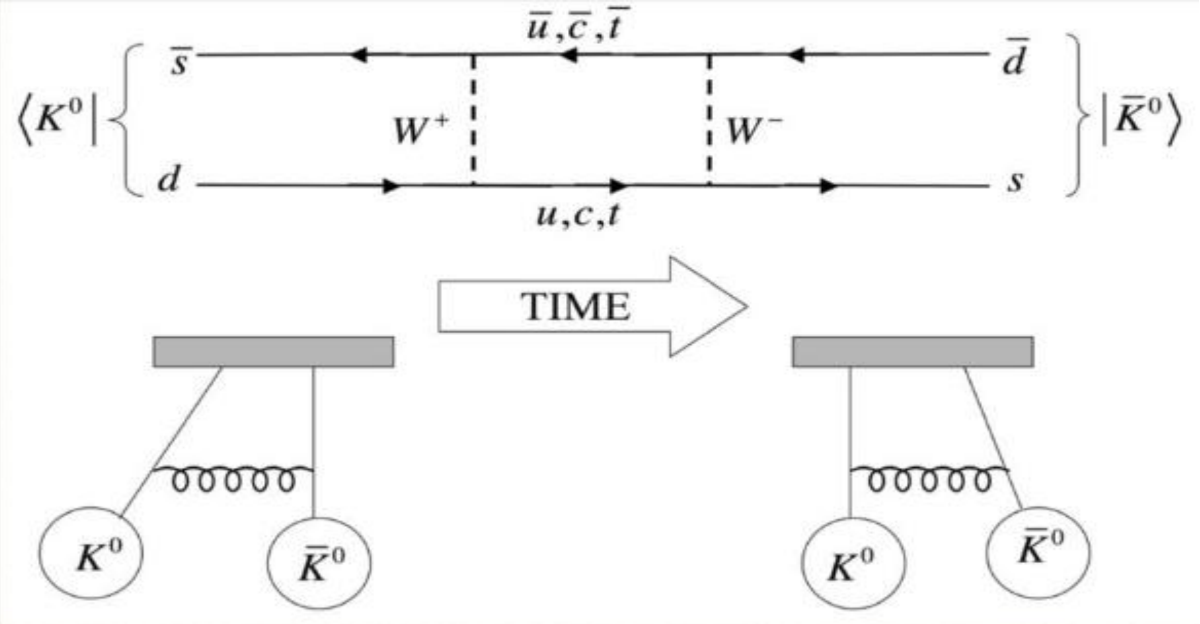
Wireless Communication

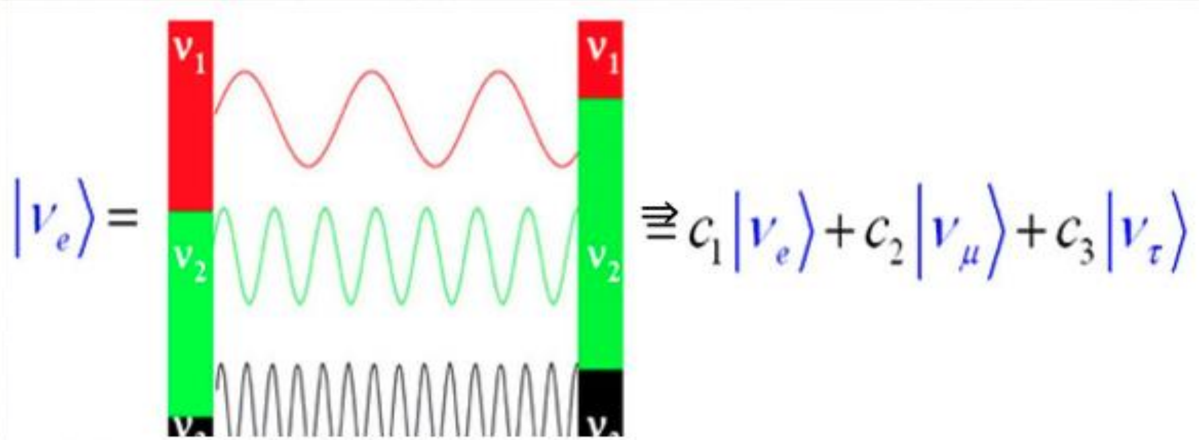
(5G, Wi-Fi)

Quantum Theory in Particle Physics

- **Neutral meson system "Flavor" physics:**
quark flavor mixing; heavy quark masses; CP violation
...

- **Neutrino mixing:**
lepton flavor mixing; neutrino masses; CP violation ...





Quantum World at highest Energies

Relativistic Quantum Mechanics

Special Relativity:

① Lorentz transformation:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

⟹

Quantum Mechanics:

Lorentz invariant equation:

Spin-0: Klein–Gordon equation

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\phi = \frac{m^2c^2}{\hbar^2}\phi$$

Spin-1/2: Dirac equation

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

② Mass–energy equivalence:

$$E = mc^2$$

⟹ Particle creation and annihilation

The second quantization:

Creation & annihilation operators $\boxed{a_{\mathbf{p}}}e^{-i\omega_{\mathbf{p}}t+i\mathbf{p}\cdot\mathbf{x}} + \boxed{a^*_{\mathbf{p}}}e^{i\omega_{\mathbf{p}}t-i\mathbf{p}\cdot\mathbf{x}}$

Bosons: $[a_\alpha, a^\dagger_\beta] = \delta_{\alpha,\beta}, \quad [a_\alpha, a_\beta] = [a^\dagger_\alpha, a^\dagger_\beta] = 0$

Fermions: $\{c_\alpha, c^\dagger_\beta\} = \delta_{\alpha,\beta}, \quad \{c_\alpha, c_\beta\} = \{c^\dagger_\alpha, c^\dagger_\beta\} = 0$

Particle number & energy operators

$$\hat{N}_p = a^\dagger_p a_p \qquad \hat{H} = (\hat{N}_p + \frac{1}{2})\hbar\omega$$

The state is
a quantum field

Some Remarks

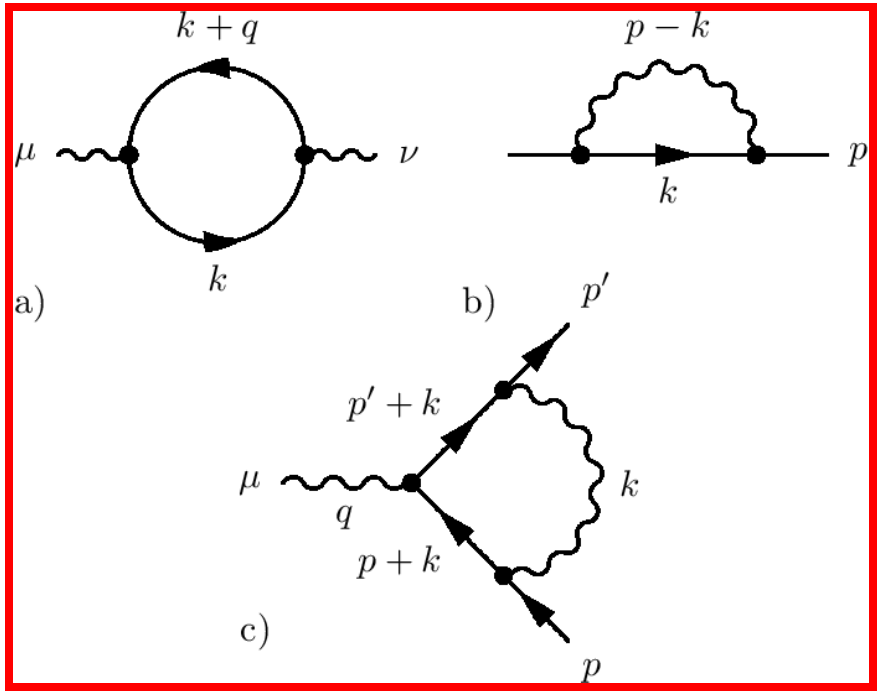
- Strictly speaking, there is no such thing as relativistic Quantum Mechanics
- Neither the Klein-Gordon nor the Dirac Equation can be interpreted as a single particle wave equation
- Quantum Field Theory = Ensemble of infinitelty many (coupled) harmonic oscillators
- Transition from a finite number of d.o.f. to an infinite number is non-trivial

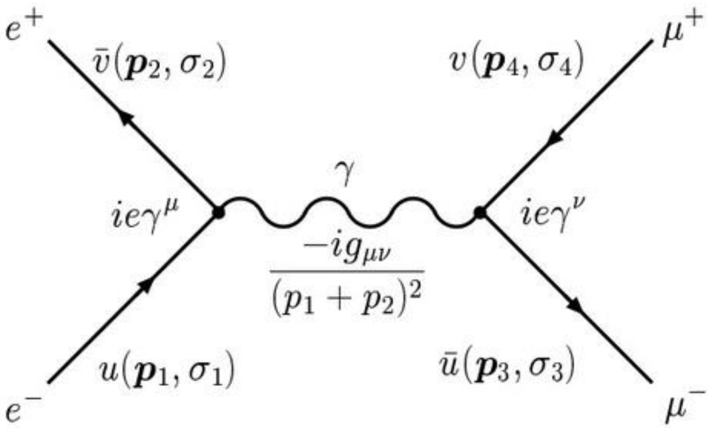
$$[a_\alpha, a^\dagger_\beta] = \delta_{\alpha,\beta}, \quad [a_\alpha, a_\beta] = [a^\dagger_\alpha, a^\dagger_\beta] = 0$$

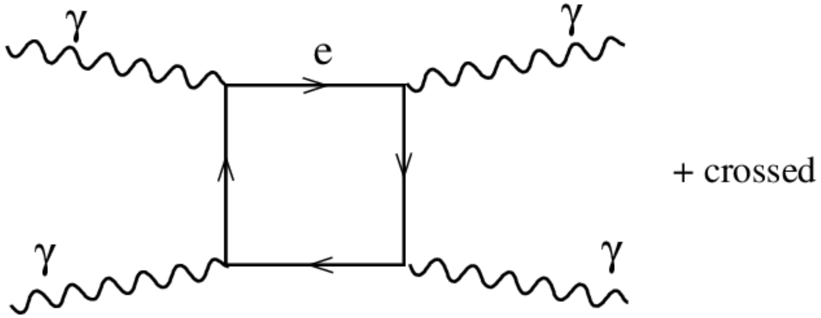
- For an infinite set, this algebra has infinitely many inequivalent reps.
- Hilbert Space is not unique

Prototype Theory: Quantum Electrodynamics

- Coupling constant = Fine structure constant α = 1/137
- Perturbative expansion in α: Feynman Diagrams
- ... corresponding to mathematical expressions
- Some diagrams yield divergent expressions:



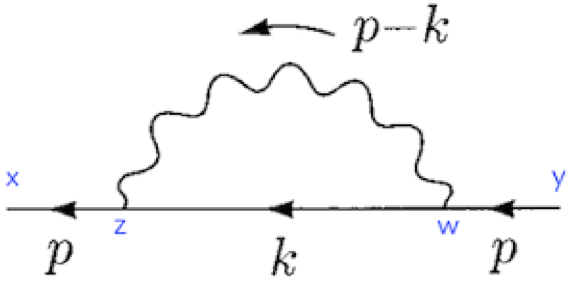




Renormalization and Renormalizibility

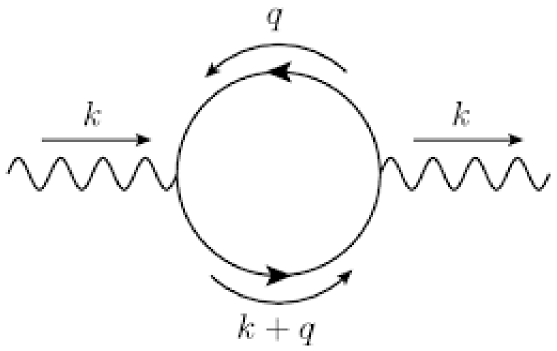
- The old problem of the electron mass re-occurs:
  How large is the contribution of the Coulomb field to the mass of the electron?

- Electron Self Energy is divergent: $\Sigma =$



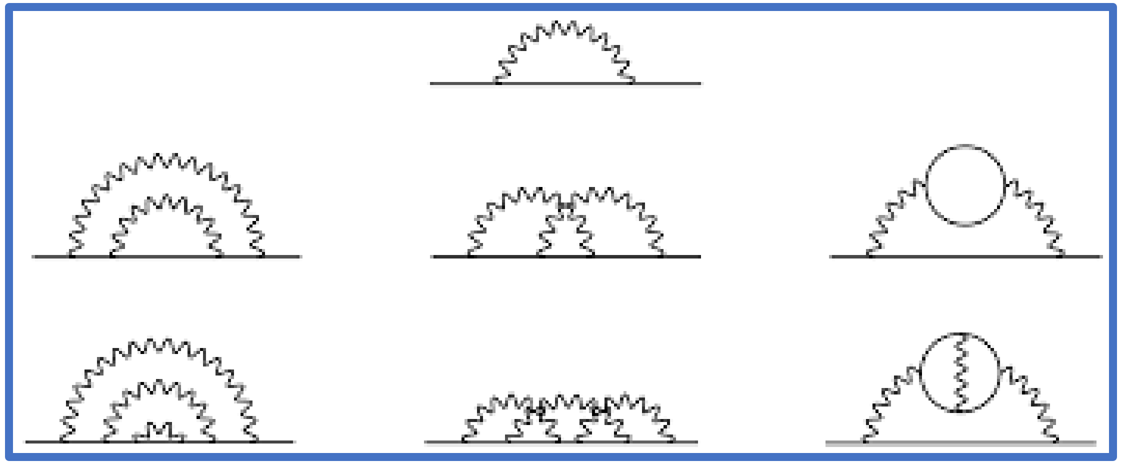
- Renormalize the mass: $m_{phys} = m_{bare} + \Sigma$ is the measured value of 511 keV

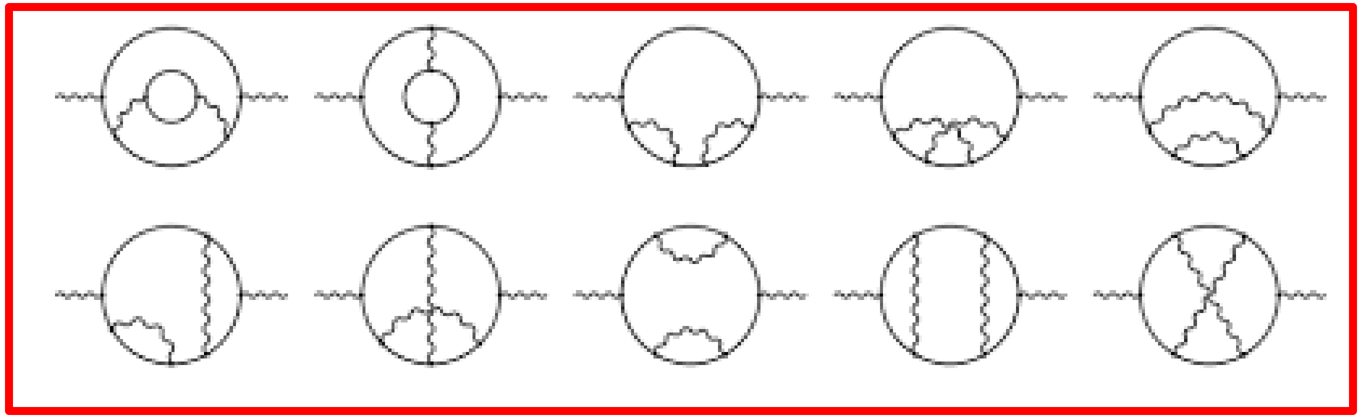
- Likewise for the coupling $\Pi =$



- $\alpha_{phys} = \alpha_{bare} + \Pi$

- This also works for higher orders in the perturbative expansion





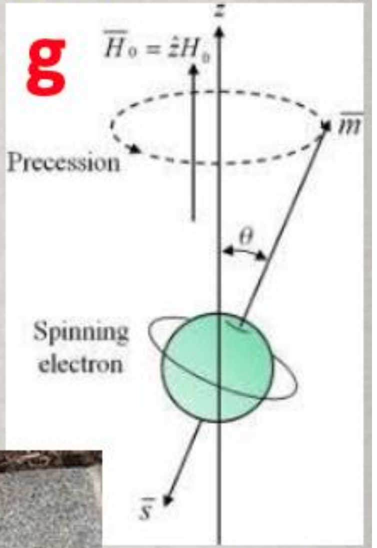
- Quantum Electrodynamics is a Renormalizable Theory:
  - The redefinition of mass and coupling is suffient to make the Feynman diagrams at all order finite,
  - The remaining finite parts of the divergent diagrams are uniquely defined.

Quantum Electro-Dynamics on trial

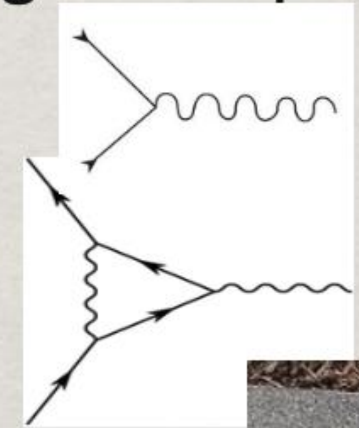
Dirac's relativistic quantum mechanics → QED

Feynman/Schwinger/Tomonaga → Renormalization

Best example: Anomalous magnetic dipole moment: **g**



Dirac equation 1928: **g = 2**



Schwinger's quantum correction in QED in 1948:

g-2= $a_e(\mathrm{Schwinger}) \approx \frac{\alpha}{2\pi} \approx 0.0011614$

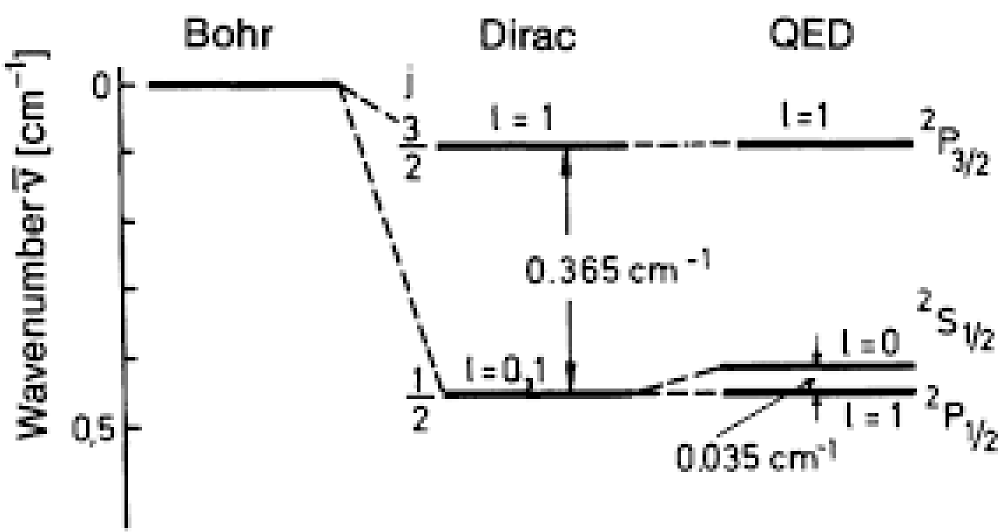
$$a_e^{theo} = 0.001159652181643(763)$$

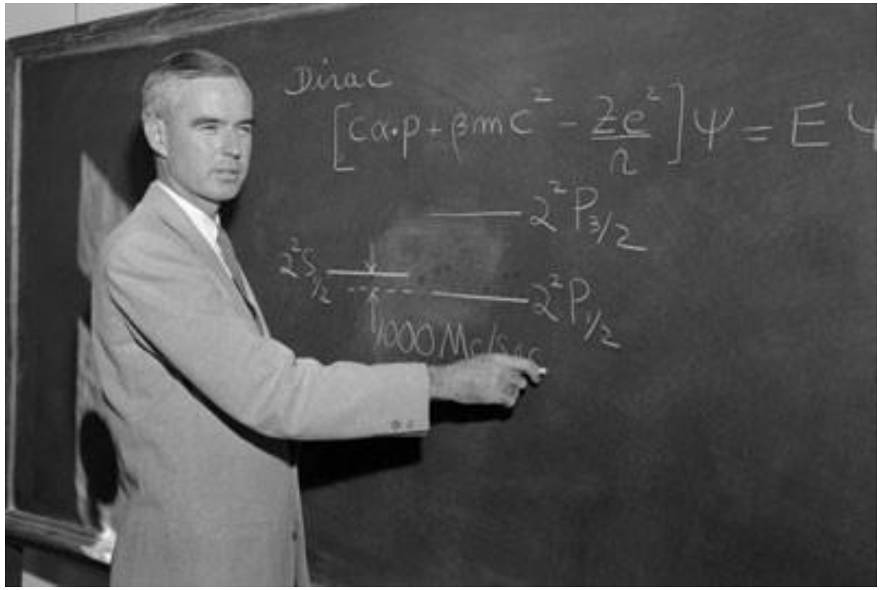
$$a_e^{exp} = 0.00115965218073(28)$$

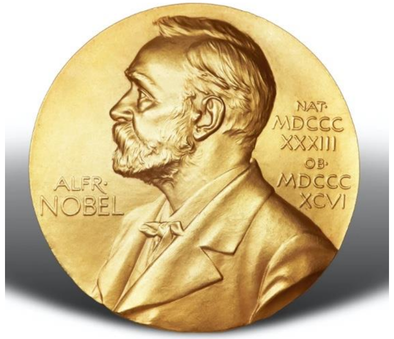
Up to 6-loops in QED → 10^{-10}

QED on Trial II: The Lamb Shift

- Self Energy of a bound electron is slightly different from the one of a free electron:



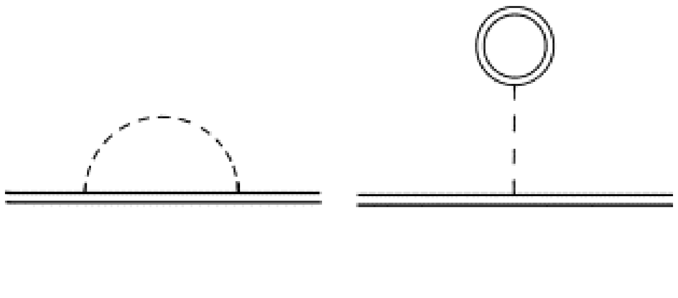




(1955)

$$\Delta E_{\text{Lamb}}(n, \ell = 0, j = \tfrac{1}{2}) = (Z\alpha)^4 \frac{4\alpha}{3\pi n^3} m_e c^2 \left(\ln\left(\frac{m_e c^2}{2R_y} \right) - \gamma(n,\ell,j) + \frac{19}{30} \right)$$

$$\Delta E_{\text{Lamb}}(n, \ell \neq 0, j) = (Z\alpha)^4 \frac{4\alpha}{3\pi n^3} m_e c^2 \left(\ln\left((Z\alpha)^2 \frac{m_e c^2}{2R_y} \right) - \gamma(n,\ell,j) + \frac{3}{8} \frac{j(j+1) - \ell(\ell+1) - \frac{3}{4}}{\ell(\ell+1)(2\ell+1)} \right)$$



(a) (b)

Some Remarks on Renormalization

- In the “early days“, renormalization was considered a “dirty trick“
- ... despite of the success of QED
- QED served as a blueprint of the Standard Model of particle physics
- ... and renormalizability became a criterion for a “good“ (i.e. predictive) theory
- Nobel price for t‘hooft and Veltman in 1999 for the proof of the renormali-zability of the Standard Model

Renormalization group in a nutshell

- Renormalization introduces a scale into the theoretical description
- Dimensional transmutation: The coupling “runs“

$$\mu \frac{d}{d\mu} \alpha_s(\mu) = \beta(\alpha_s(\mu))$$

- Scale Change including the change in α_s: Renormlization group transformation
- This changes the dimensional analysis:
  - The coupling also changes once the scale is changed.
  - Breaking of conformal invariance
- Renormalization theory opened the road to understand also critical phenomena and phase transitions

Photo from the Nobel Foundation archive.

Kenneth G. Wilson

Prize share: 1/1

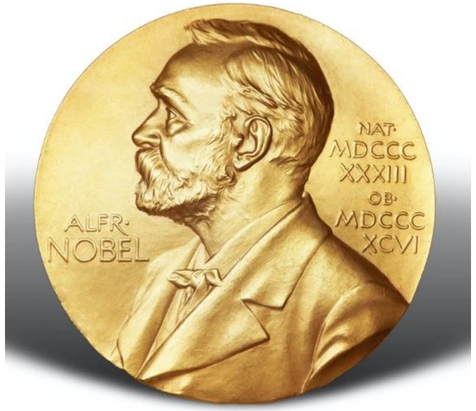
The renormalization group: Critical phenomena and the Kondo problem*†

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

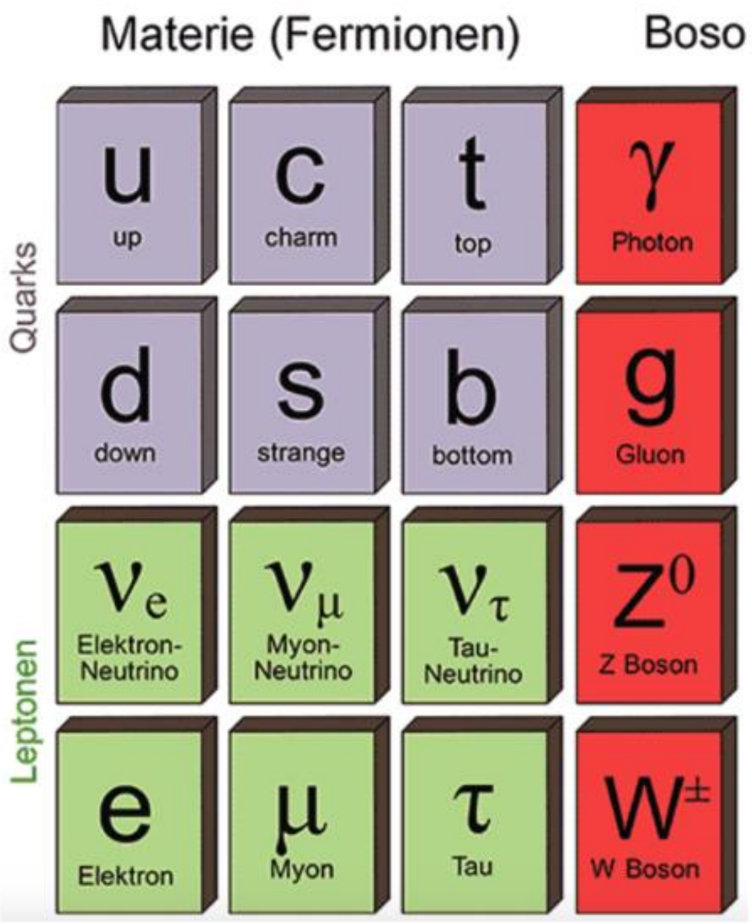
This review covers several topics involving renormalization group ideas. The solution of the *s*-wave Kondo Hamiltonian, describing a single magnetic impurity in a nonmagnetic metal, is explained in detail. See Secs. VII–IX. "Block spin" methods, applied to the two dimensional Ising model, are explained in Sec. VI. The first three sections give a relatively short review of basic renormalization group ideas, mainly in the context of critical phenomena. The relationship of the modern renormalization group to the older problems of divergences in statistical mechanics and field theory and field theoretic renormalization is discussed in Sec. IV. In Sec. V the special case of "marginal variables" is discussed in detail, along with the relationship of the modern renormalization group to its original formulation by Gell-Mann and Low and others.

The Nobel Prize in Physics 1982 was awarded to Kenneth G. Wilson "for his theory for critical phenomena in connection with phase transitions"



Putting the Standard model to trial

Before the Higgs Discovery

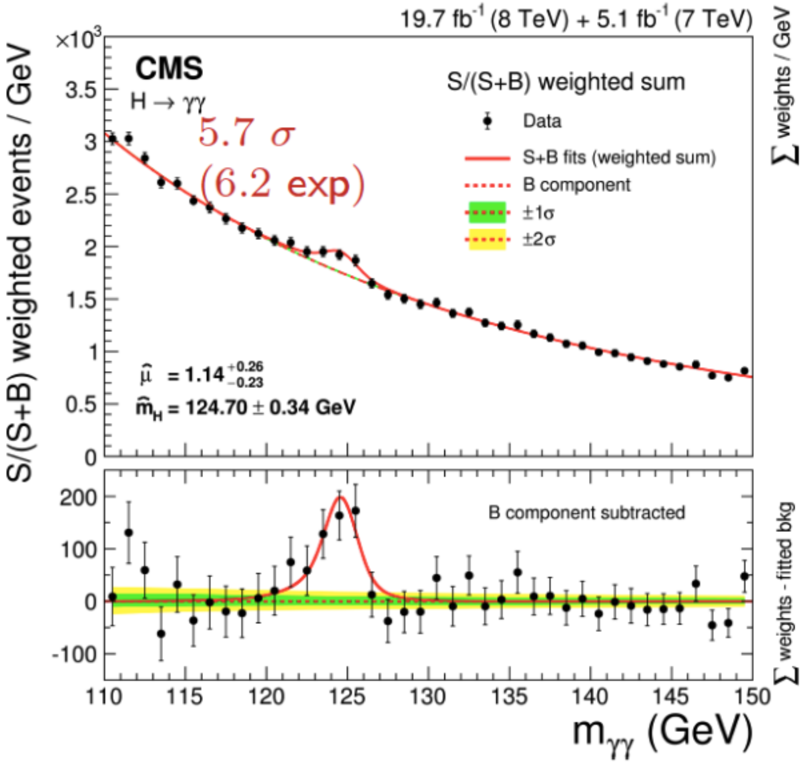


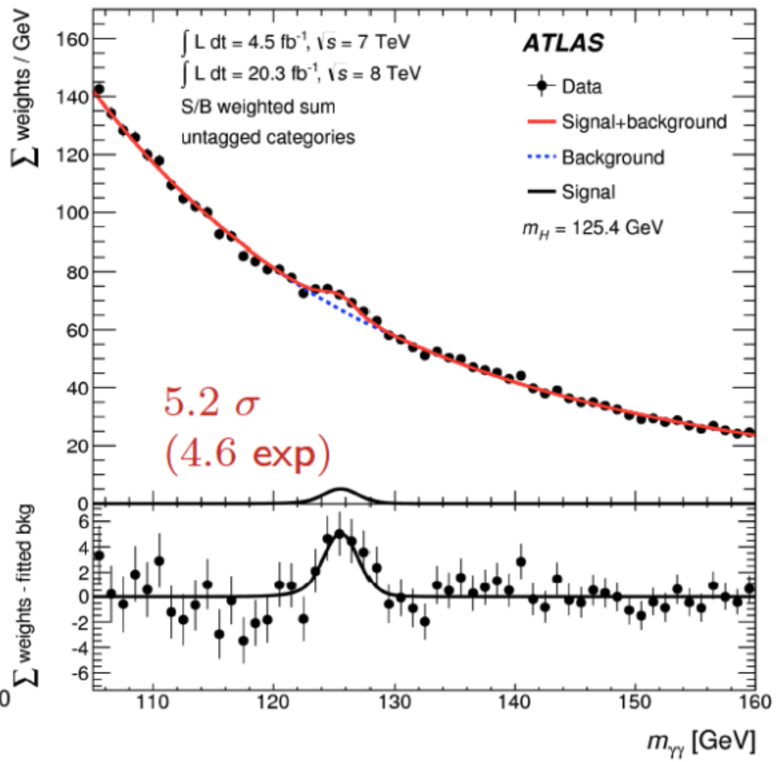
Theoretical Problems before the Higgs Discovery:

- The weak bosons $W^{\pm}$ and Z^0 have to be very massive:
  Interactions of longitudinal modes generate problems
  Model without (something like) the Higgs becomes invalid at the TeV scale
- Quark and Leptons have masses
  Incompatible with a consistent description of parity violation
- This theory fails at high energy scales of about $\mathcal{O}(1)$ TeV

The Standard Model without the Higgs particle is not renormalizable!

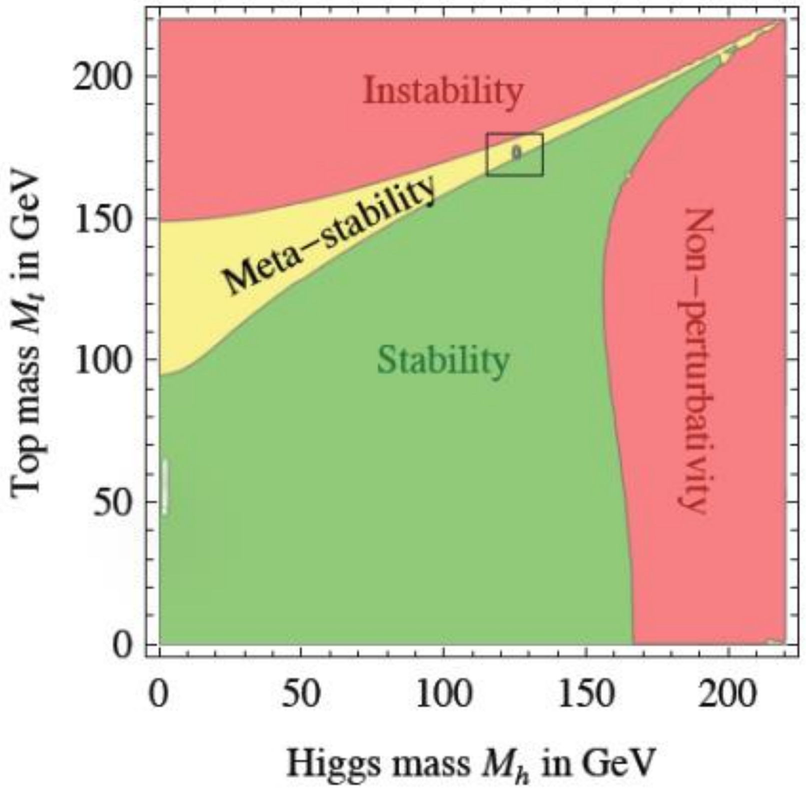
The Discovery of the Higgs Boson

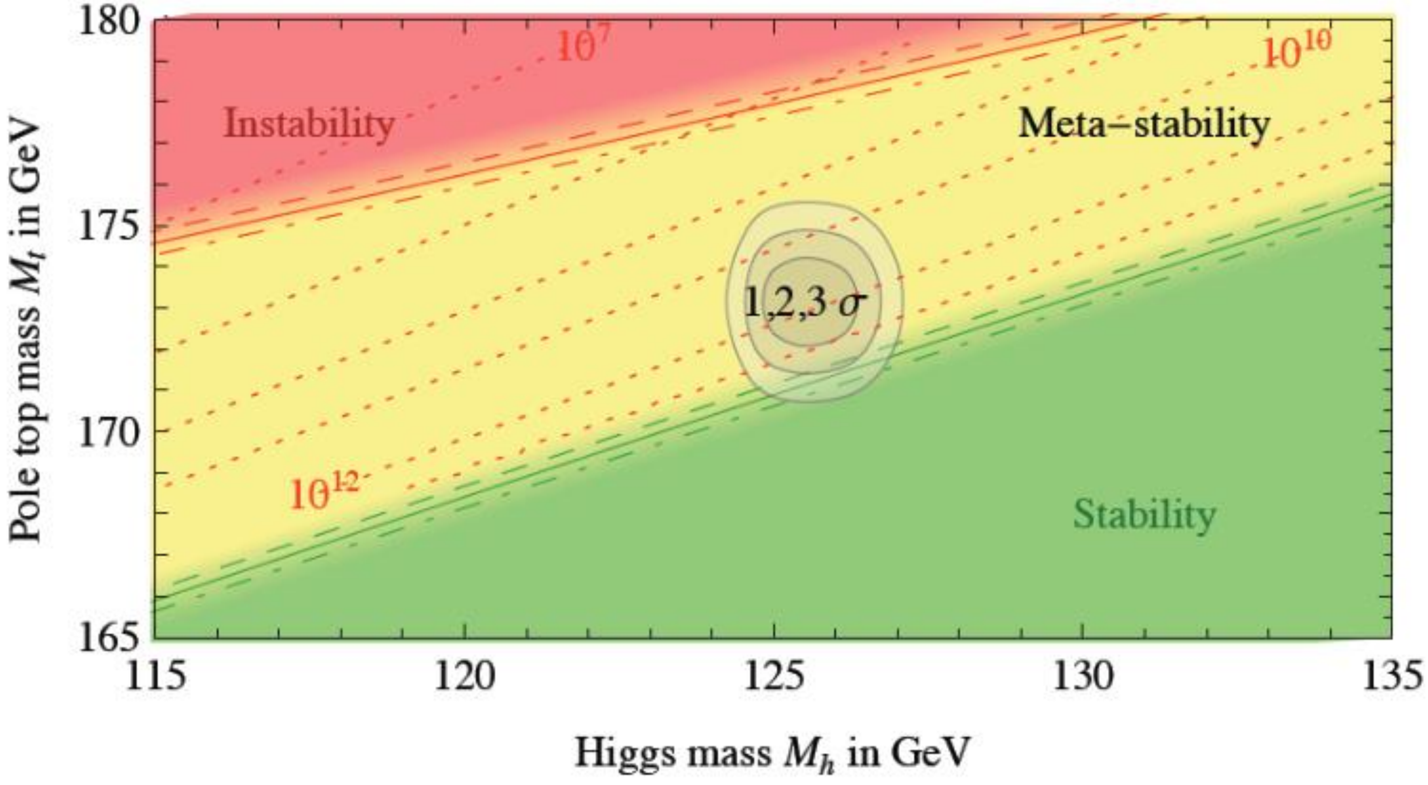




From the theory point of view:

- Including the Higgs this becomes a renormalizable theory: This model can be valid up to extremely high scales $\mu \sim 10^{12}$ GeV!





Second Remark on Renormalizability

- A renormalizable theory can be scaled up to (almost) arbitrarily high energies
- ... It is constructed such that it is insensitive to effects from high scales
- Thus it is a parametrization of our ignorance about the physics at high scales
- The Standard model alone cannot give us a hint on a scale where it fails
- This motivates the searches for searches for effects beyond the SM
- No significant deviations have been found in particle physics experiments
- ... Despite of intensive searches!

Where do we go from here?

- The Standard Model is one of the most successful theories ever
- Its observed spectrum and interactions corresponds to a renomalizable quantum field theory
- ... So it does not tell us, at what scale it fails!
- The situation resembles a bit the one at the end of the 19th century
- There are various elefants in the room:
  - Dark matter
  - Dark Energy
  - Matter-Antimatter Asymmetry

A word on Gravitation …

- Gravitation is not included in the Standard Model
- General Relativity is formulated as a theory of space-time geometry:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- Left hand side: Space-time geometry
- Right hand side: Energy momentum operator(?)
- Add in a “cosmological constant“?
- Do we need to quantize Gravitation, and if yes, how does a consistent Quantum Theory of Gravitation look like?

Outlook ...

- There are in total three constants of nature characterizing the progress in fundamental physics of the last century:
  - $\hbar$: Plancks constant, relating particles and waves
  - c: The speed of light, relating mass with energy
  - G: Newtons gravitational constant, relating space-time geometry with energy density

- Quantum Field Theory has sucessfully combined quantum mechanics and special relativity
- Gravitation remains „isolated“
- Can a theory that combines $\hbar$, c and G solve the „Dark problems“?