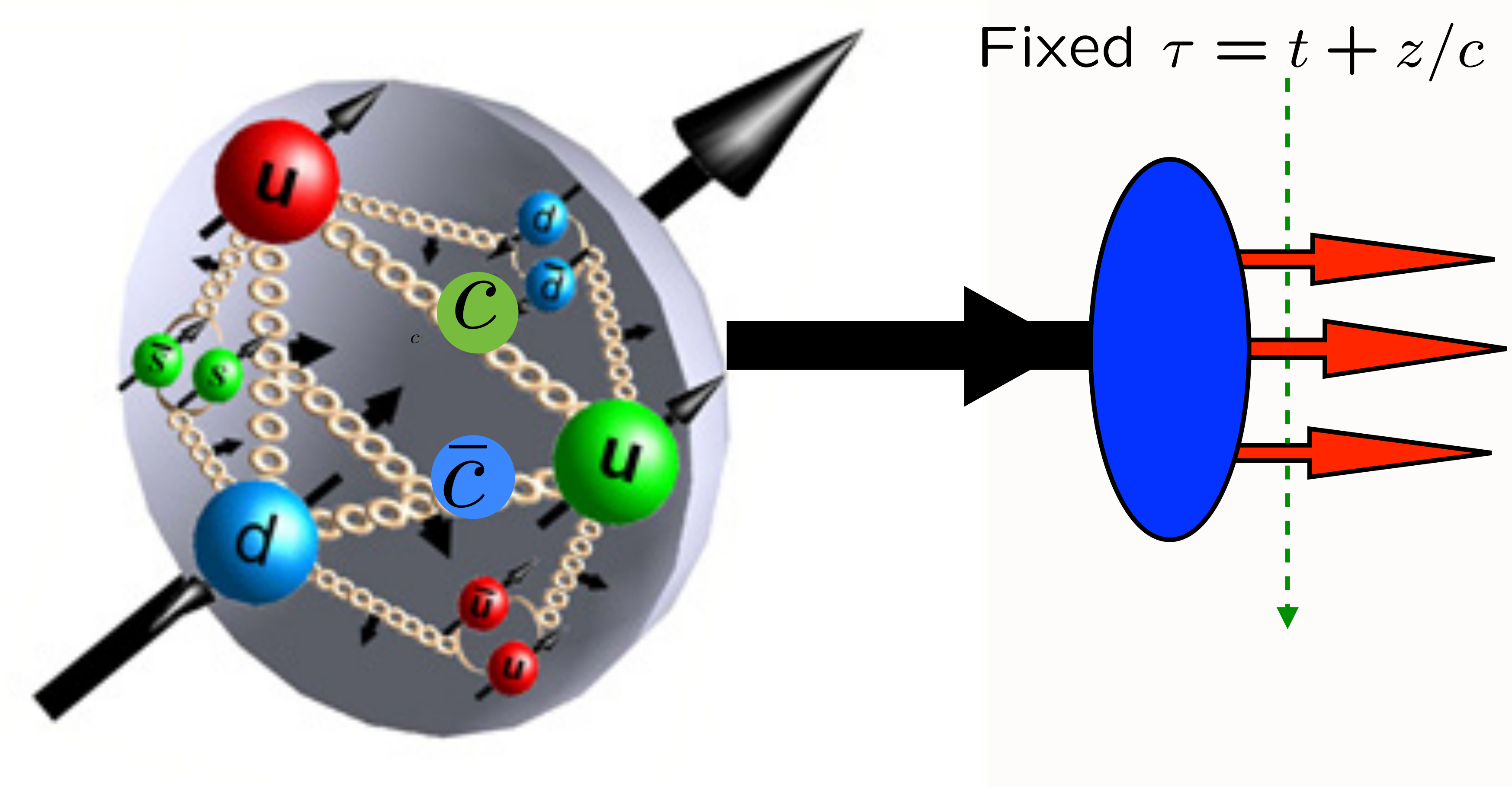
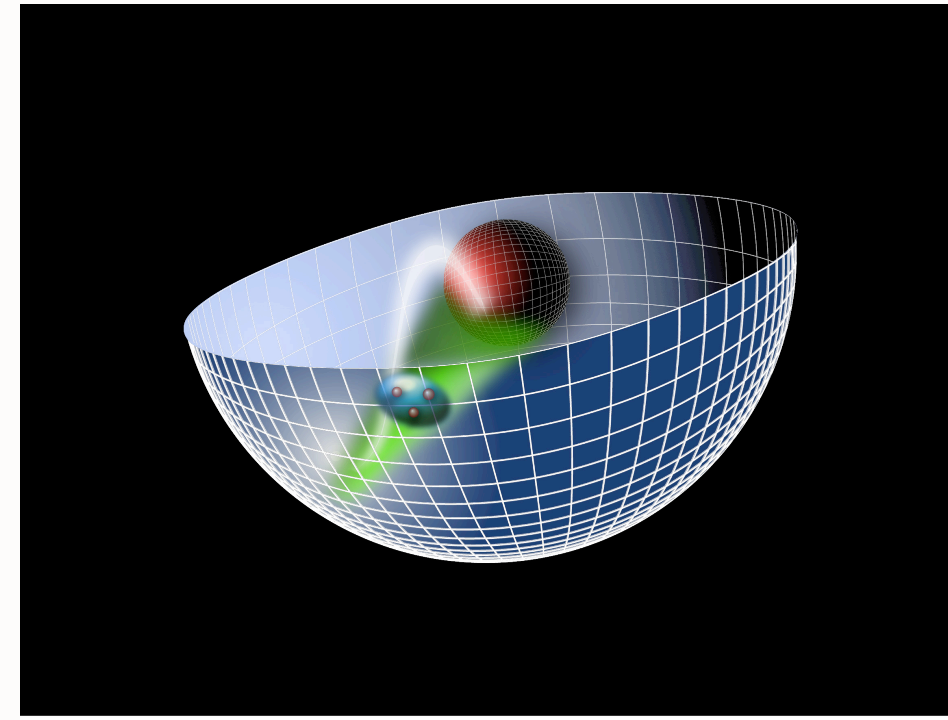
# AdS/QCD, Light-Front Holography, and Color Confinement




Stan Brodsky

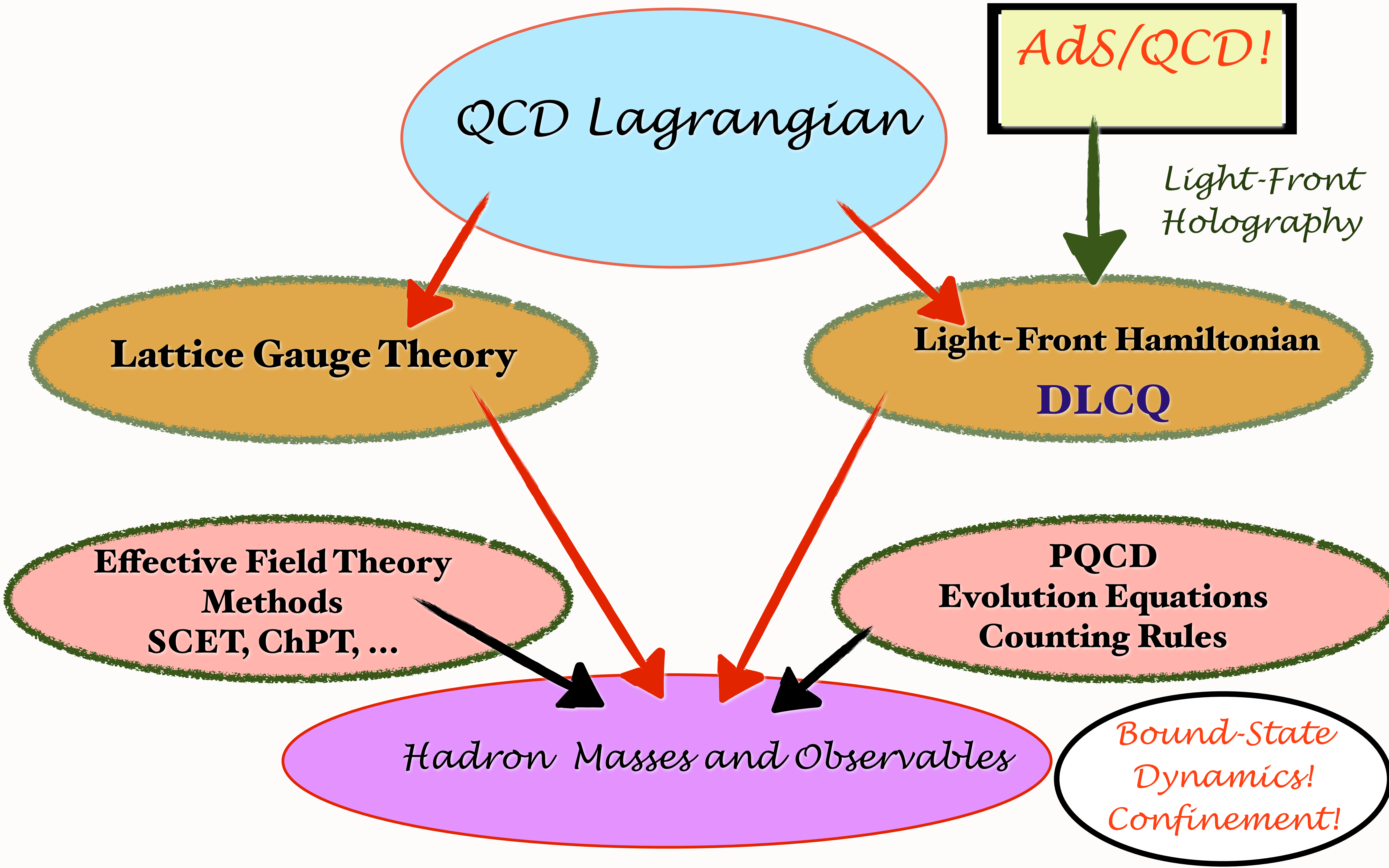
SLAC NATIONAL ACCELERATOR LABORATORY

Technische Universität Wien
Vienna University of Technology


**November 6, 2012**

**Institute for Theoretical Physics**

# *Predict Hadron Properties from First Principles!*



# Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insights into QCD Condensates**
- **Systematically improvable**
- **Eliminate scale ambiguities**

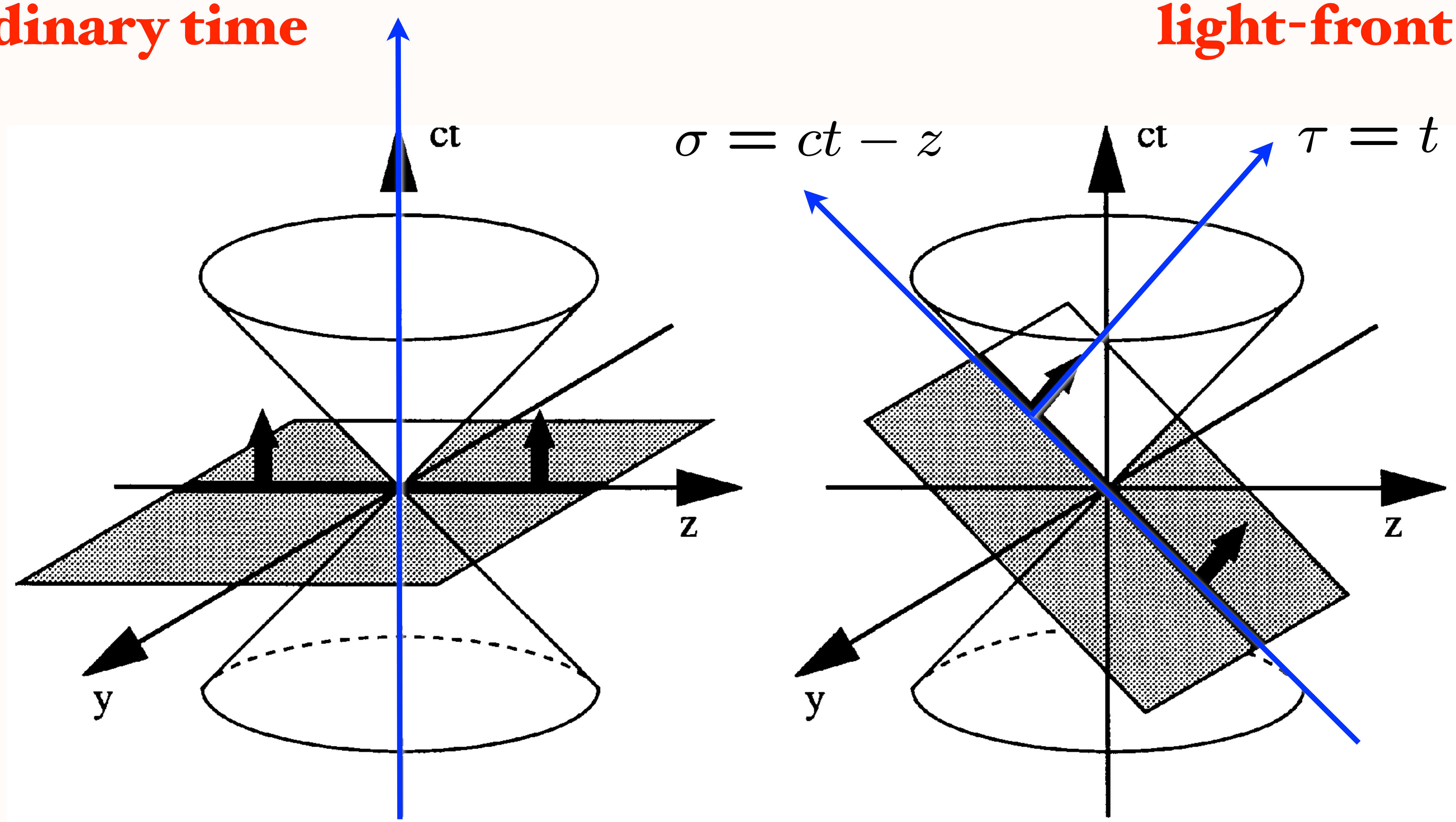
**Guy de Teramond,**
**Xing-Gang Wu**
**Leonardo di Giustino**
**Matin Mojaza**
**Joseph Day**
**sjb**

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

# Dirac's Amazing Idea: The Front Form

**Evolve in ordinary time**

**Evolve in light-front time!**



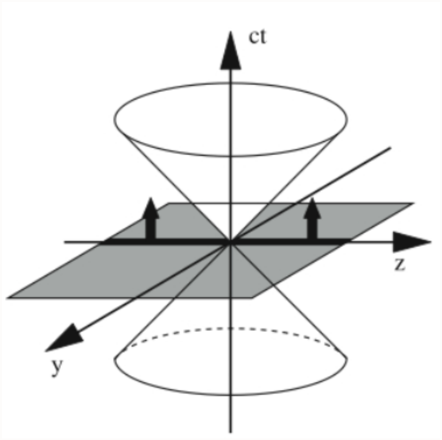

**Instant Form**

**Front Form**

- Different possibilities to parametrize space-time [Dirac (1949)]

- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results

- *Instant form*: hypersurface defined by $t = 0$, the familiar one



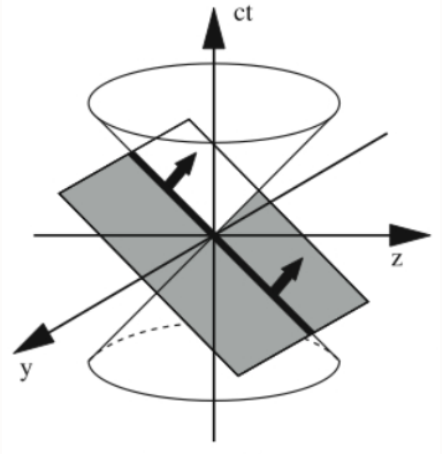

- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

$k^- = k^0 - k^3$ light-front energy




$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

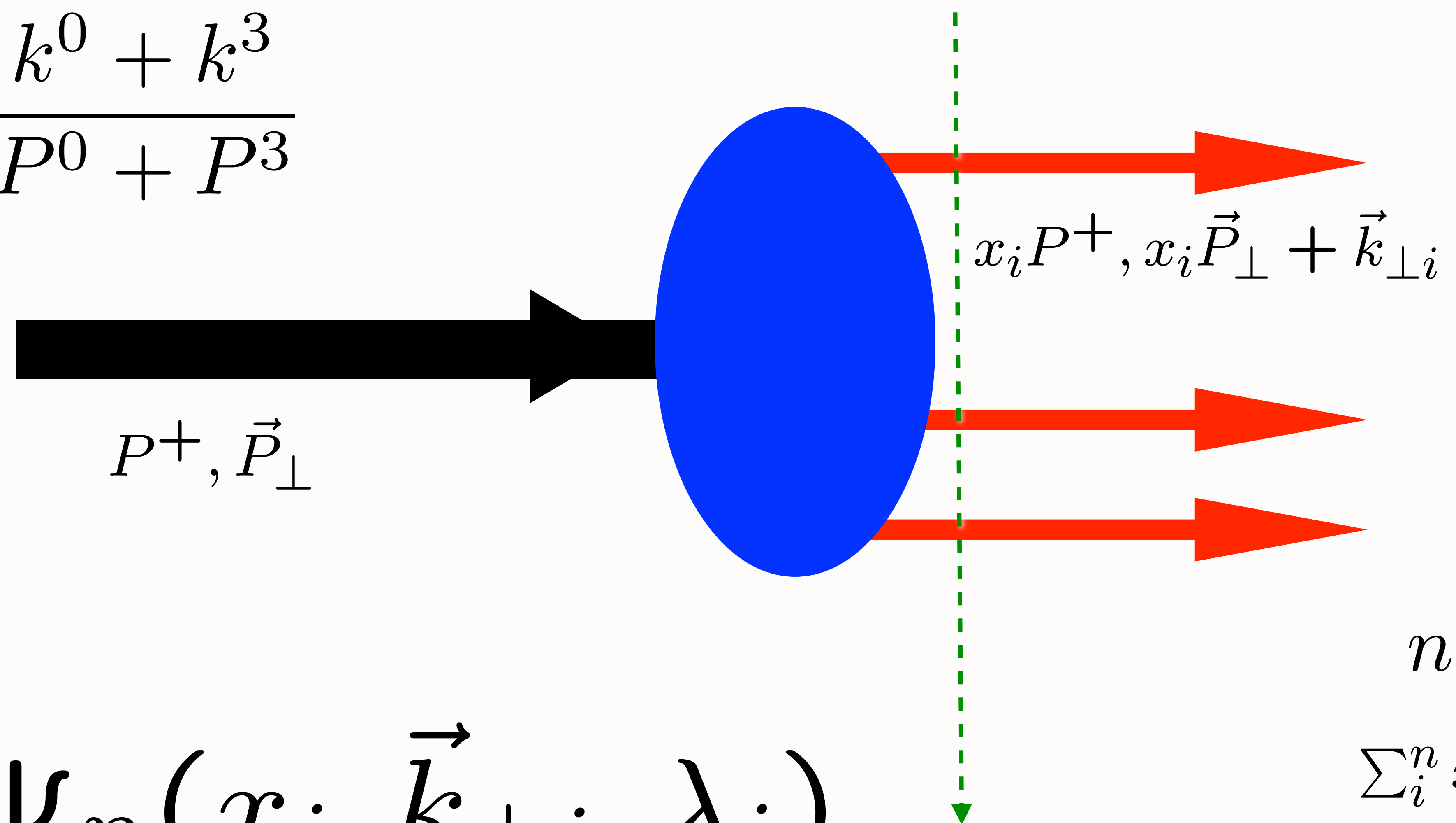
**Quantum chromodynamics and other field theories on the light cone.**
Stanley J. Brodsky (SLAC), Hans-Christian Pauli (Heidelberg, Max Planck Inst.),
Stephen S. Pinsky (Ohio State U.). SLAC-PUB-7484, MPIH-V1-1997. Apr 1997. 203 pp.
Published in **Phys.Rept. 301 (1998) 299-486**
e-Print: **hep-ph/9705477**

# Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$k^{\pm} = k^0 \pm k^z$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$




$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$n = 3$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

*Invariant under boosts! Independent of $P^\mu$*

**Bethe-Salpeter WF integrated over $k^-$**

*Square: Structure Functions Measured in DIS*

*Each element of flash photograph illuminated at same Light-Front time*

$$\tau = t + z/c$$

**Causal, frame-independent**

*Evolve in LF time*

$$P^- = i \frac{d}{d\tau}$$

*Eigenstate -- independent of* $\tau$

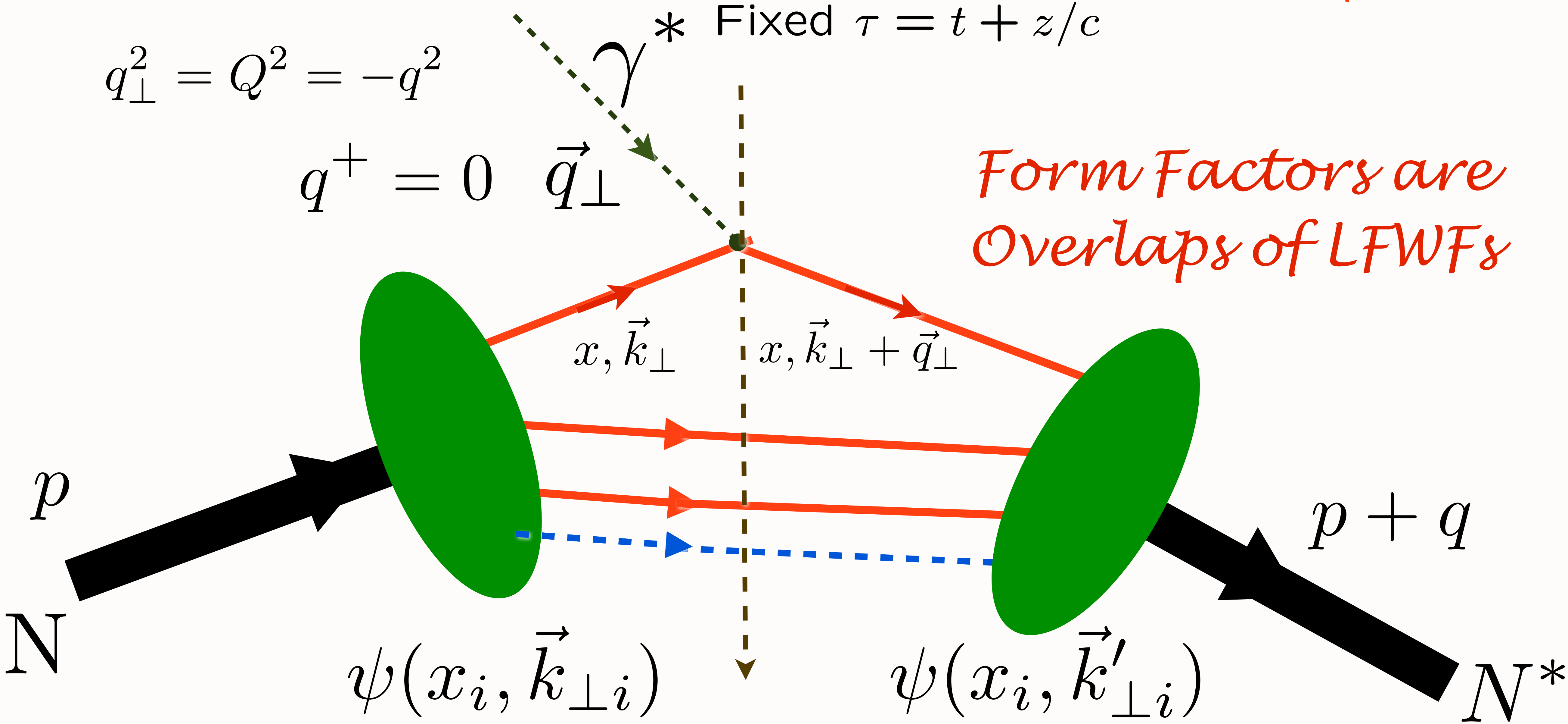
$$H_{LF} = P^+ P^- - \vec{P}_\perp^2$$

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture




**Drell &Yan, West**
*Exact LF formula*
**Drell, sjb**

*struck* $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

*spectators* $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

# Exact LF Formula for Pauli Form Factor

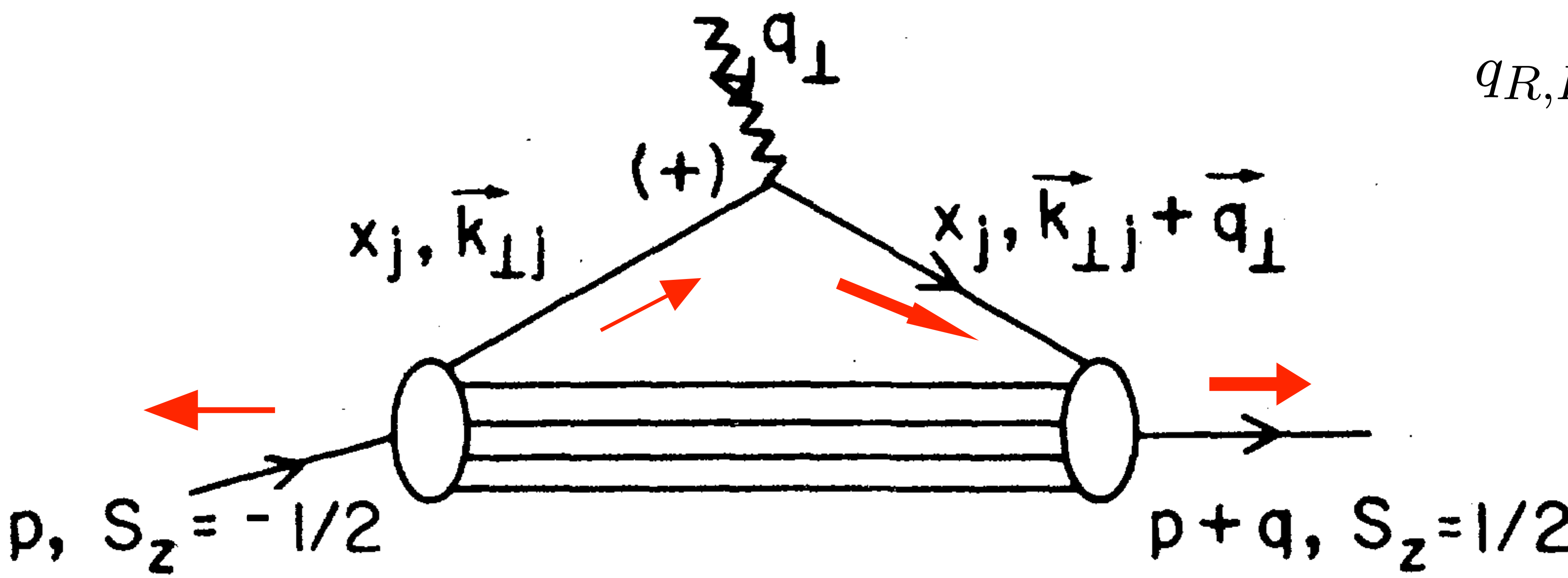
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \; \times$$

$$\left[ -\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

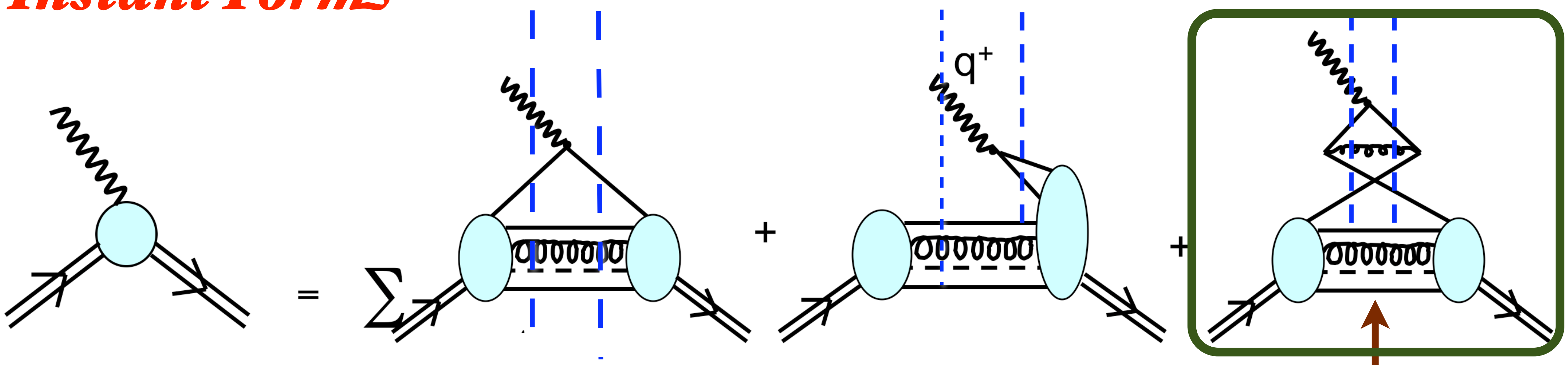



Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

## Calculation of Form Factors in Equal-Time Theory

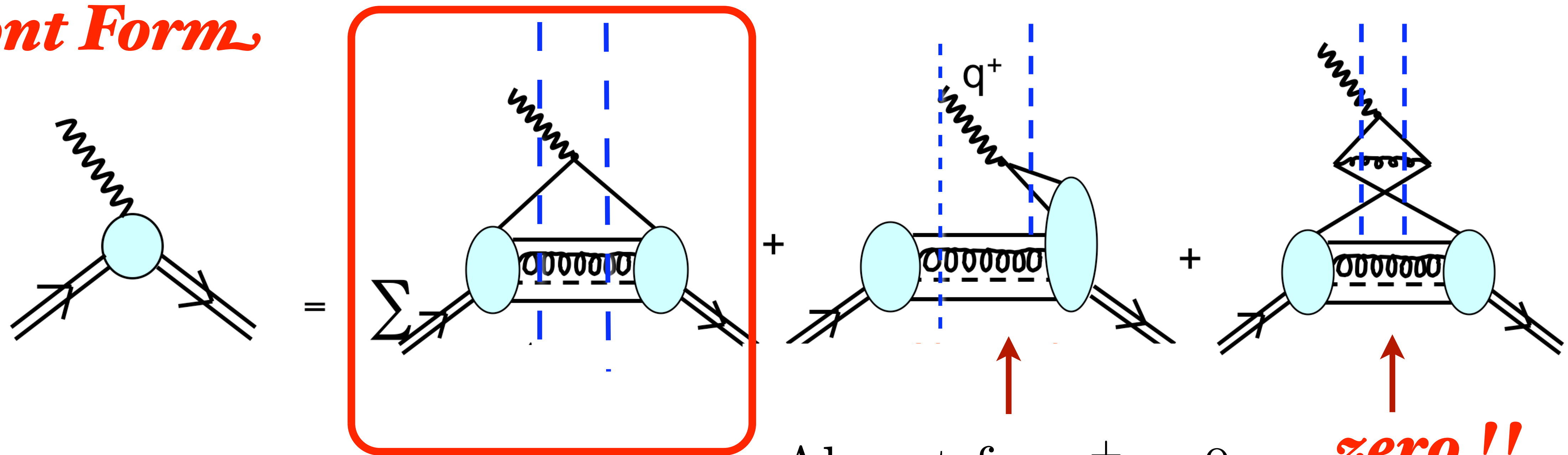
### Instant Form




**Need vacuum-induced currents**

## Calculation of Form Factors in Light-Front Theory

### Front Form




Complete Answer

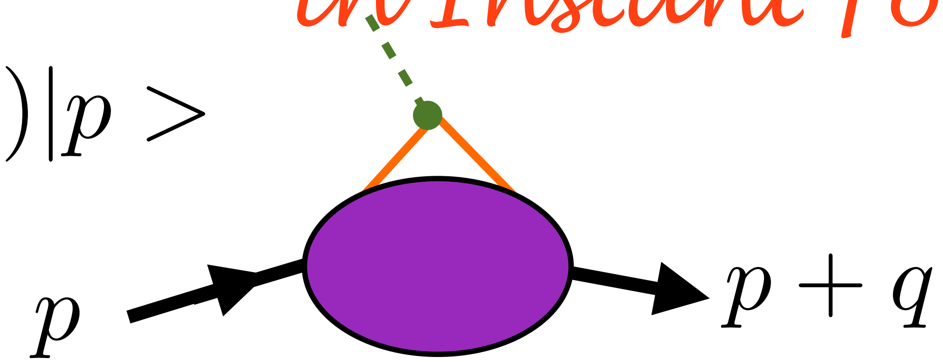
Absent for $q^+ = 0$

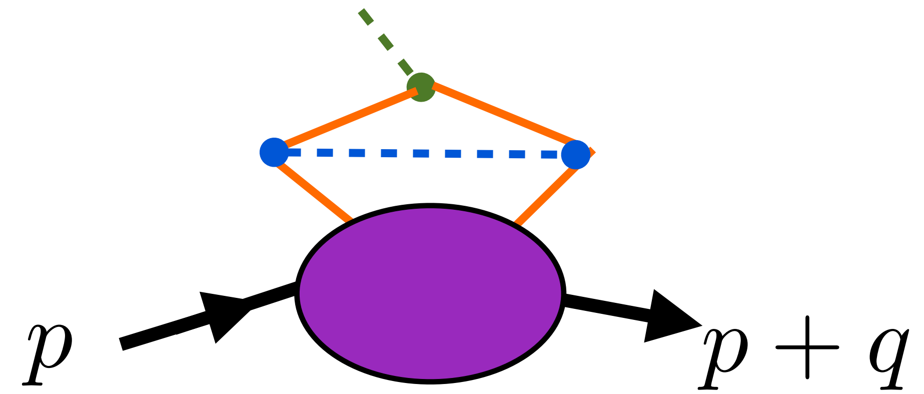
**zero !!**

No vacuum graphs

# Compare with calculation of proton form factor in Instant Form

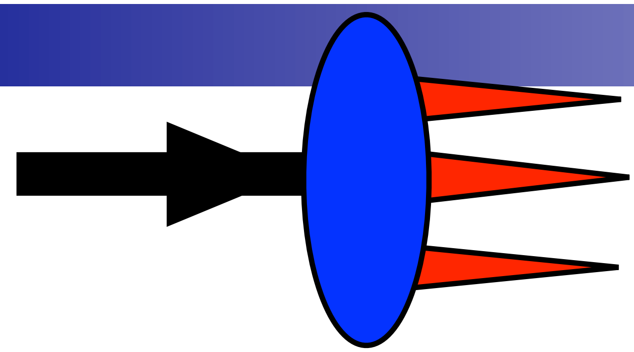
$$< p + q|J^{\mu}(0)|p >$$



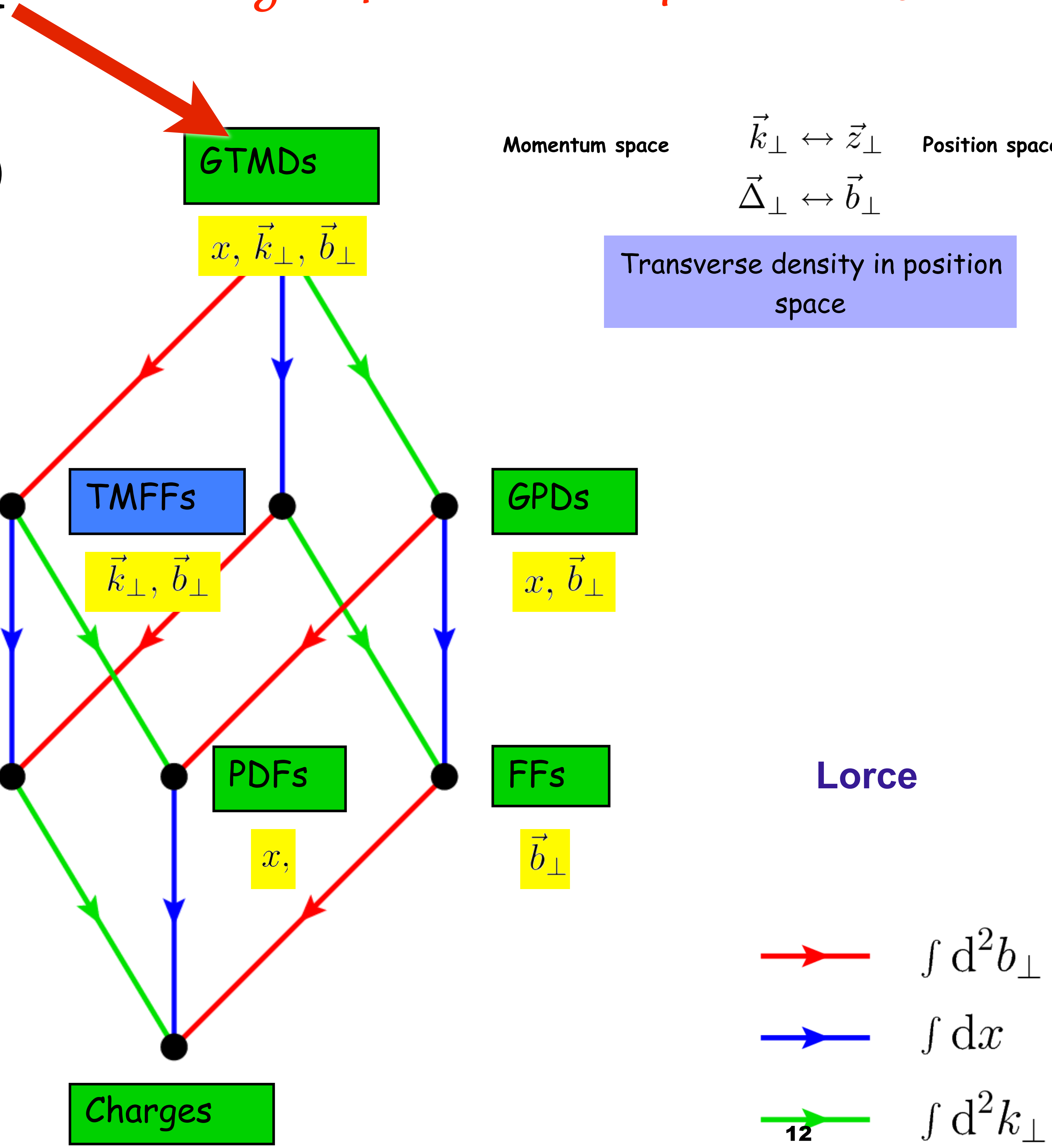




- **Need to boost proton wavefunction: p to p+q. Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!! Remain even after normal-ordering**
- **Instant-form WFs insufficient to calculate form factors**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**

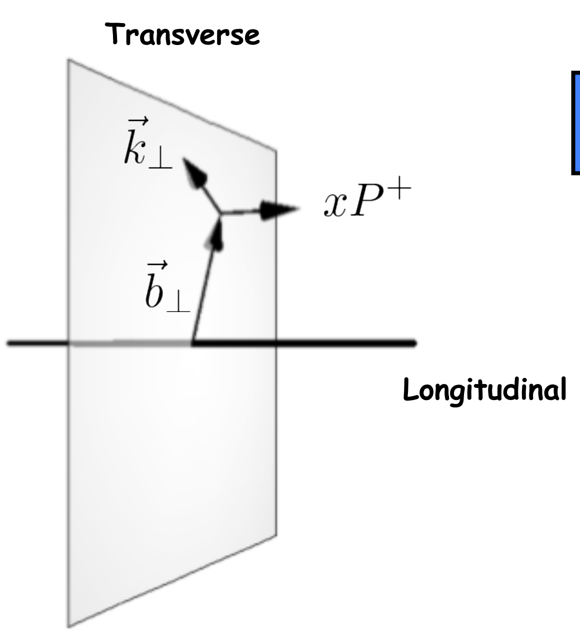
# • *Light Front Wavefunctions:*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$




Lorce



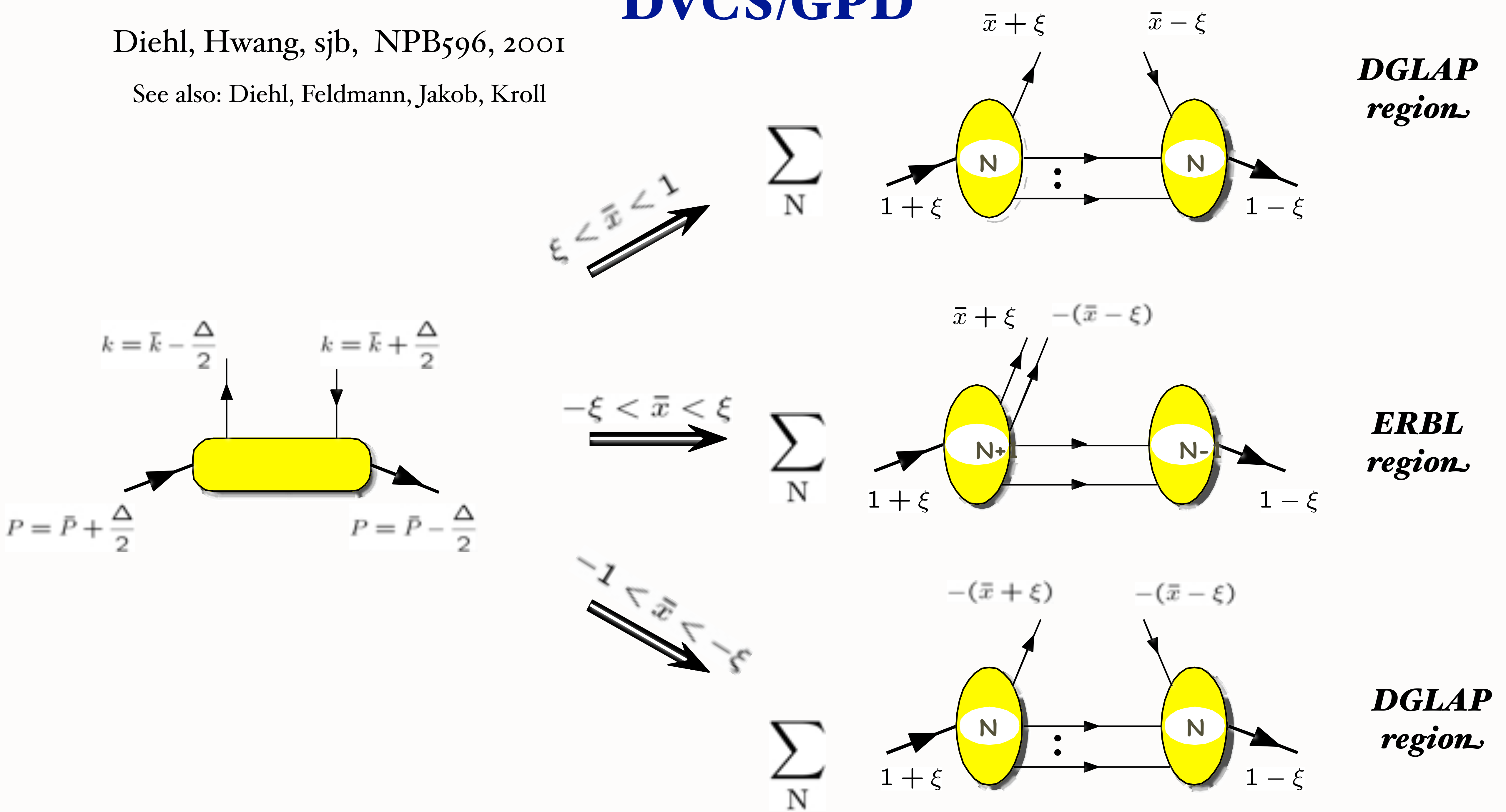

*Sivers, T-odd from lensing*

# Light-Front Wave Function Overlap Representation

## DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

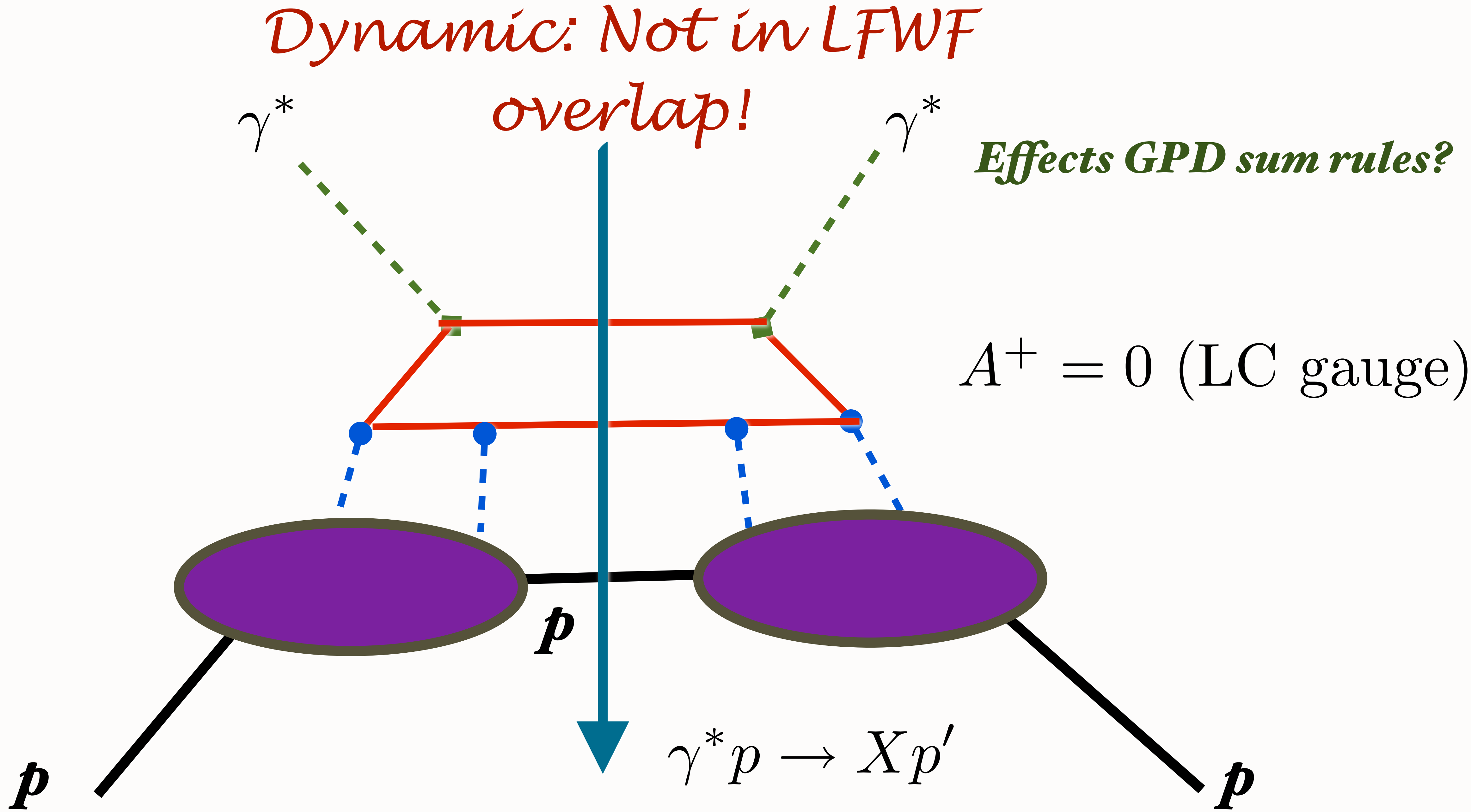
See also: Diehl, Feldmann, Jakob, Kroll




**Bakker & JI**
**Lorce**

# Leading-Twist Contribution to DVCS

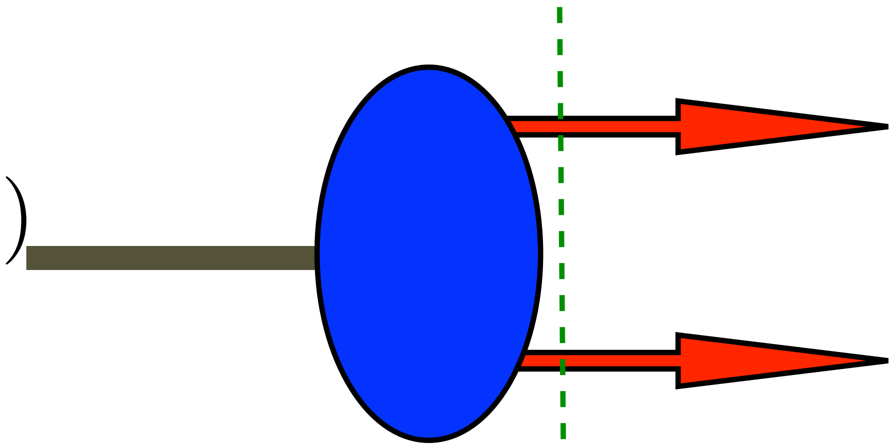
***Interactions occur between the LF times of the two virtual photon!!***




**Cut is Leading-Twist Diffractive DIS**

# Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$




$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

*Lepage, sjb*

- Evolution Equations from PQCD, OPE
- Conformal Expansions
- Compute from valence light-front wavefunction in light-cone gauge

*Lepage, sjb*

*Efremov, Radyushkin*

*Sachrajda, Frishman Lepage, sjb*

*Braun, Gardi*

# Single-spin asymmetries

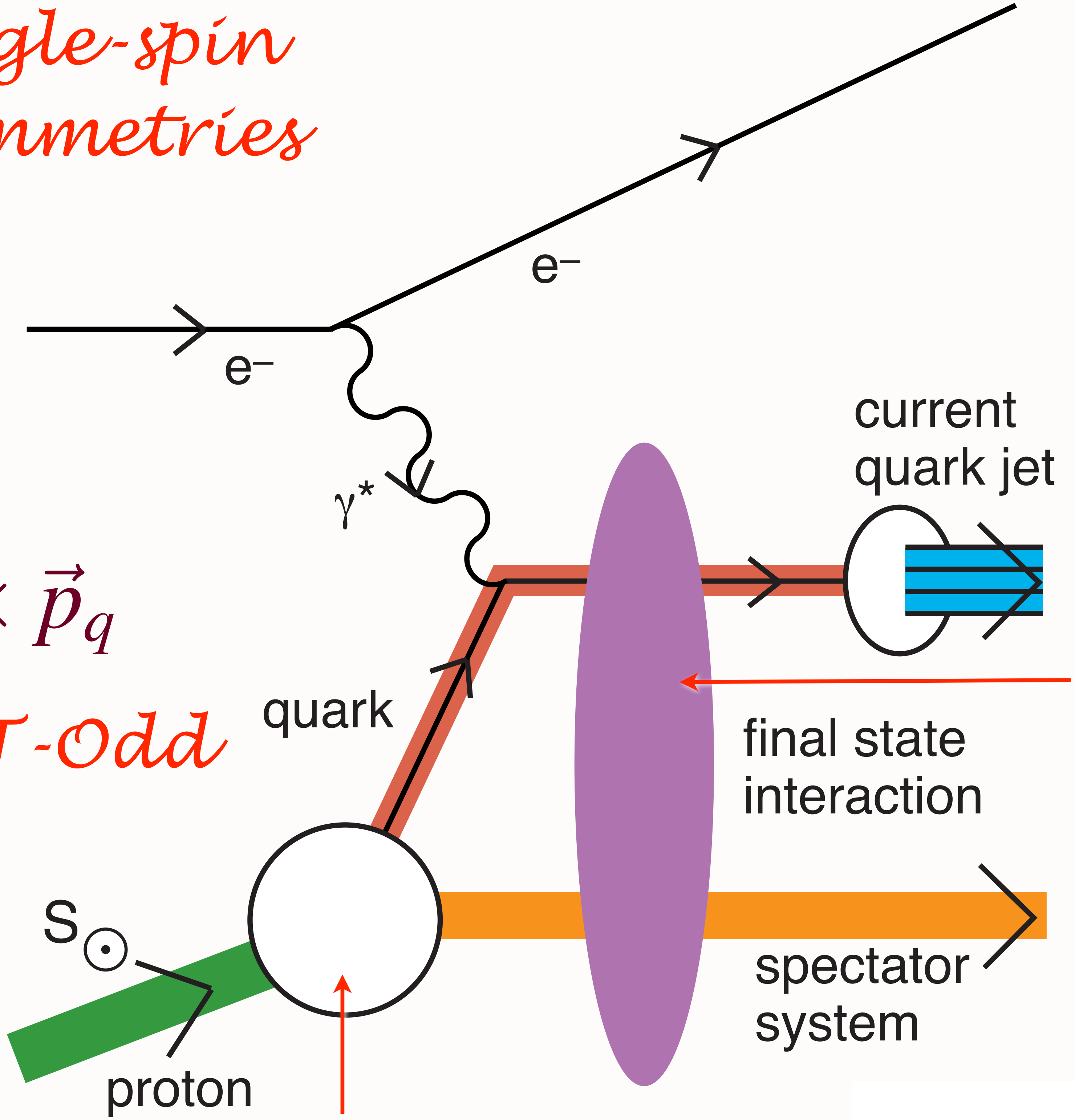
# Leading Twist Sivers Effect

**Hwang, Schmidt, sjb**

**Collins, Burkardt, Ji, Yuan. Pasquini, ...**

$$i \; \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

*Pseudo- T-Odd*




*QCD S- and P-Coulomb Phases --Wilson Line*

**"Lensing Effect"**

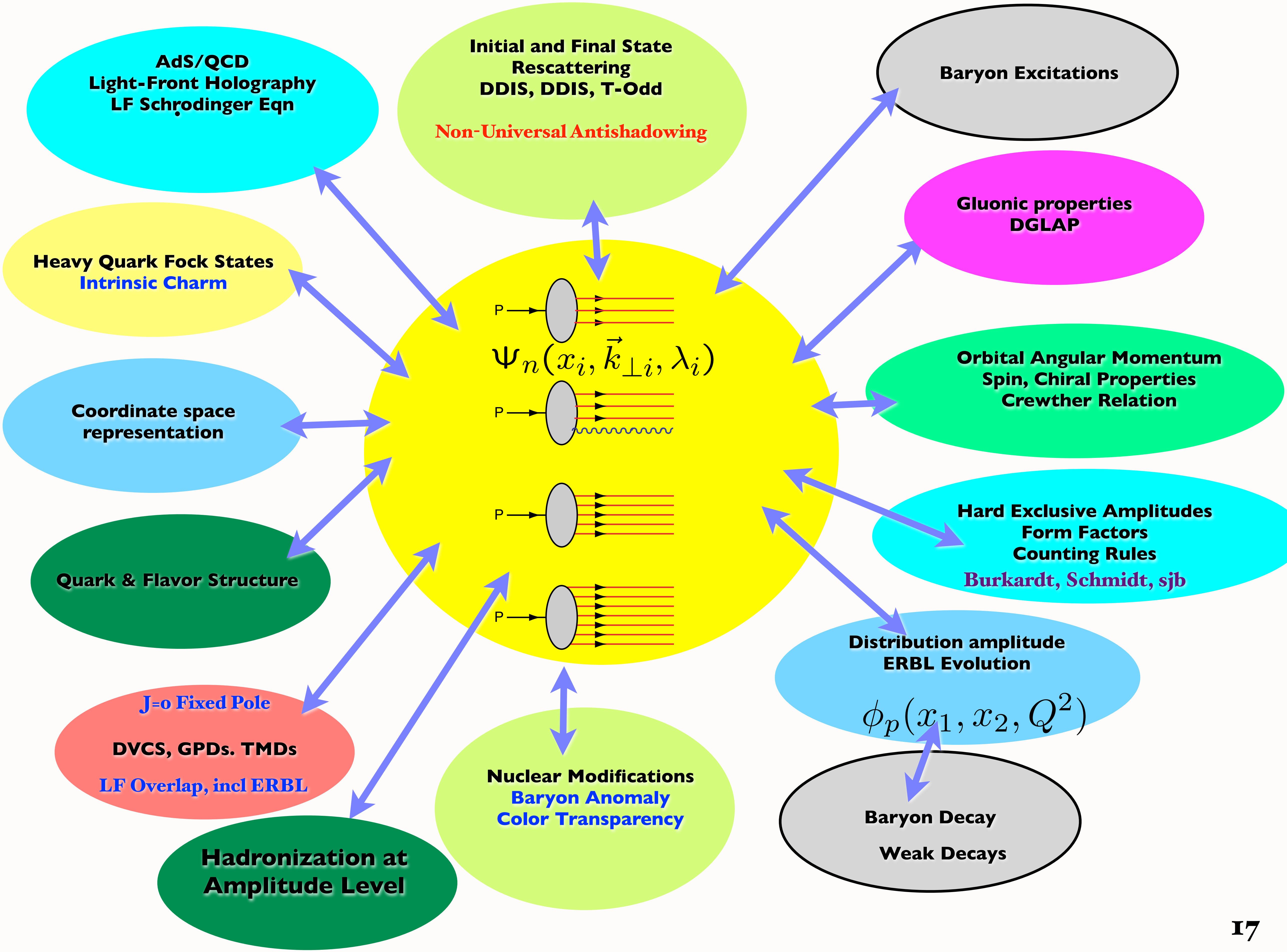
**QED: Lensing involves soft scales**

*Light-Front Wavefunction S and P- Waves!*

*Leading-Twist Rescattering Violates pQCD Factorization!*

*Sign reversal in DY!*

# QCD and the LF Hadron Wavefunctions



Fixed $\tau = t + z/c$

# Light-Front QCD

*Physical gauge:* $A^+ = 0$

**Exact frame-independent formulation of nonperturbative QCD!**

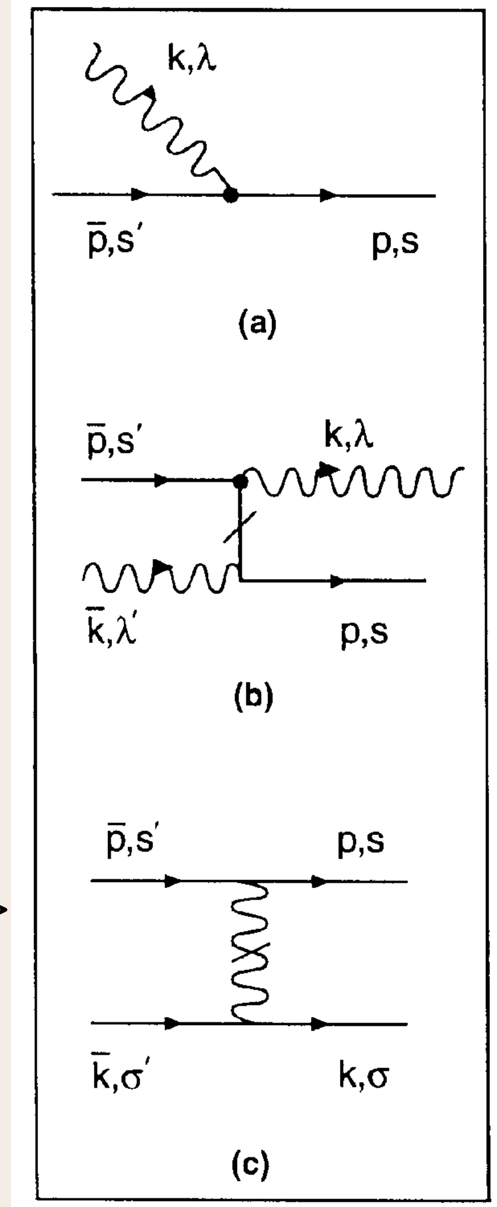
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

$H^{int}_{LF}$: Matrix in Fock Space

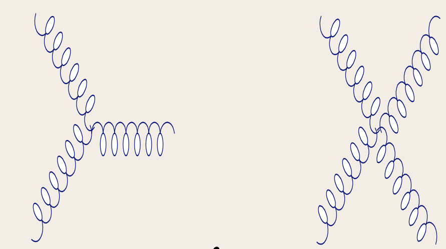
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; x_i, \vec{k}_{\perp i}, \lambda_i>$$




**Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions**

**LFWFs: Off-shell in P- and invariant mass**



$H^{int}_{LF}$

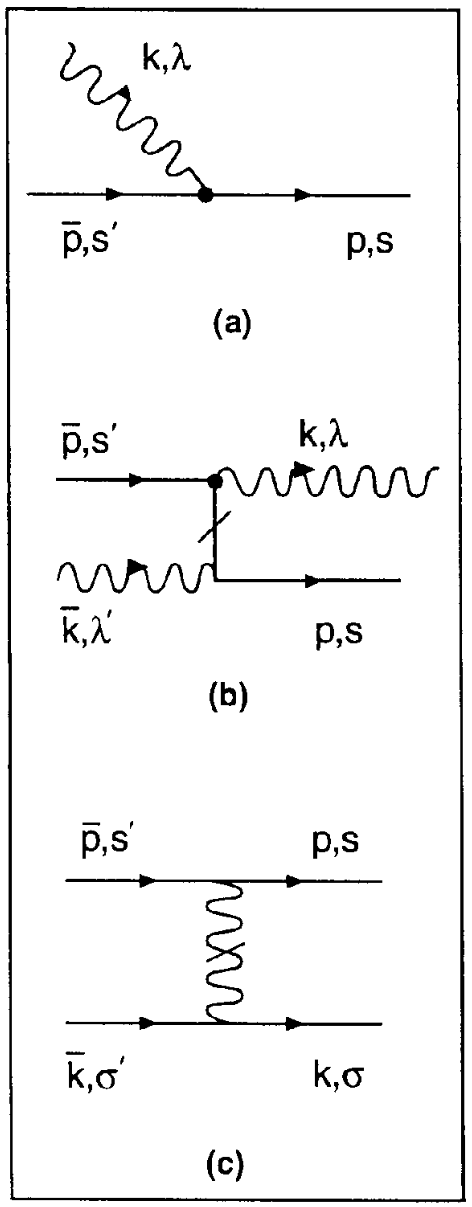
*Light-Front QCD*

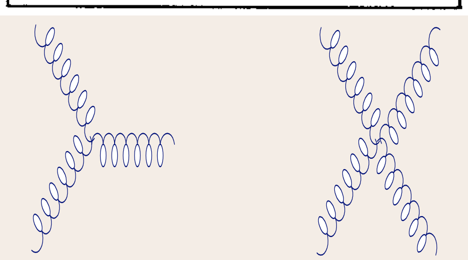
*Heisenberg Equation*

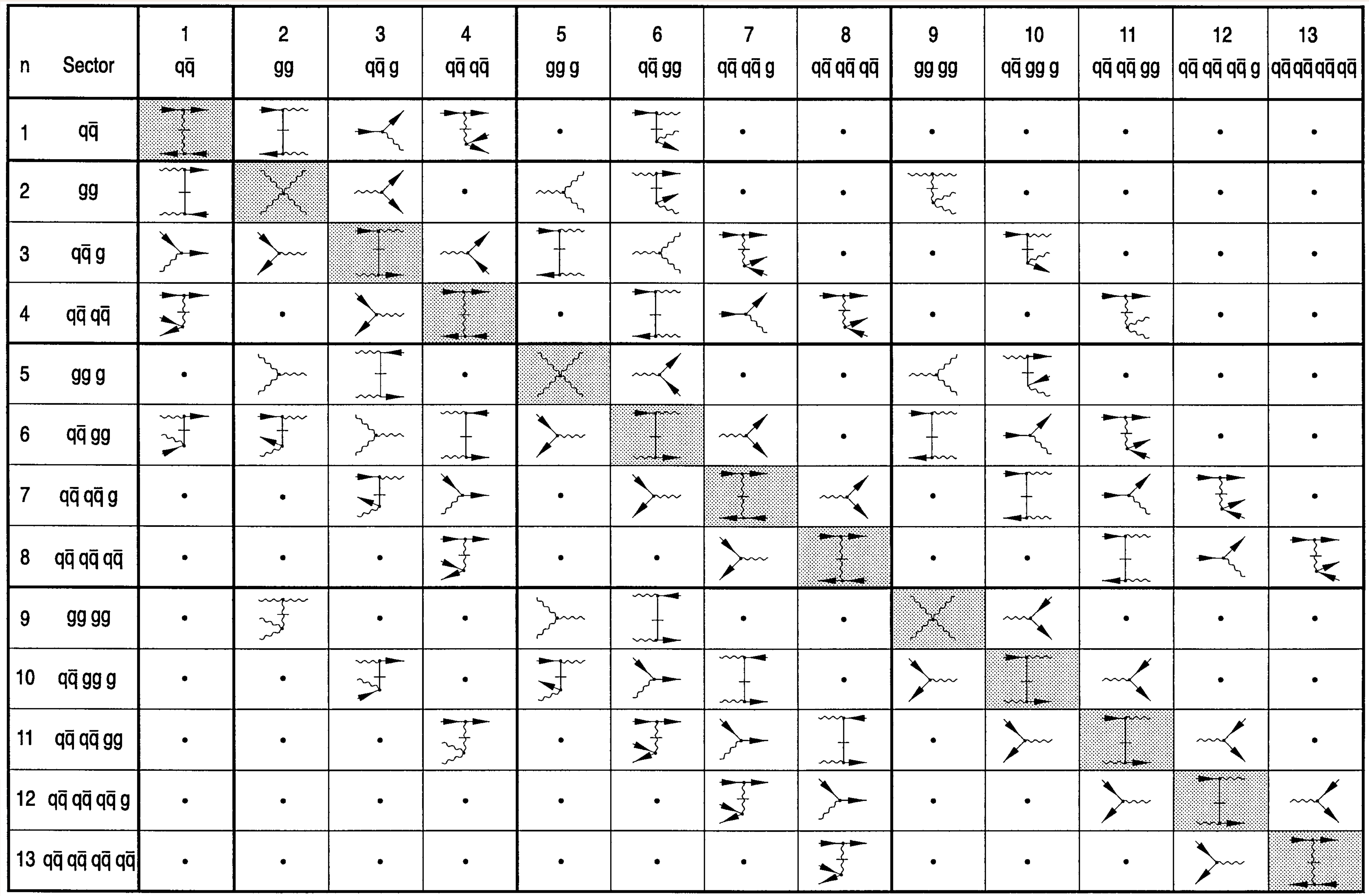
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$$

*DLCQ*

**Pauli, Hornbostel, sjb**






*Minkowski space; frame-independent; no fermion doubling; no ghosts*

# Light-Front vs. Instant Form

- **Light-Front Wavefunctions are frame-independent**
- **Boosting an instant-form wavefunctions dynamical problem -- extremely complicated even in QED**
- **Need to couple to all currents arising from vacuum (Remain even after normal-ordering)**
- **Vacuum state is lowest energy eigenstate of Hamiltonian**
- **Light-Front Vacuum same as vacuum of free Hamiltonian**
- **Zero anomalous gravitomagnetic moment**
- **Instant-Form Vacuum infinitely complex even in QED**
- **n! time-ordered diagrams in Instant Form**
- **Causal commutators using LF time; cluster decomposition**

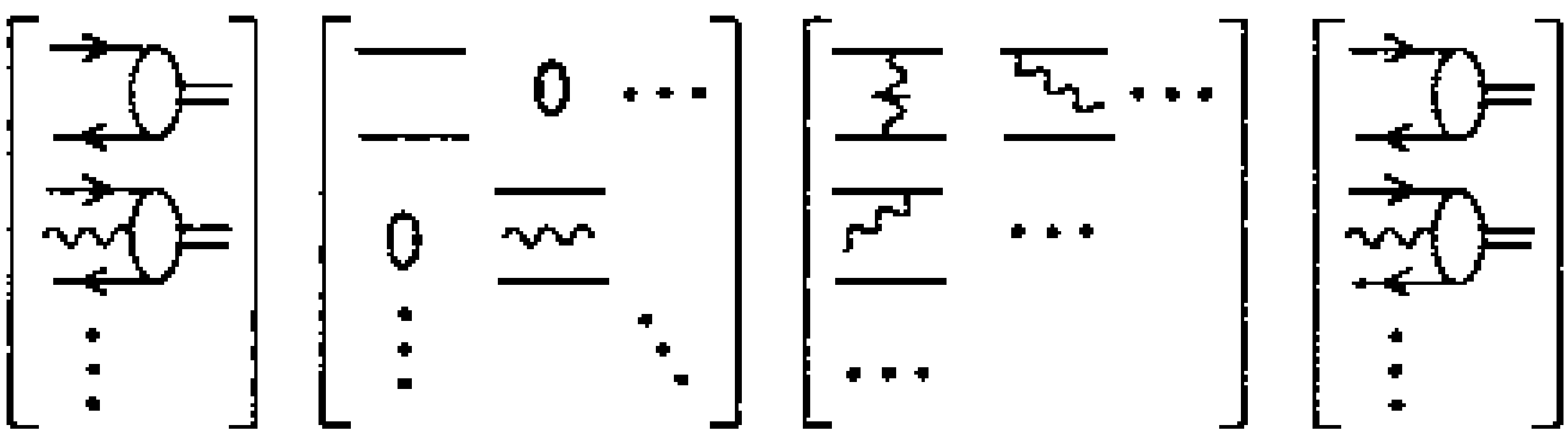
# LIGHT-FRONT MATRIX EQUATION

G.P. Lepage, sjb

*Rigorous Method for Solving Non-Perturbative QCD!*

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

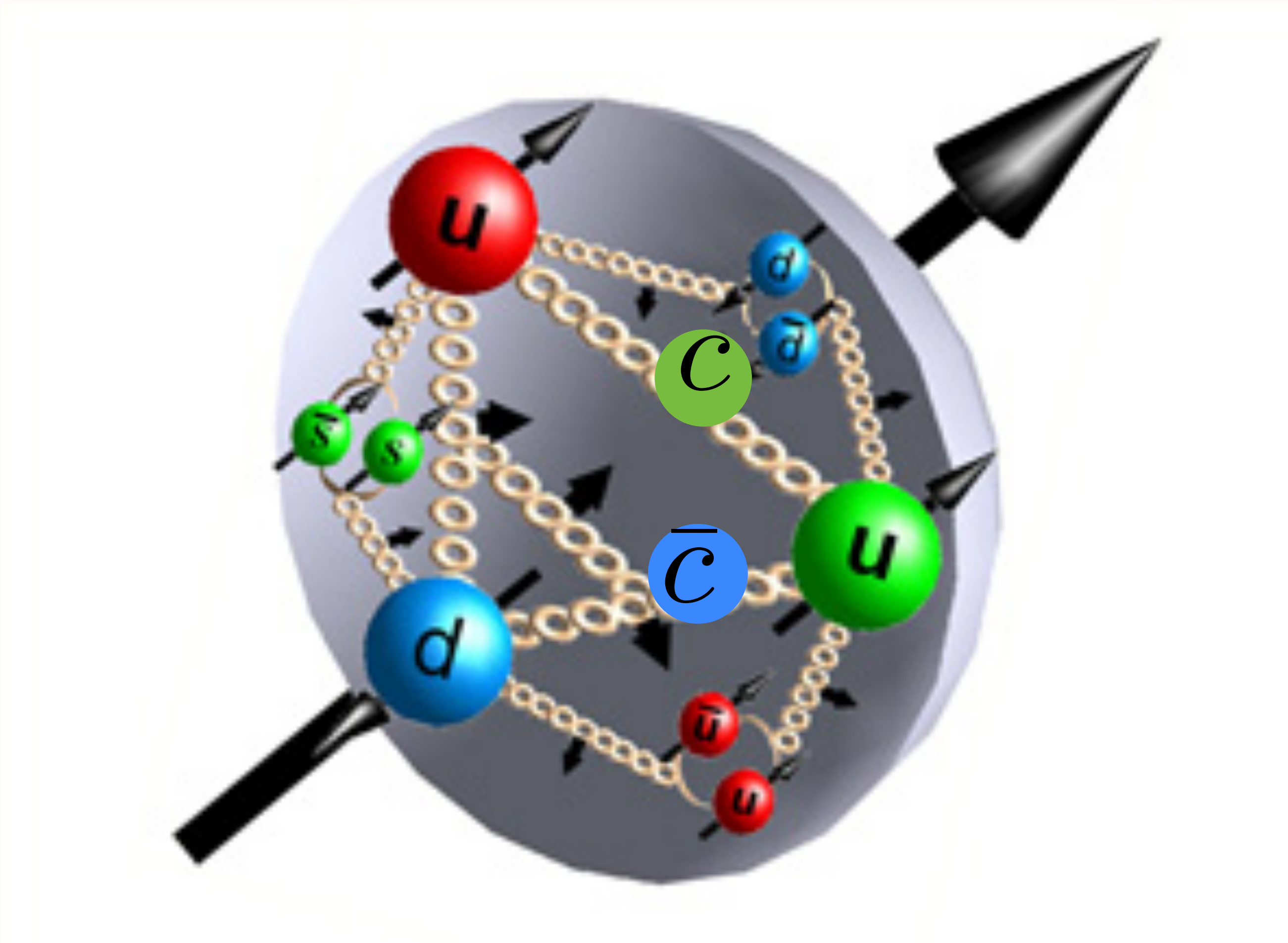
$A^+ = 0$

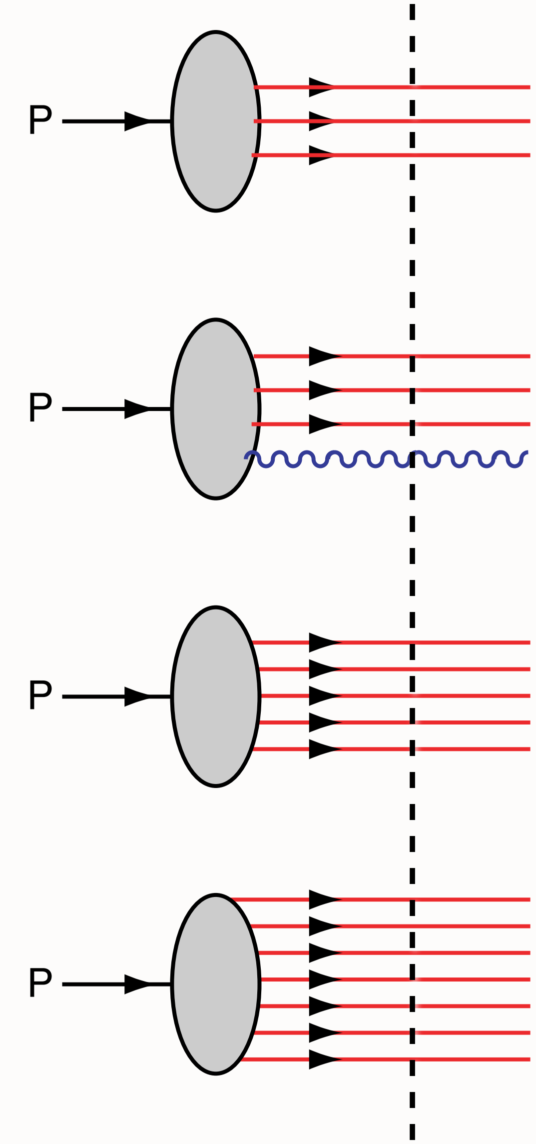


- ***Light-Front Vacuum = Vacuum of Free Hamiltonian!***

***Causal, Frame-Independent***

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$




## *Higher Fock States of the Proton*

*Fixed LF time*

*Intrinsic heavy quarks*

***s(x), c(x), b(x) at high x !***

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

*sum over states with n=3, 4, ...constituents*

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

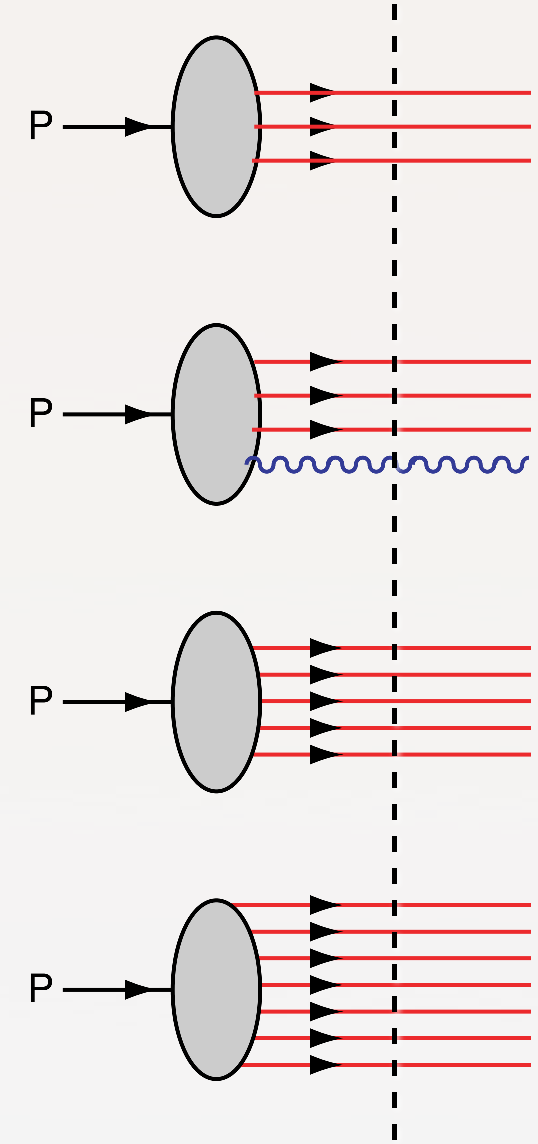
are boost invariant; they are independent of the hadron's energy and momentum $P^\mu$.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

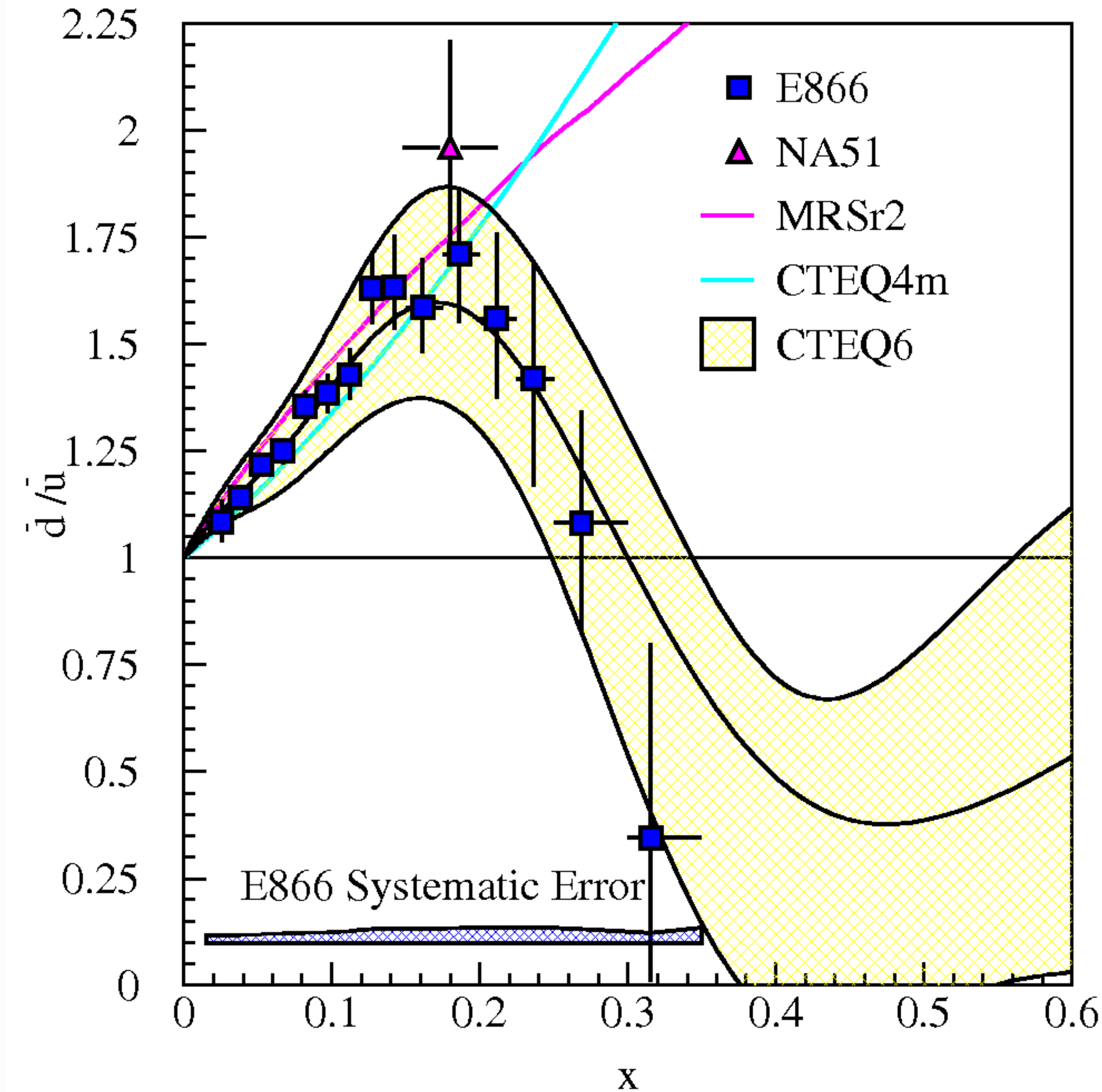



*Fixed LF time*

**Intrinsic heavy quarks,** $\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$




- E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

$$s(x) \neq \bar{s}(x)$$

*Intrinsic glue, sea, heavy quarks*

$$H_{QED}$$

*QED atoms: positronium and muonium*

Fixed time t ("instant form")

$$(H_0 + H_{int})\ |\Psi> = E\ |\Psi>$$

*Coupled Fock states*

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

*Effective two-particle equation*

**Includes Lamb Shift, quantum corrections**

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

*Spherical Basis* $r, \theta, \phi$

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

*Coulomb potential*

**Bohr Spectrum**

*Semiclassical first approximation to QED*

## QCD Meson Spectrum

$$H^{LF}_{QCD}$$

**Fixed Light-Front Time (Front form)**

Fixed $\tau = t + z/c$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Light-Front Fock states

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis $\zeta, \phi$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

# Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

**Change variables** $(\vec{\zeta},\varphi)$, $\vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp$: $\nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

# Light-Front Schrödinger Equation

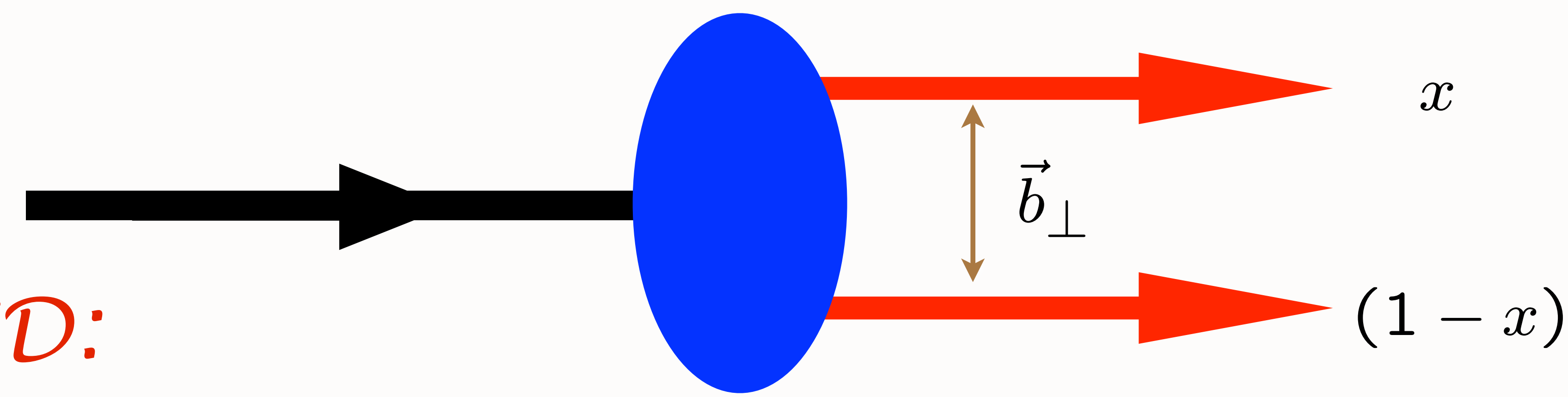
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

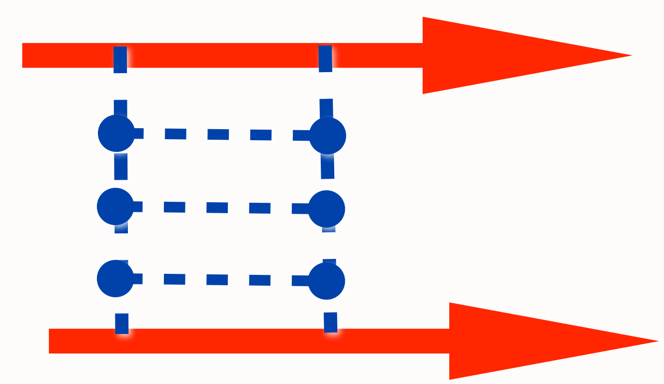
$$\left[-\frac{d^2}{d\zeta^2}+\frac{m^2}{x(1-x)}+\frac{-1+4L^2}{\zeta^2}+U(\zeta,S,L)\right]\psi_{LF}(\zeta)=M^2\,\psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$




AdS/QCD:

$$U(\zeta,S,L)=\kappa^2\zeta^2+\kappa^2(L+S-1/2)$$

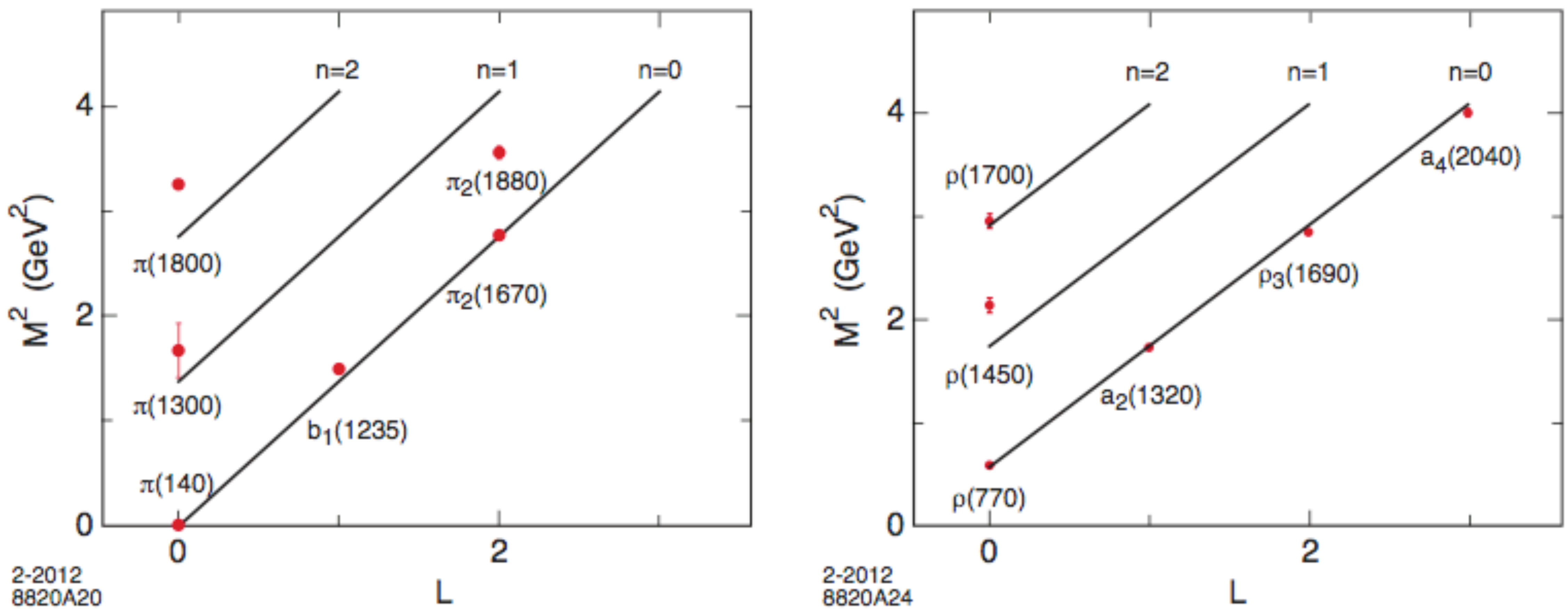


**U is the exact QCD potential**
**Conjecture: 'H'-diagrams generate**

- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

***Same slope in n and L*** ***Massless pion***




I=1 orbital and radial excitations for the $\pi$ ($\kappa = 0.59$ GeV) and the $\rho$-meson families ($\kappa = 0.54$ GeV)

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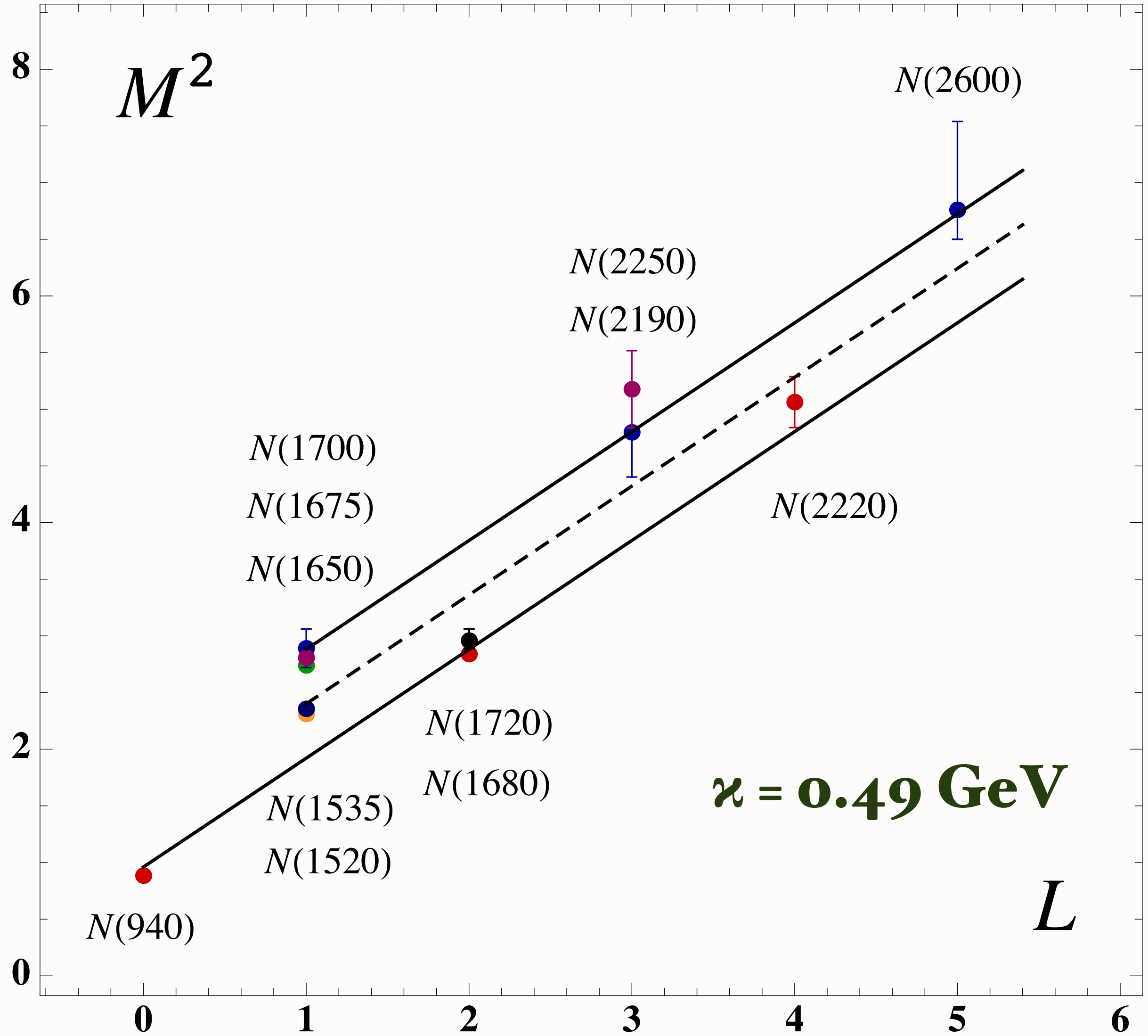
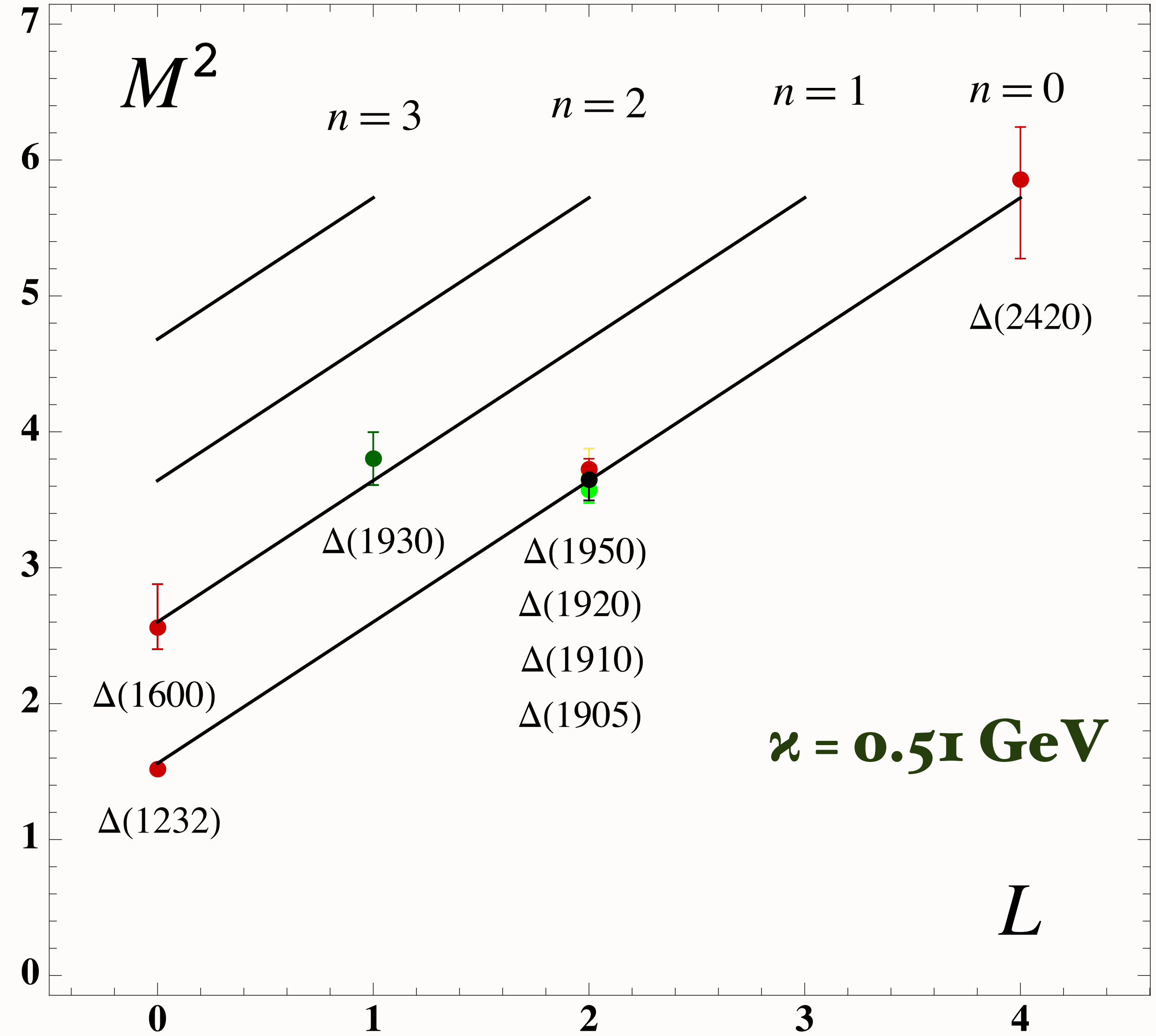
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

# Baryon Spectroscopy from AdS/QCD and Light-Front Holography




**de Teramond, sjb**

**All confirmed resonances from PDG 2012**

$$\mathcal{M}^{2\,(+)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{3}{4}\right),$$ **positive parity**

$$\mathcal{M}^{2\,(-)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{5}{4}\right),$$ **negative parity**

**See also Forkel, Beyer, Federico, Klempt**

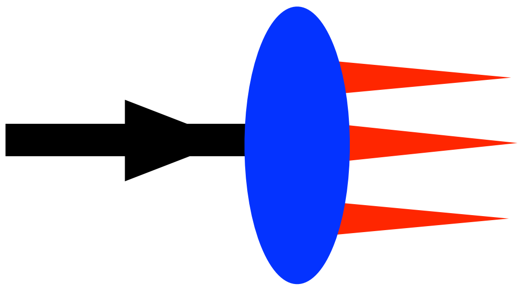
- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**

- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**

- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, .... modulo `lensing' from ISIs, FSIs**

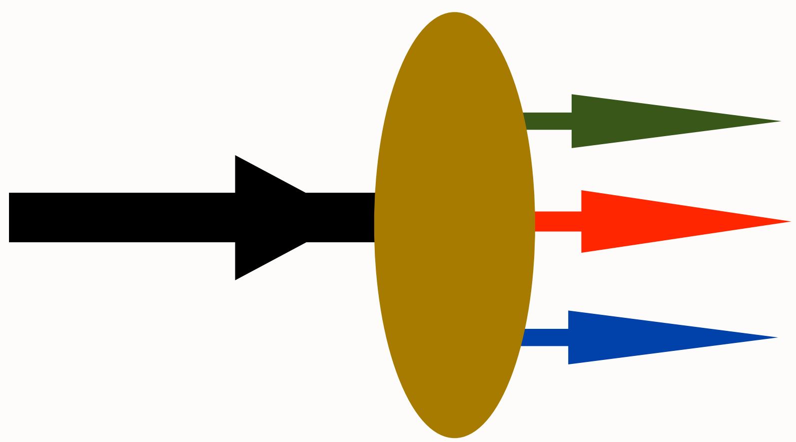
- **Cannot compute current matrix elements using instant or point form from eigensolutions alone -- need to include vacuum currents!**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- *Hadron Physics without LFWFs is like Biology without DNA!*

- *Hadron Physics without LFWFs is like Biology without DNA!*

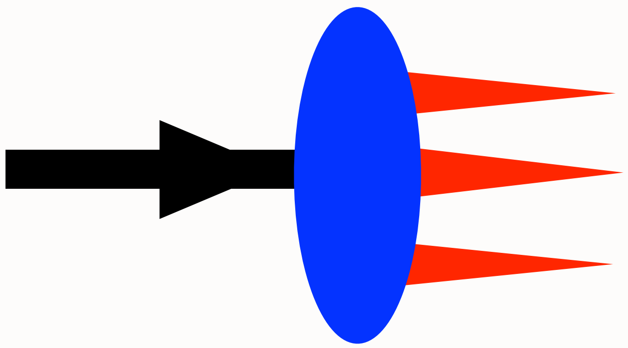


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

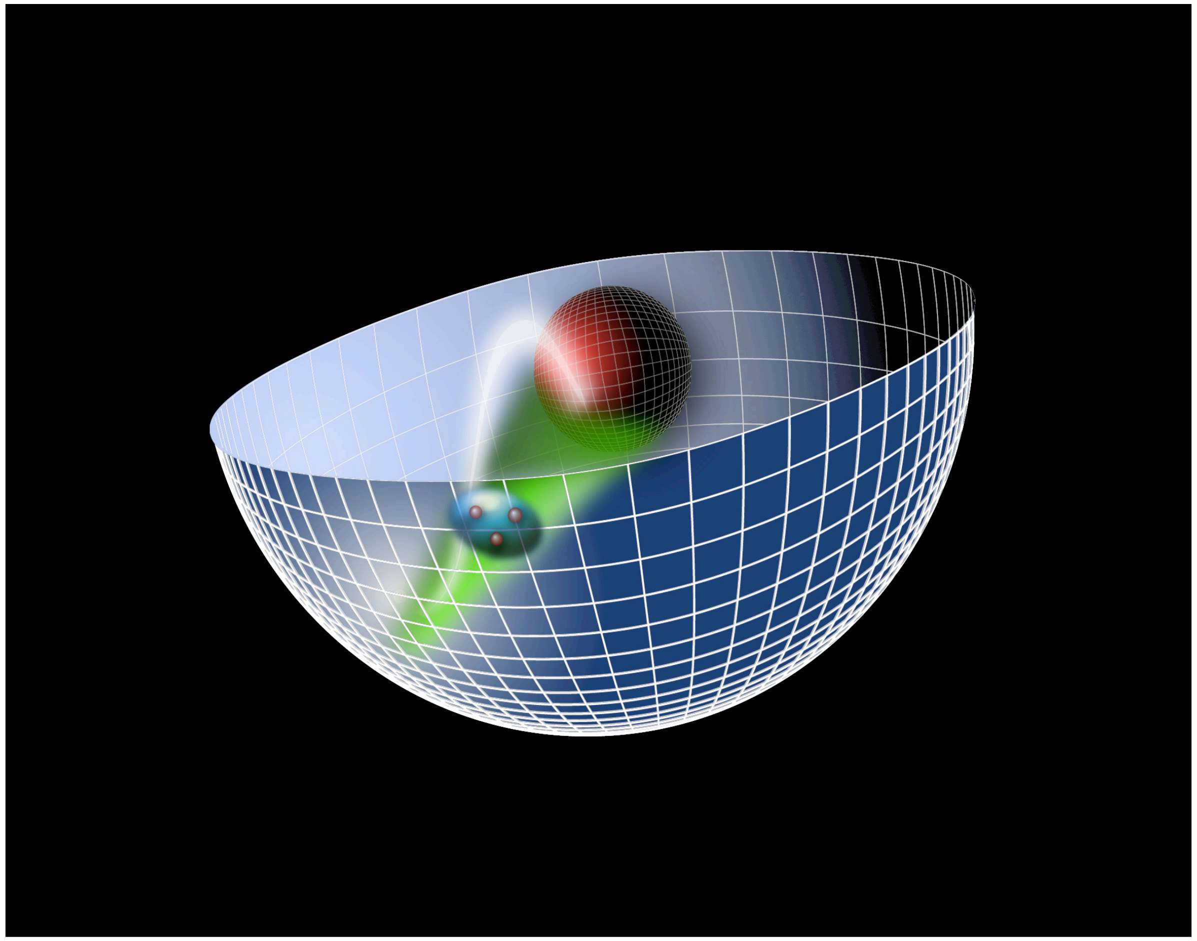
# Light-Front Holography and Non-Perturbative QCD

**Goal:**
**Use AdS/QCD duality to construct a first approximation to QCD**

*Hadron Spectrum*
*Light-Front Wavefunctions,*
*Running coupling in IR*

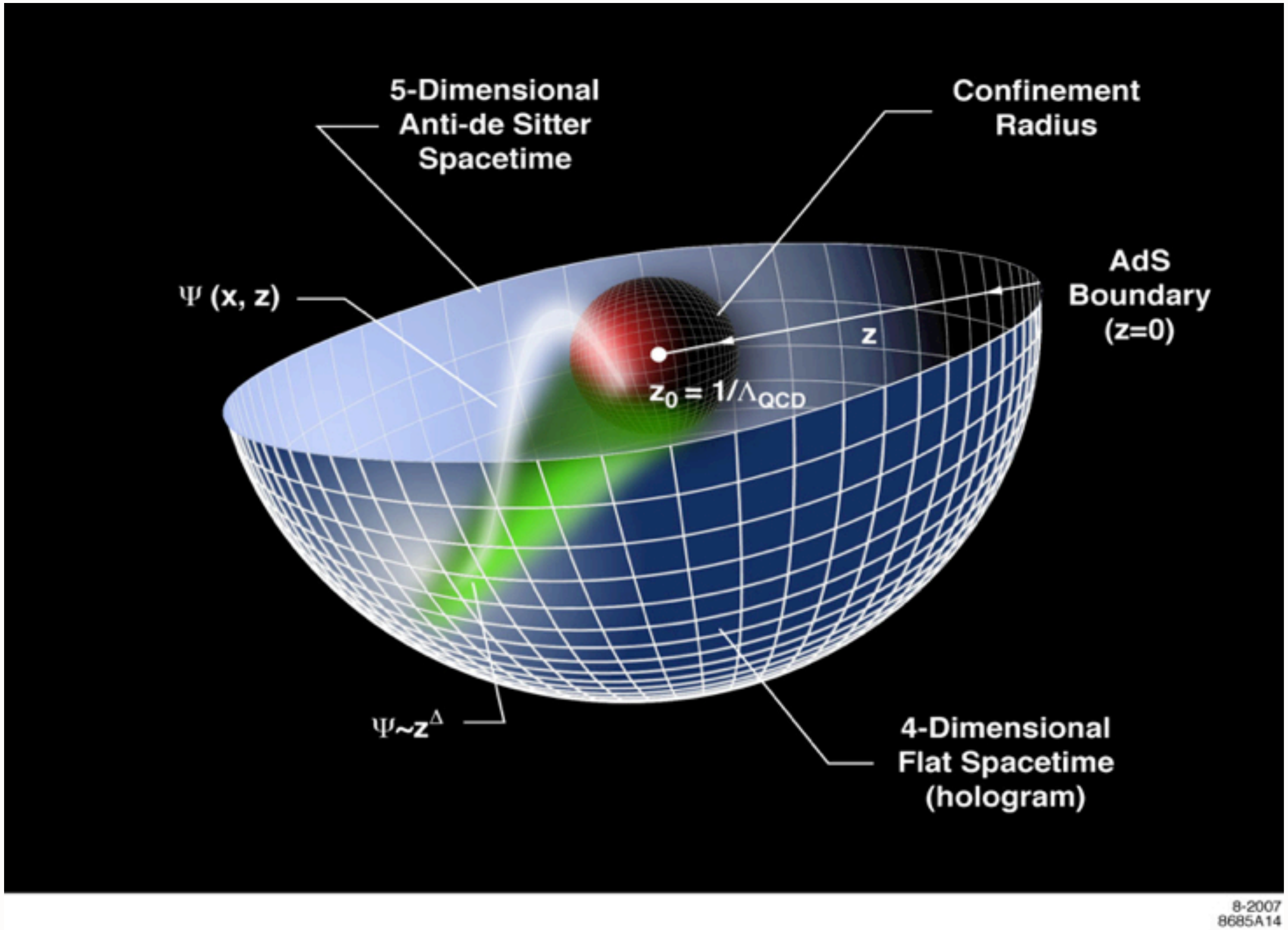


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with
Guy de Teramond

***Central problem for strongly-coupled gauge theories***




*Changes in physical length scale mapped to evolution in the 5th dimension z*

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

# Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

*invariant measure*

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.
- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

# Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$ .
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along $x^\mu$-coordinates , $\Phi_P(x, z) = e^{-iP \cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$ :

$$\left[ z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z \mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}]\phi(z) = \mathcal{M}^2\phi(z)$$

**L = $L^z$ : Light-Front orbital angular momentum**

Derived from variation of Action in $AdS_5$

Hard wall model: truncated space

$$\phi(\mathbf{z} = \mathbf{z_0} = \frac{1}{\Lambda_c}) = 0.$$

## *Match fall-off at small z to conformal twist-dimension at short distances*

*twist*

$$\Delta = 2 + L$$

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z =$ $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$

$$\Delta = 2 + L$$

*Twist = equivalent to dimensions of chiral superfields*

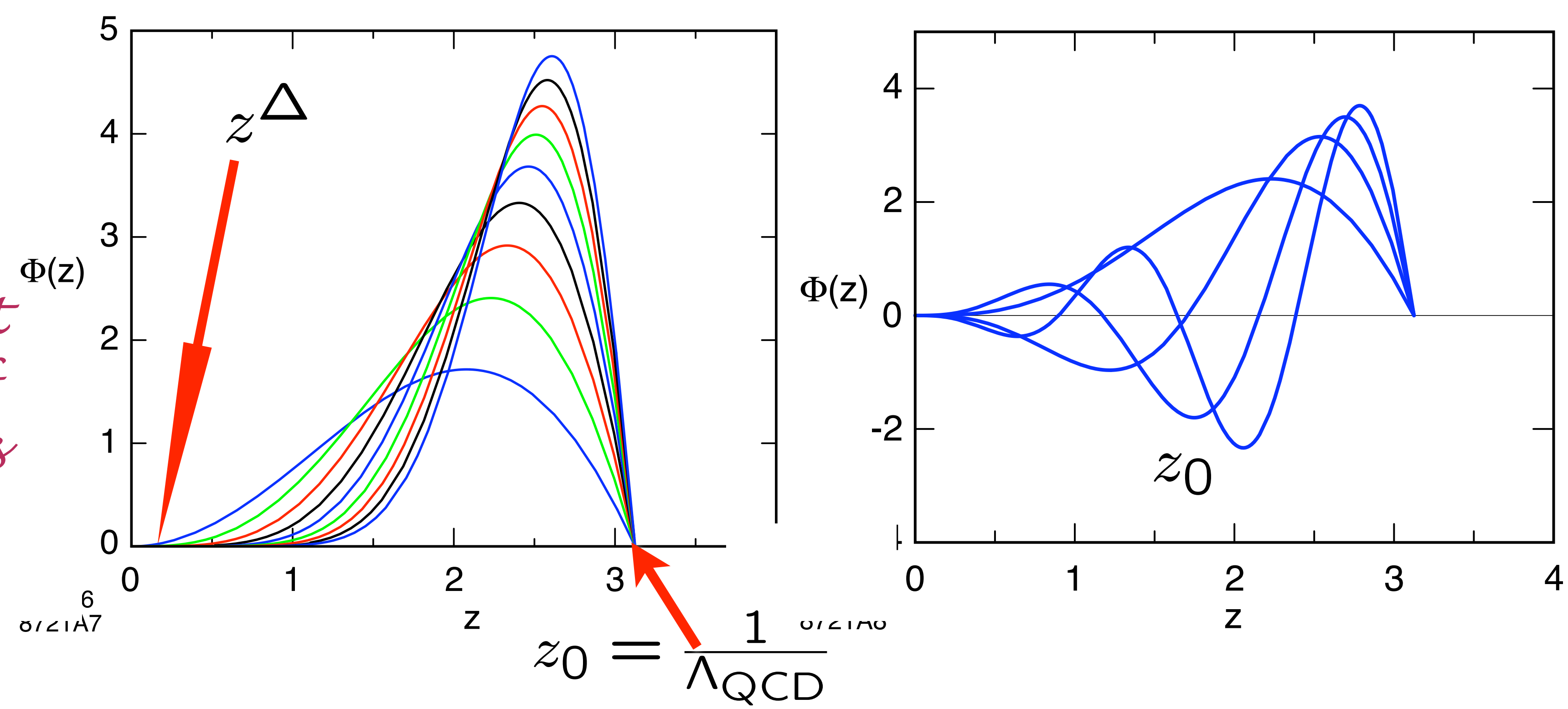
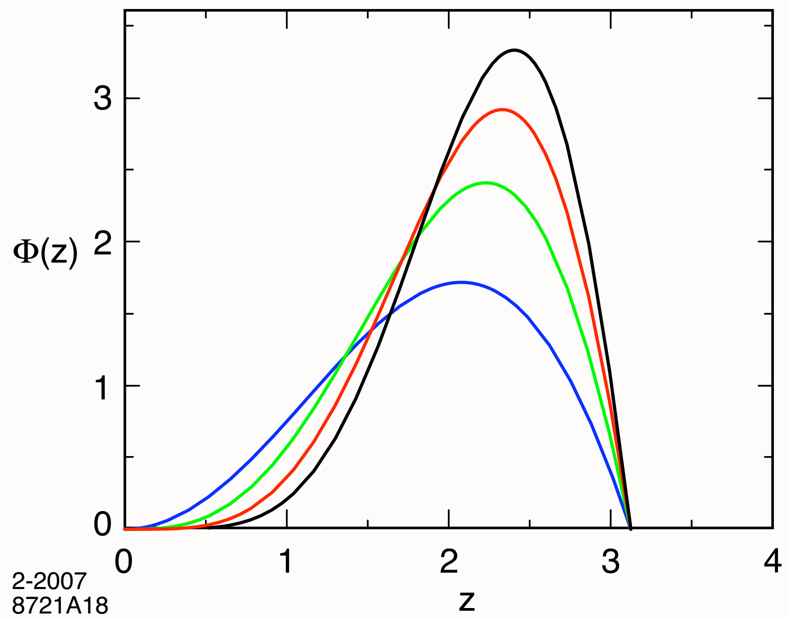



Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

**Identify hadron by its interpolating operator at z --> 0**



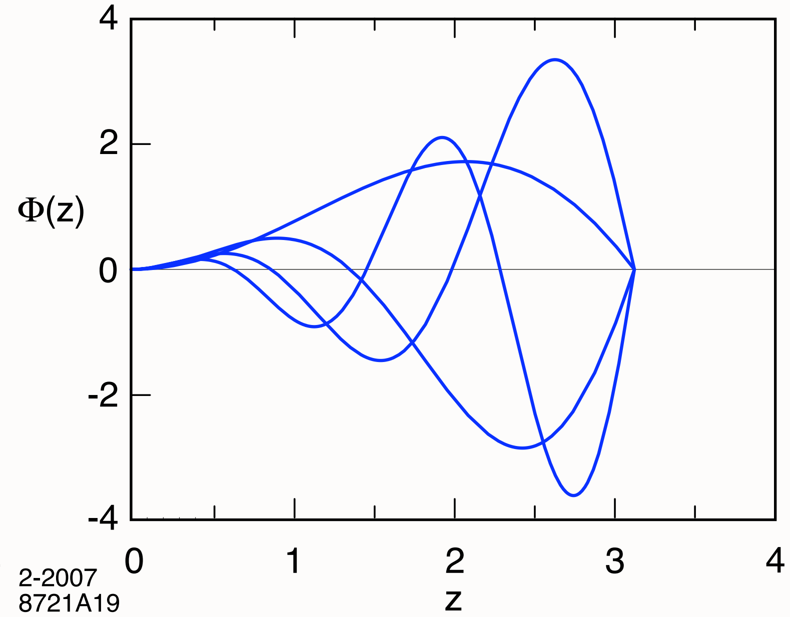
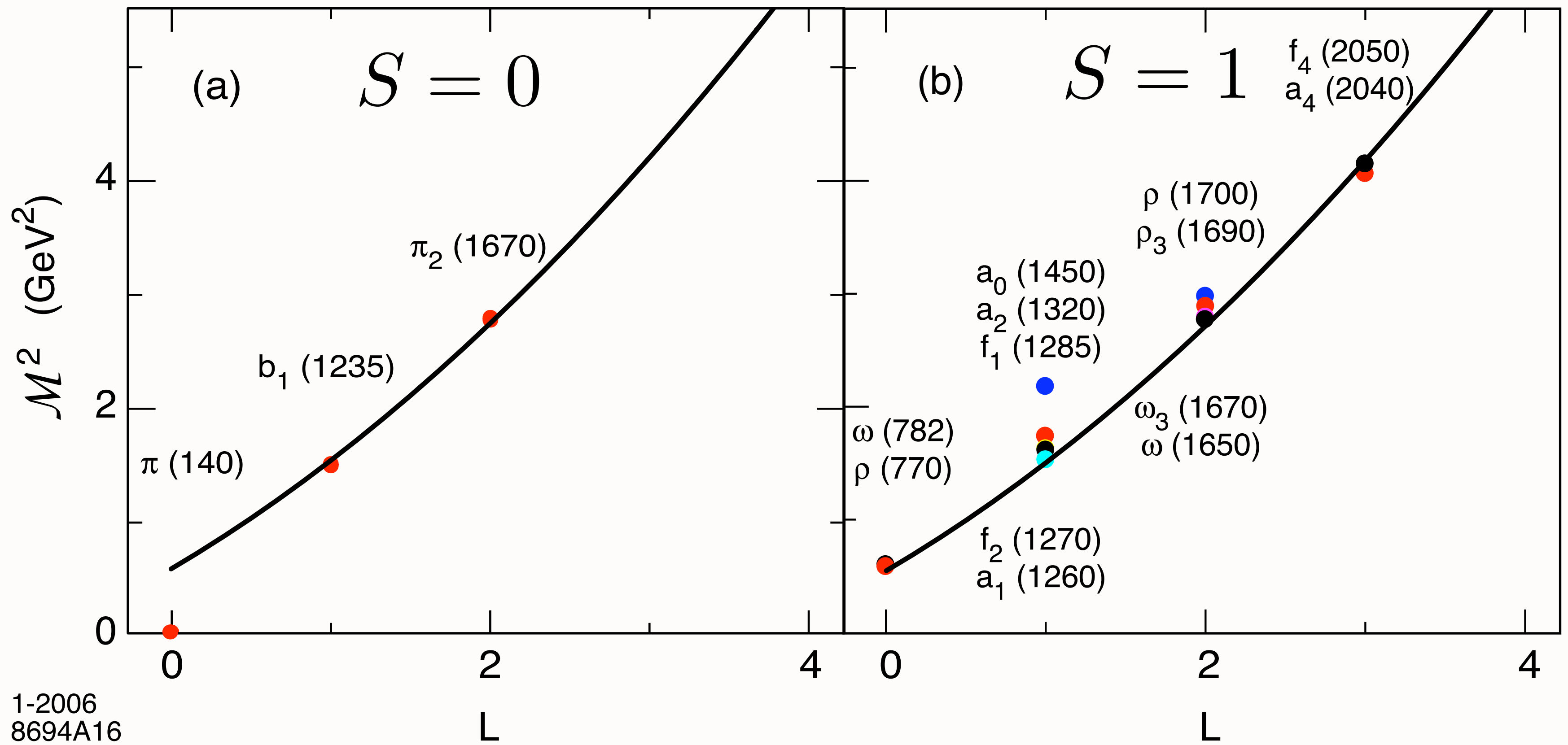




Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .




***Hard Wall***

$$m_\pi = m_\rho$$

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32 \text{ GeV}$

## Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

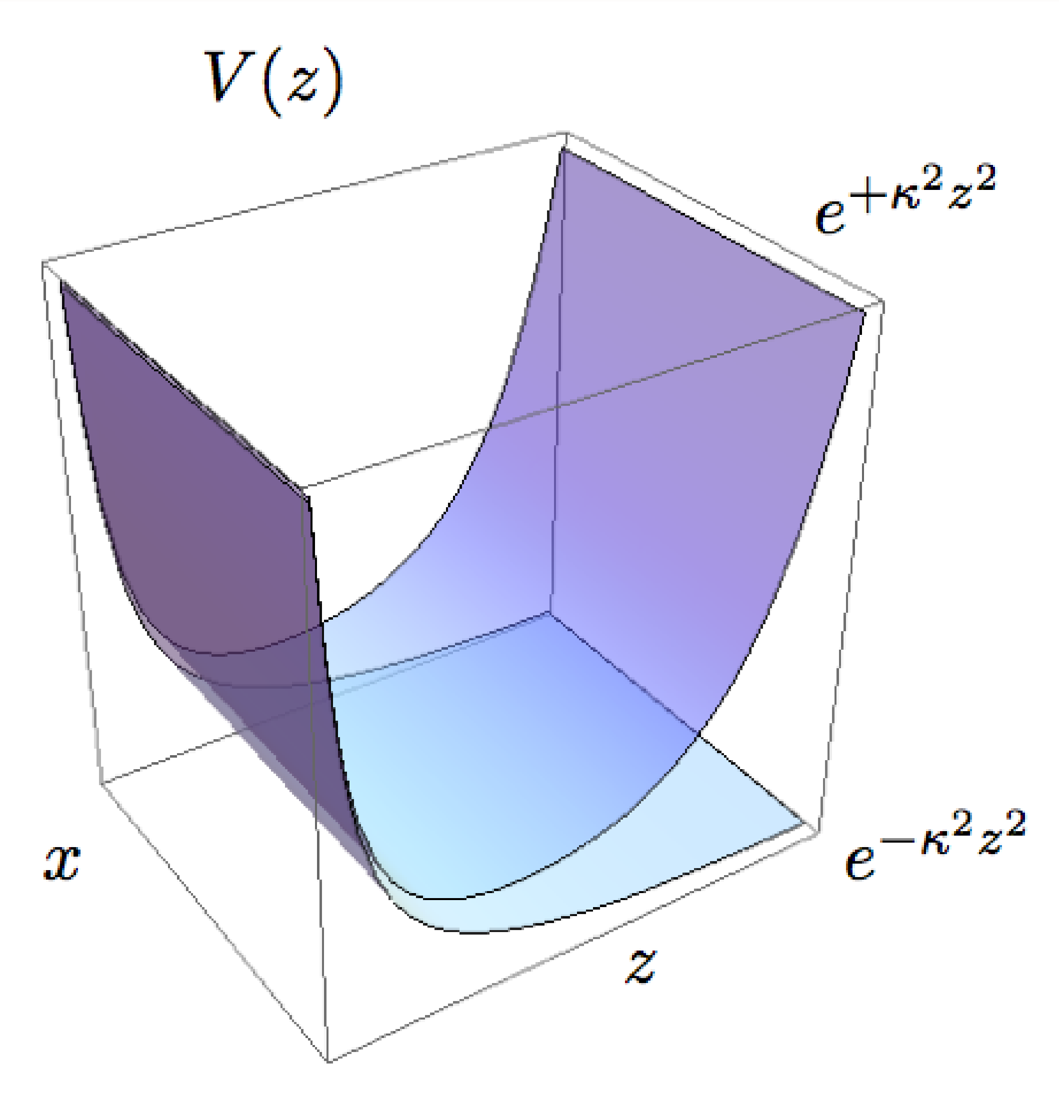
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small $z$ for geometries which are asymptotically $AdS_5$

- Gravitational potential energy for object of mass $m$

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$




Klebanov and Maldacena

## Dual QCD Light-Front Wave Equation

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE $(d=4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension $\tau$ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

# General-Spin Hadrons

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for $\Phi$

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2} \, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2-J)^2 + L^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2}$$

- de Teramond, sjb

**Positive-sign dilaton**

# AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified $AdS_5$

***Identical to Light-Front Bound State Equation!***

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

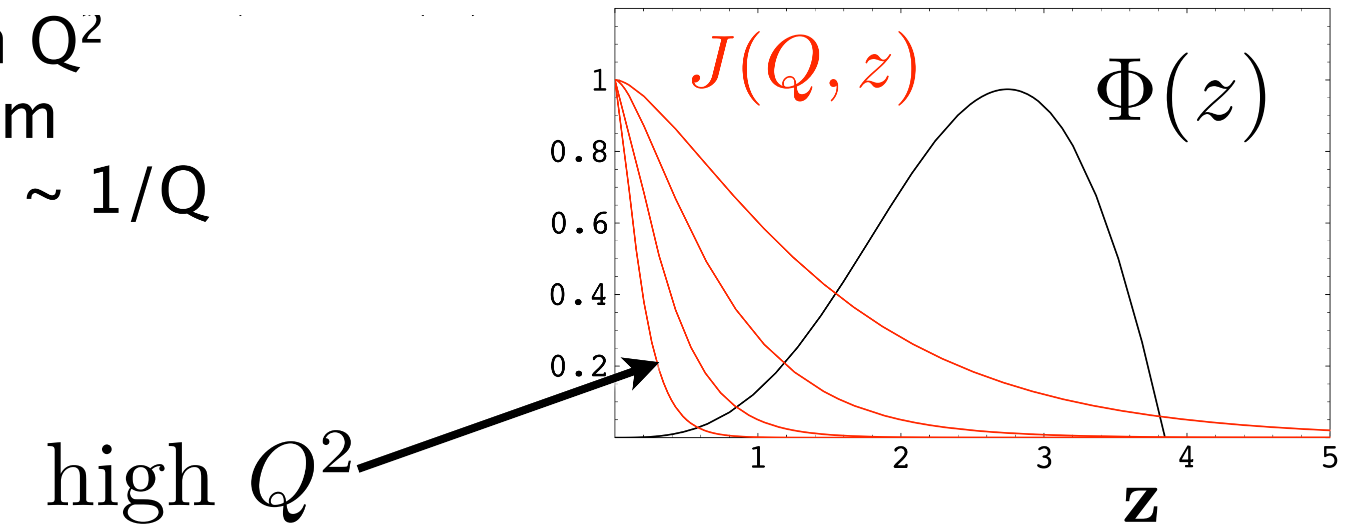
# Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQK_1(zQ)$$

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High $Q^2$ from small z ~ 1/Q




Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an $n$ partonic Fock state $|n\rangle$. At small $z$, $\Phi$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau - 1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

## Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary $Q$ using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$ !

# Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} \, H(Q^2, z) \, |\Phi_\pi(z)|^2 \, ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary $Q$

$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},}$$

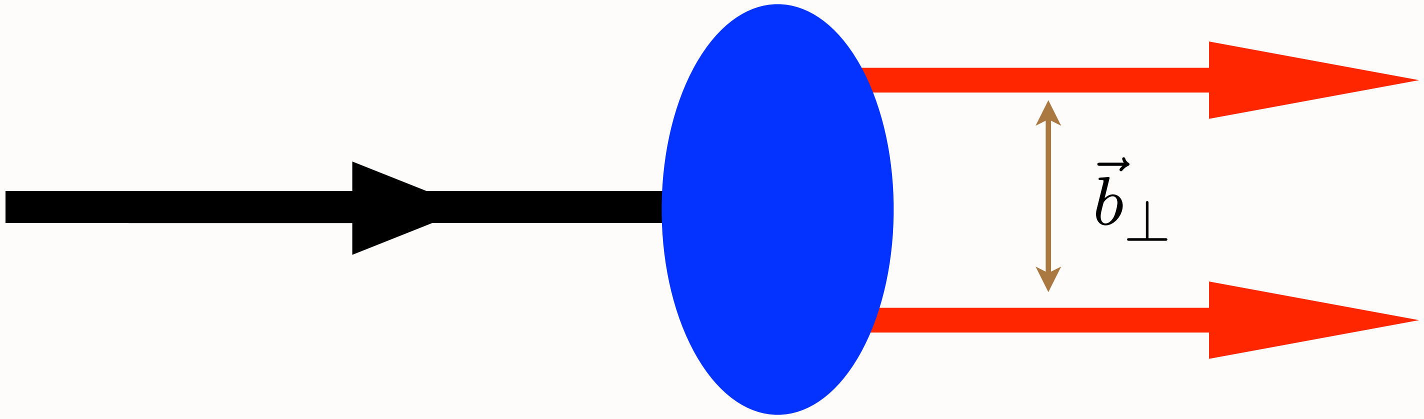
*Identical to LF Holography obtained from electromagnetic current*

de Teramond, sjb

LF(3+1) ⟷ $AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$




$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

*Light Front Holography: Unique mapping derived from equality of LF and AdS formulae for bound-states and form factors*

# QCD Meson Spectrum

$$H^{LF}_{QCD}$$

**Fixed Light-Front Time (Front form)**

Fixed $\tau = t + z/c$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Light-Front Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis $\zeta, \phi$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

# Light-Front Schrödinger Equation

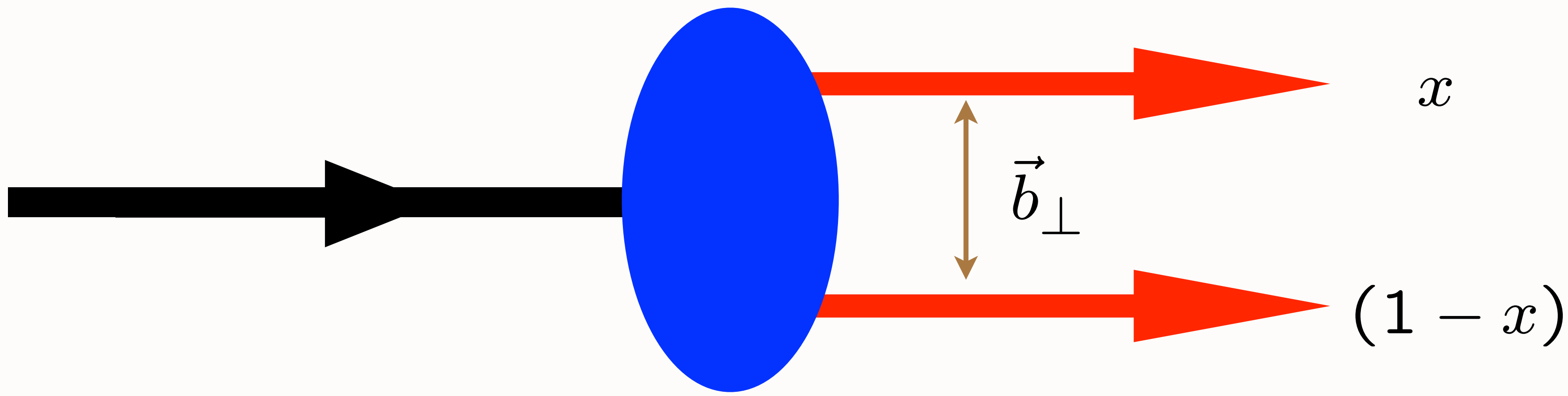
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$




## AdS/QCD:

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

## Meson Spectrum in Soft Wall Model

- Linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]
- Dilaton profile $\varphi(z) = +\kappa^2 z^2$
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

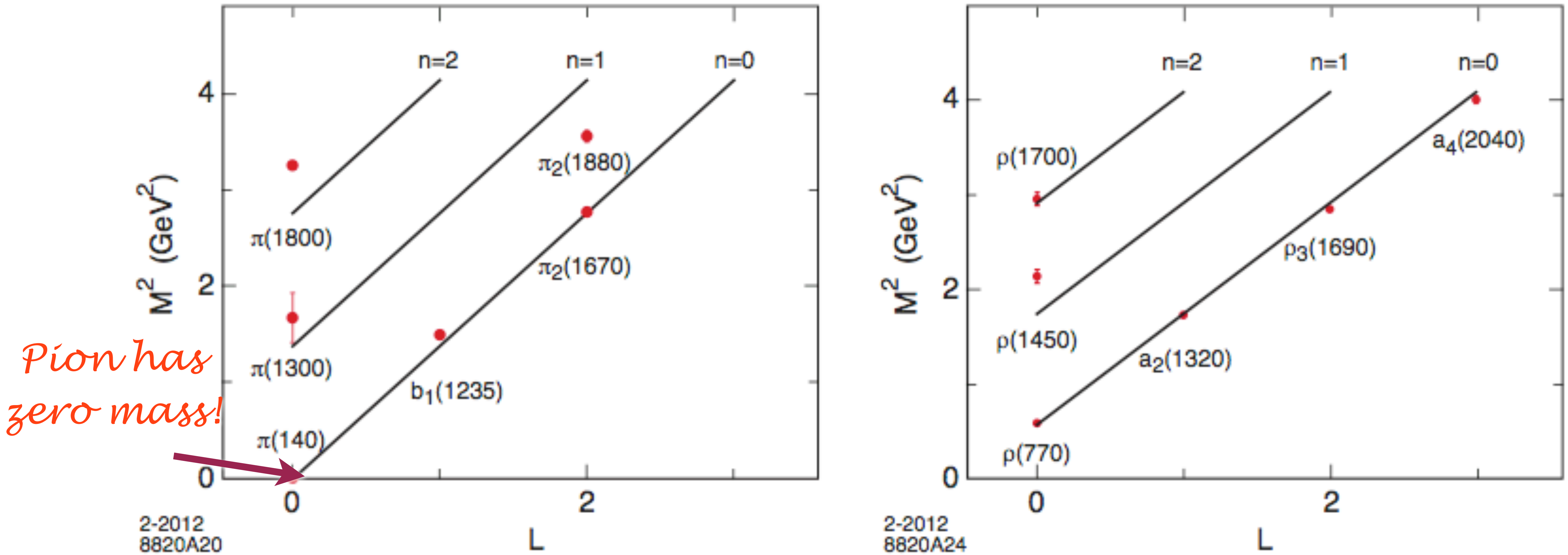
$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$




I=1 orbital and radial excitations for the $\pi$ ($\kappa = 0.59$ GeV) and the $\rho$-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1$, $L = 1$, $J = 0, 1, 2$, vector meson $a$-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

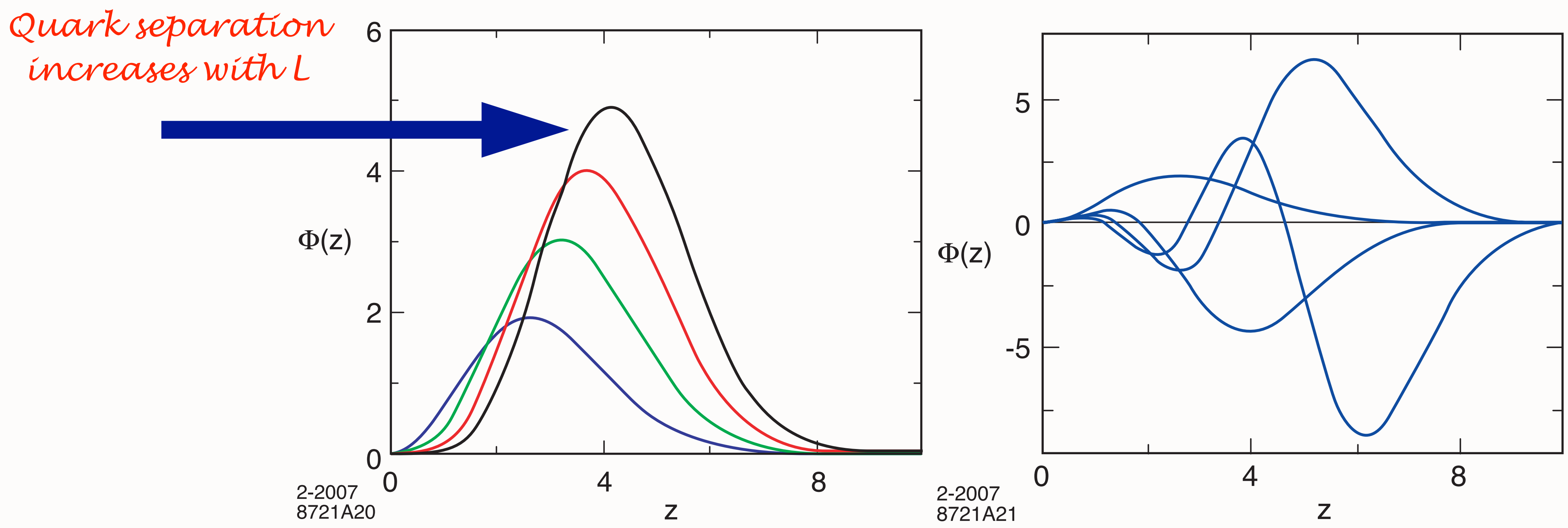
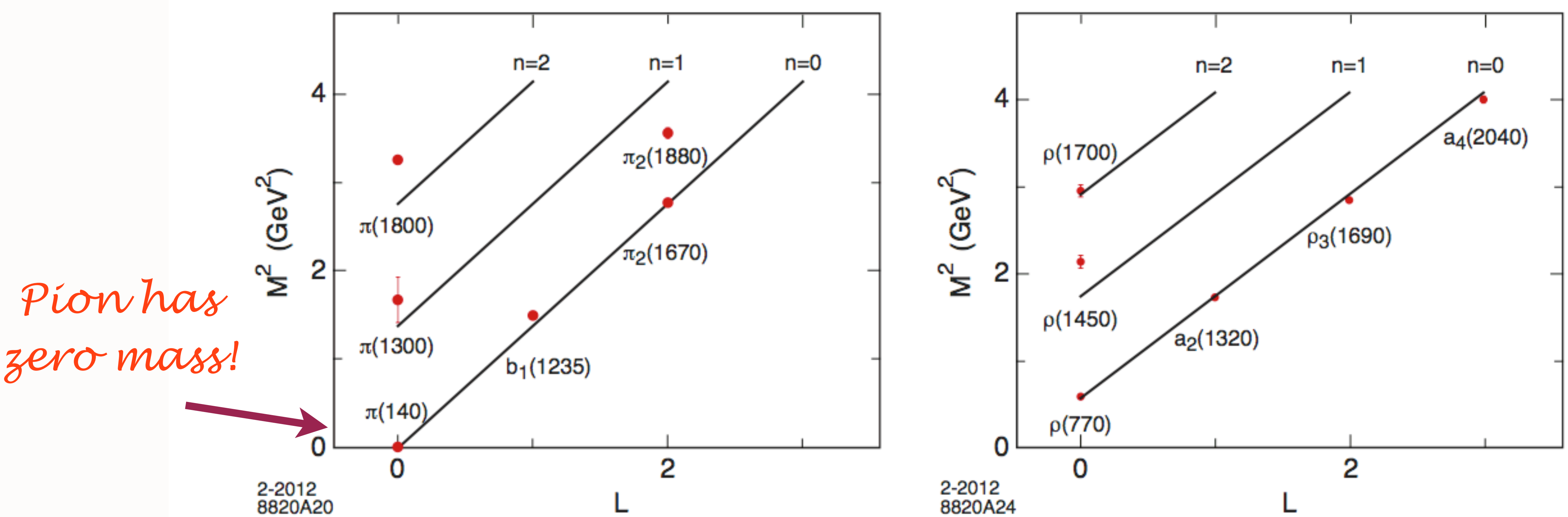



Fig: Orbital and radial AdS modes in the soft wall model for $\kappa$ = 0.6 GeV .

# Soft Wall Model




Orbital and radial excitations for the $\pi$ ($\kappa = 0.59$ GeV) and the $\rho$ I=1meson families ($\kappa = 0.54$ GeV)

*Quark separation increases with L*

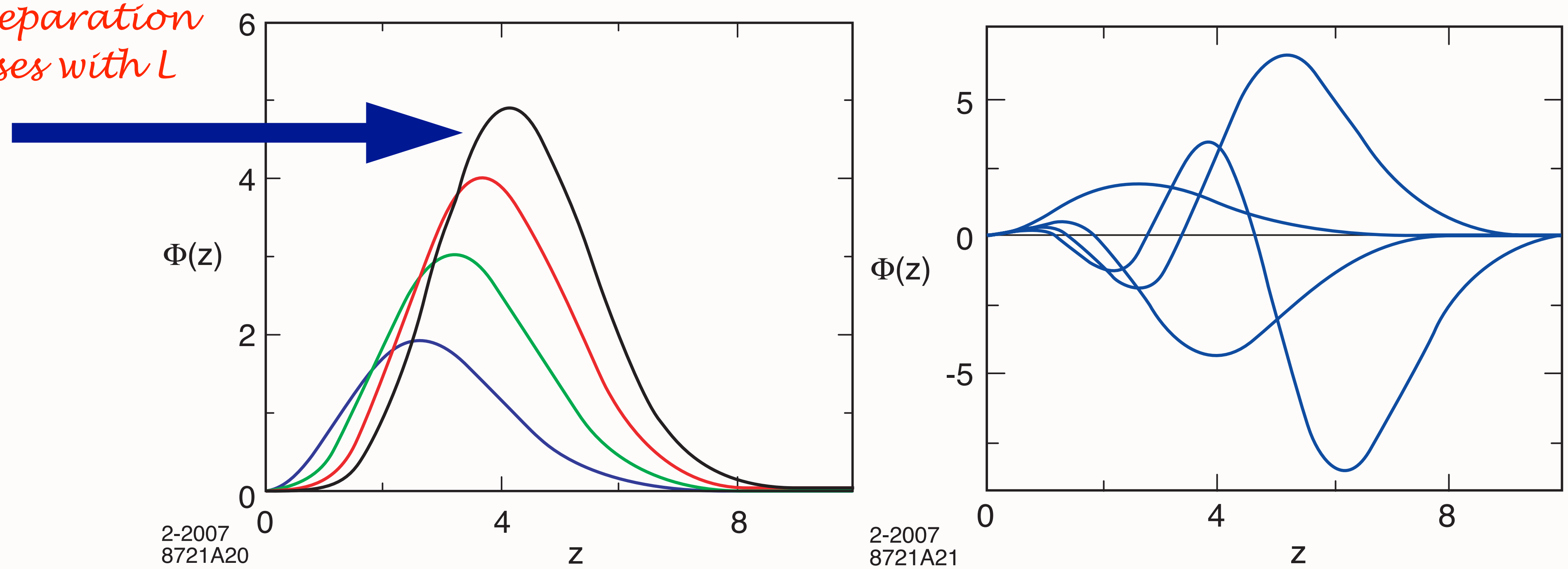
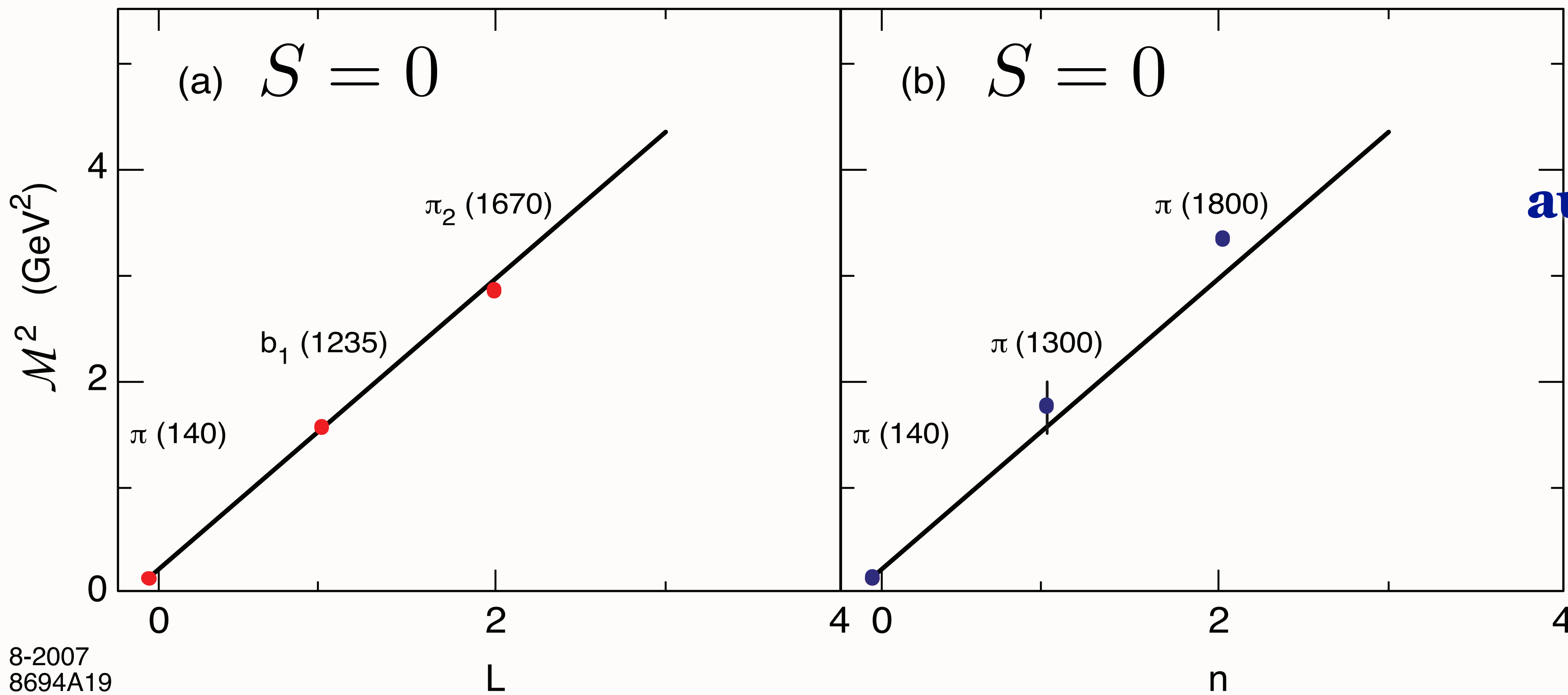



Fig: Orbital and radial AdS modes in the soft wall model for $\kappa$ = 0.6 GeV .

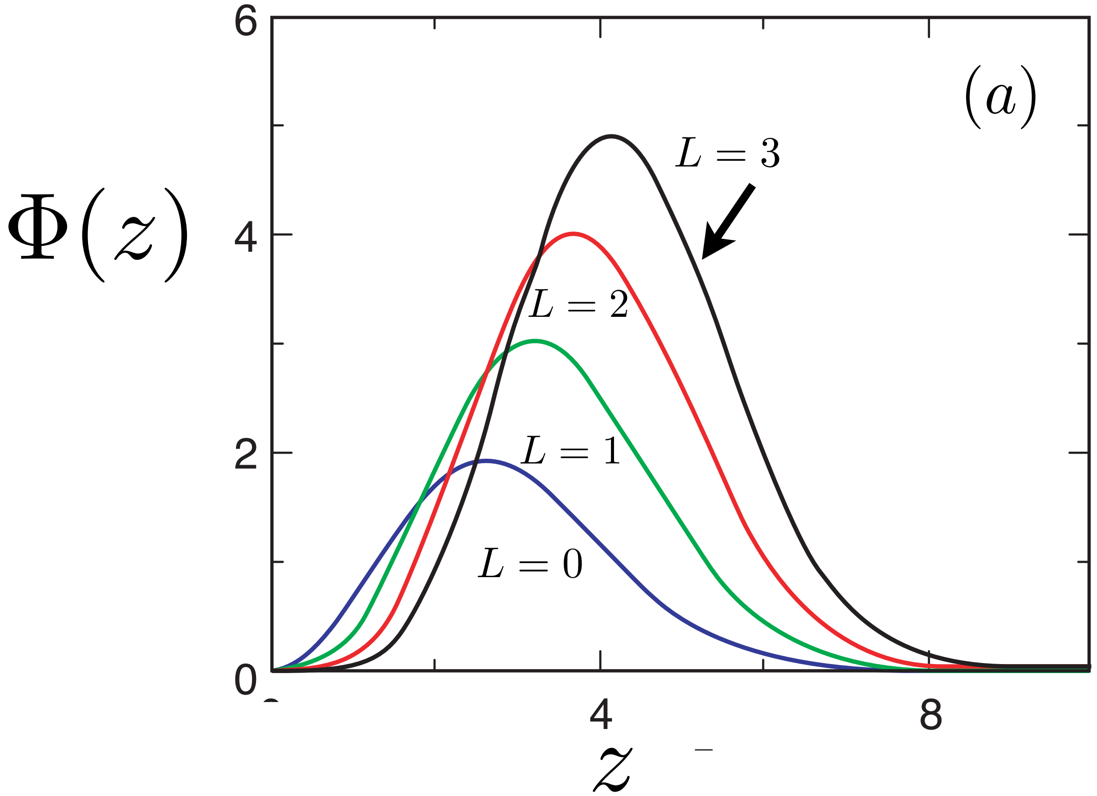
# *Soft Wall Model*

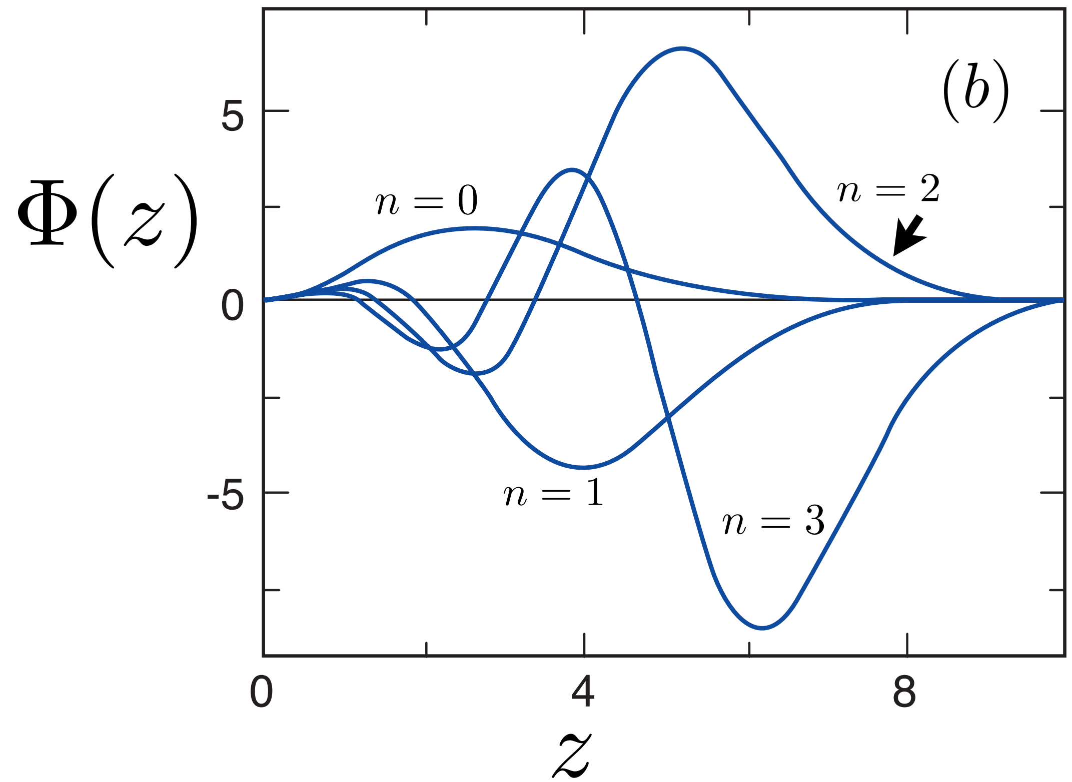


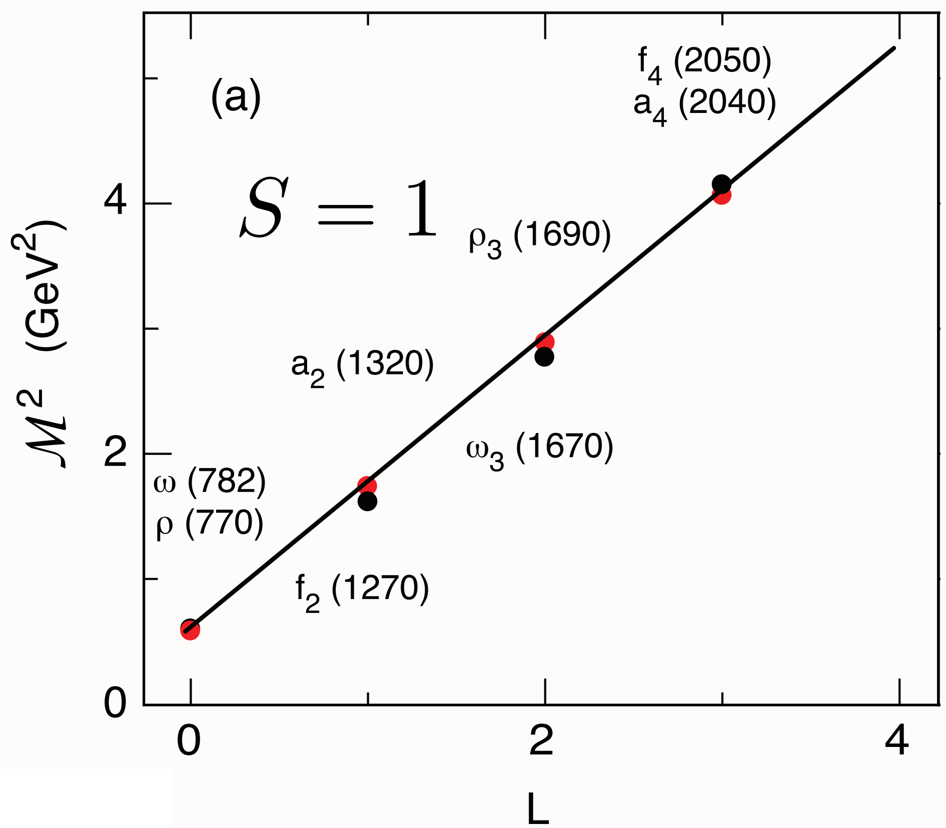

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

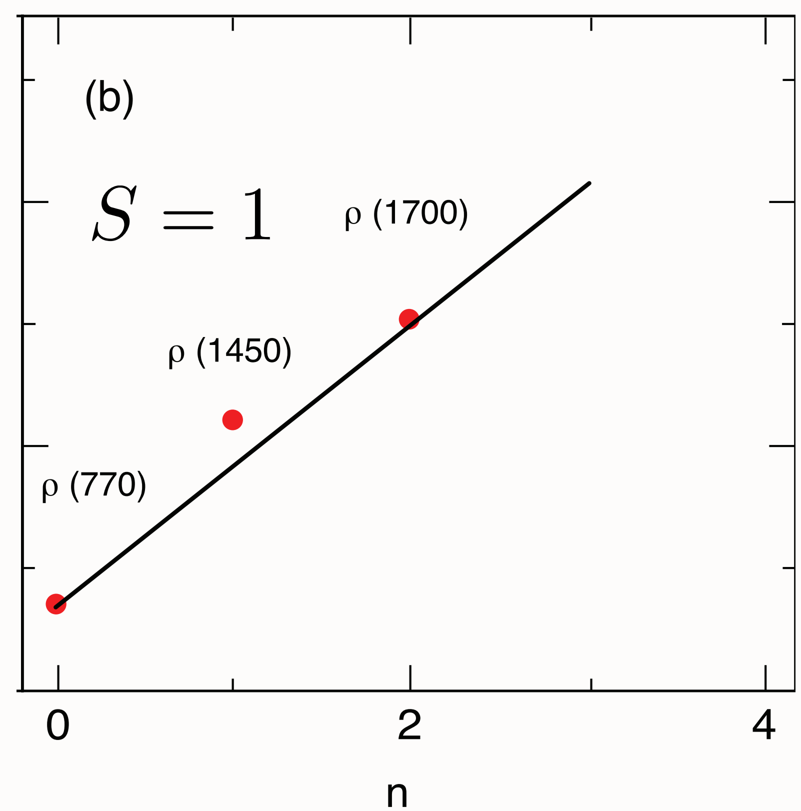
**Pion mass automatically zero!**

$$m_q = 0$$


6
4
2
0
4
8
(a)
$\Phi(z)$
$L = 3$
$L = 2$
$L = 1$
$L = 0$
$z$


5
0
-5
0
4
8
(b)
$\Phi(z)$
$n = 0$
$n = 2$
$n = 1$
$n = 3$
$z$


(a)
$S = 1$
f$_4$ (2050)
a$_4$ (2040)
ρ$_3$ (1690)
a$_2$ (1320)
ω$_3$ (1670)
ω (782)
ρ (770)
f$_2$ (1270)
$\mathcal{M}^2$ (GeV$^2$)
4
2
0
0
2
4
L


(b)
$S = 1$
ρ (1700)
ρ (1450)
ρ (770)
0
2
4
n

# Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2}\Phi(z) J_\kappa(Q,z)\Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

## Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension $\tau$, $\Phi_\tau$ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1+\frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau+\frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N, \quad \Gamma(N+z) = (N-1+z)(N-2+z)\ldots(1+z)\Gamma(1+z).$

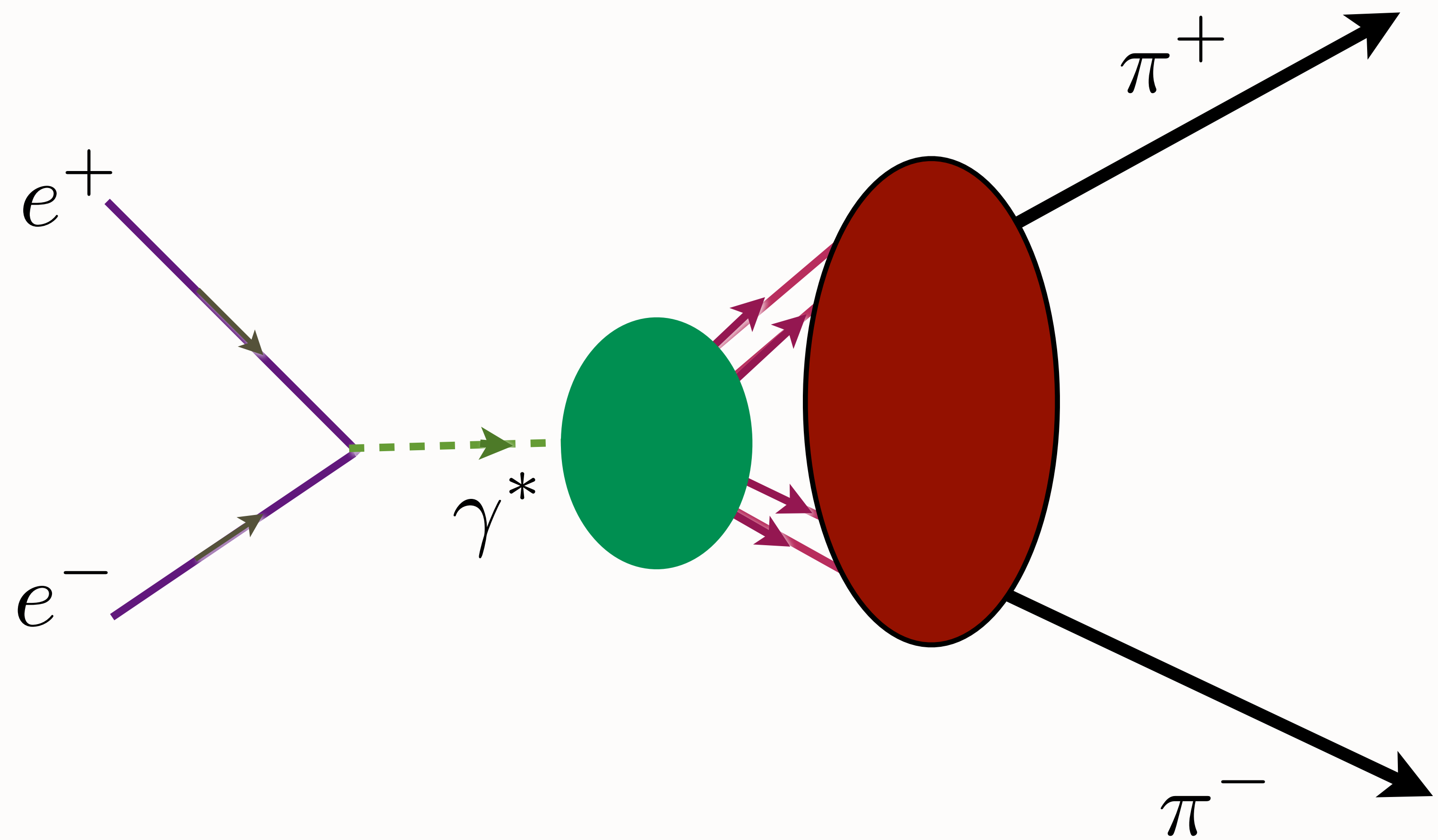
- Form factor expressed as $N-1$ product of poles

$$\begin{aligned}
F(Q^2) &= \frac{1}{1+\frac{Q^2}{4\kappa^2}}, \quad N=2, \\
F(Q^2) &= \frac{2}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)}, \quad N=3, \\
&\cdots \\
F(Q^2) &= \frac{(N-1)!}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)\cdots\left(N-1+\frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

- For large $Q^2$:

$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

# Dressed soft-wall current brings in higher Fock states and more vector meson poles

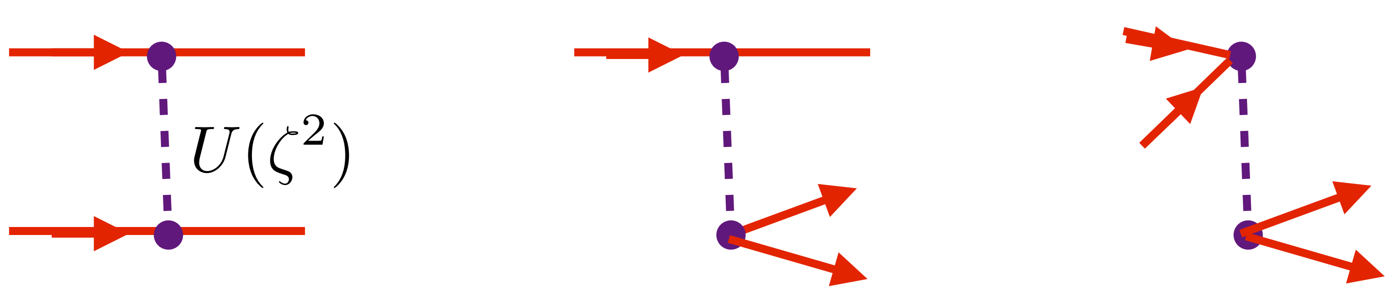


# Higher Fock States

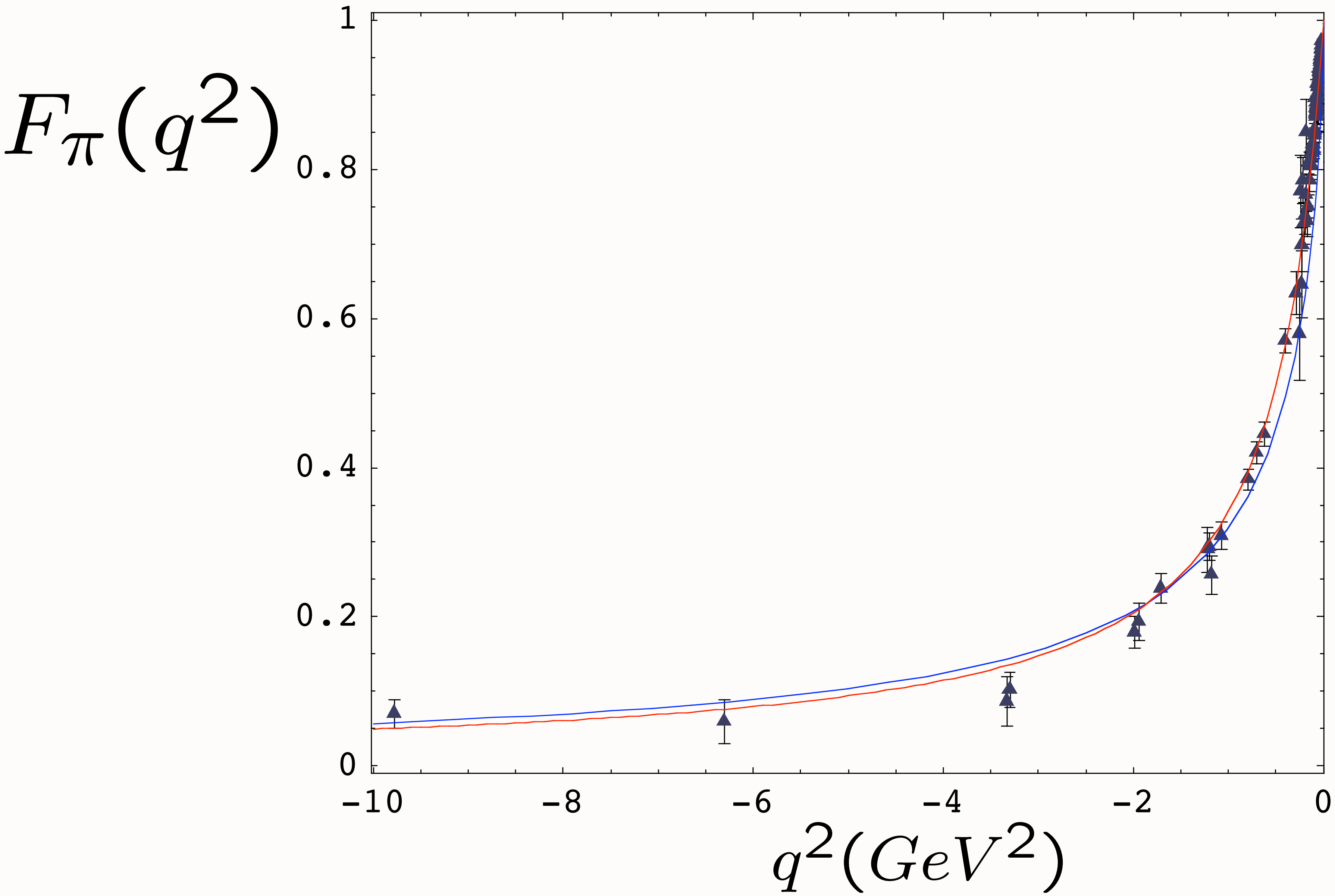
- Exposed by timelike form factor through dressed current.
- Created by confining interaction

$$P^-_{\text{confinement}} \simeq \kappa^4 \int dx^- d^2\vec{x}_\perp \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+}$$

- Similar to QCD(1+1) in lcg



# Spacelike pion form factor from AdS/CFT



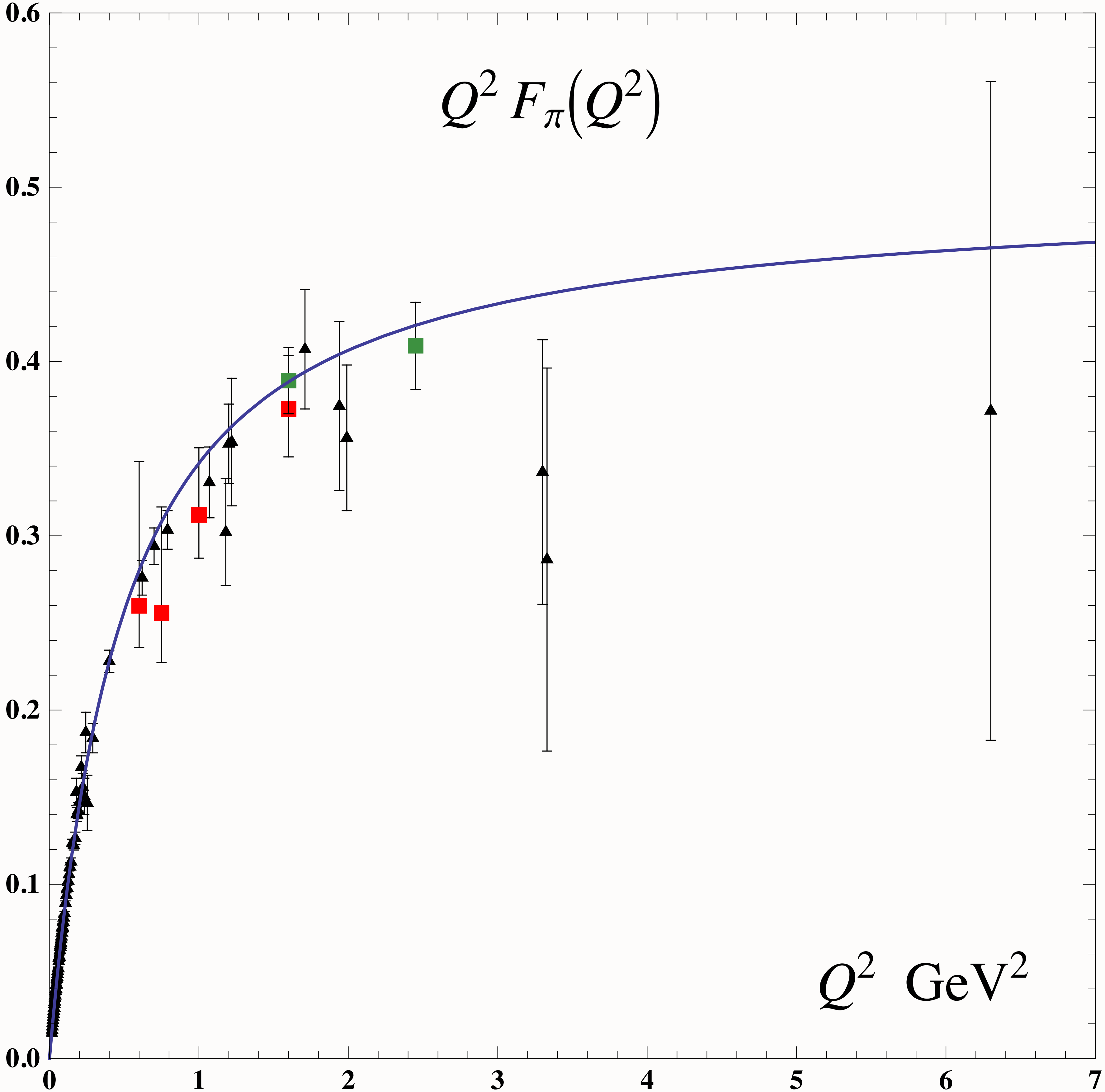

Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

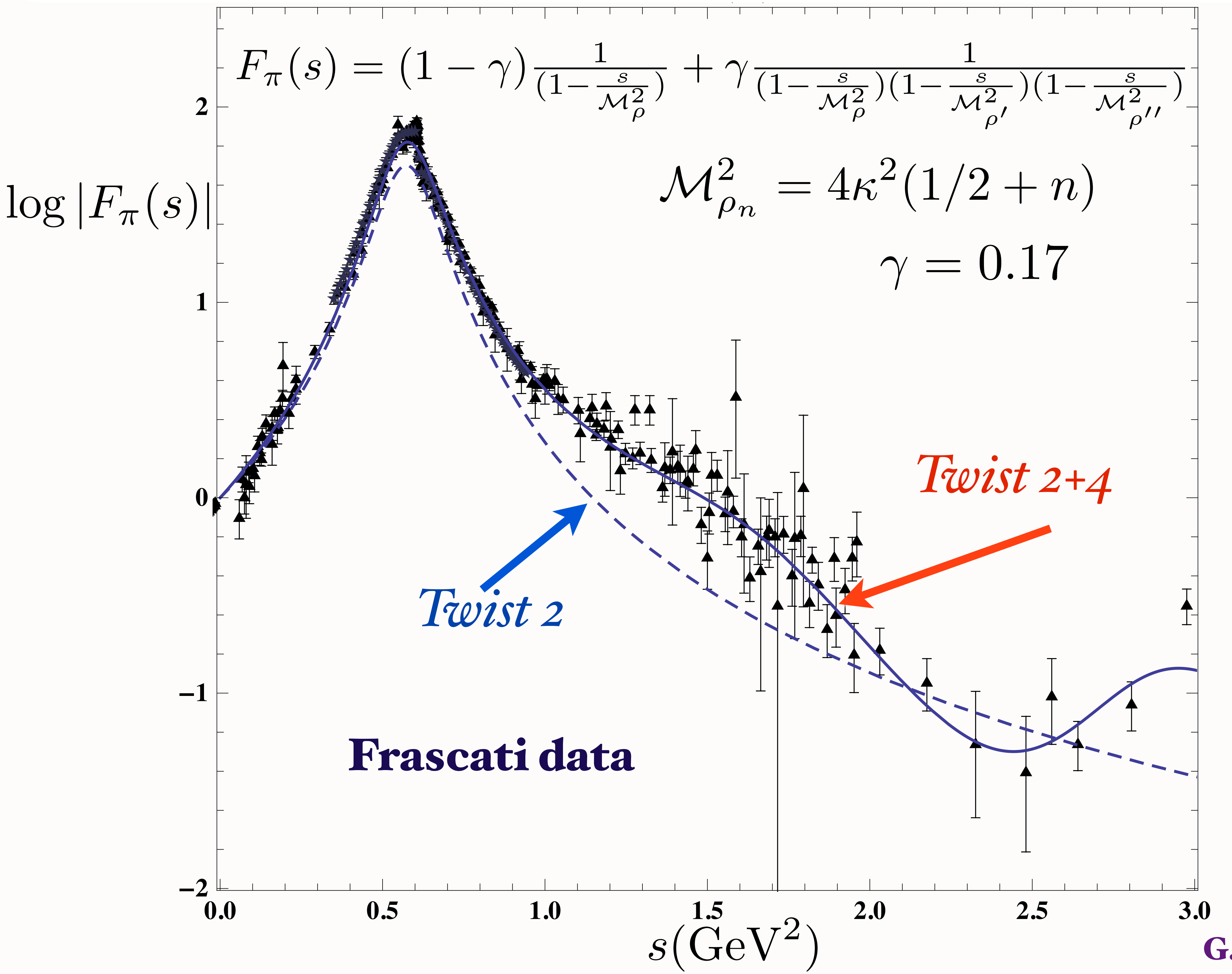
Hard Wall: Truncated Space Confinement

*One parameter - set by pion decay constant*

de Teramond, sjb
See also: Radyushkin


$Q^2 F_\pi(Q^2)$
$Q^2$ GeV$^2$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

# Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



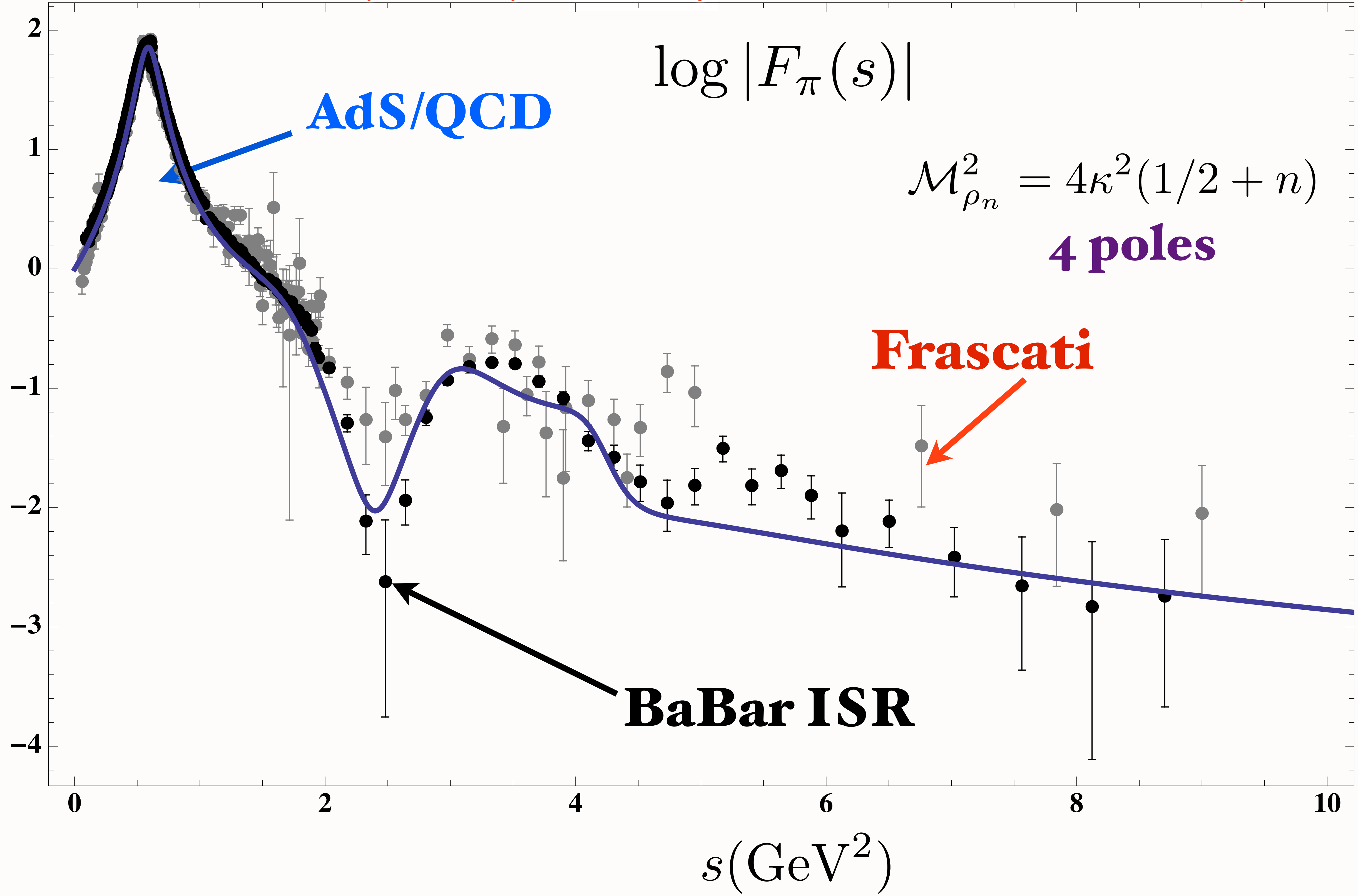

***Prescription for Timelike poles :***

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

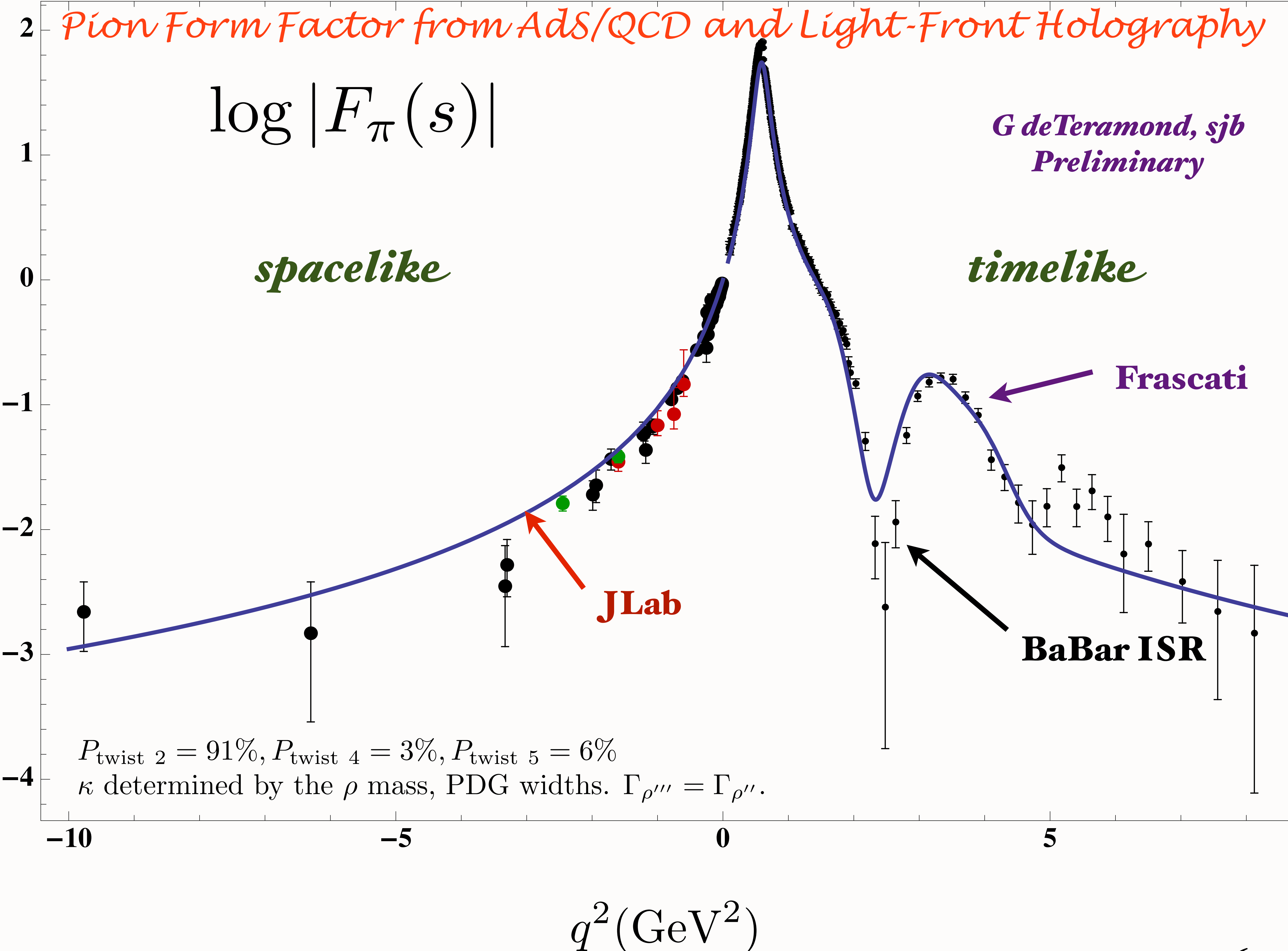
**14% four-quark probability**

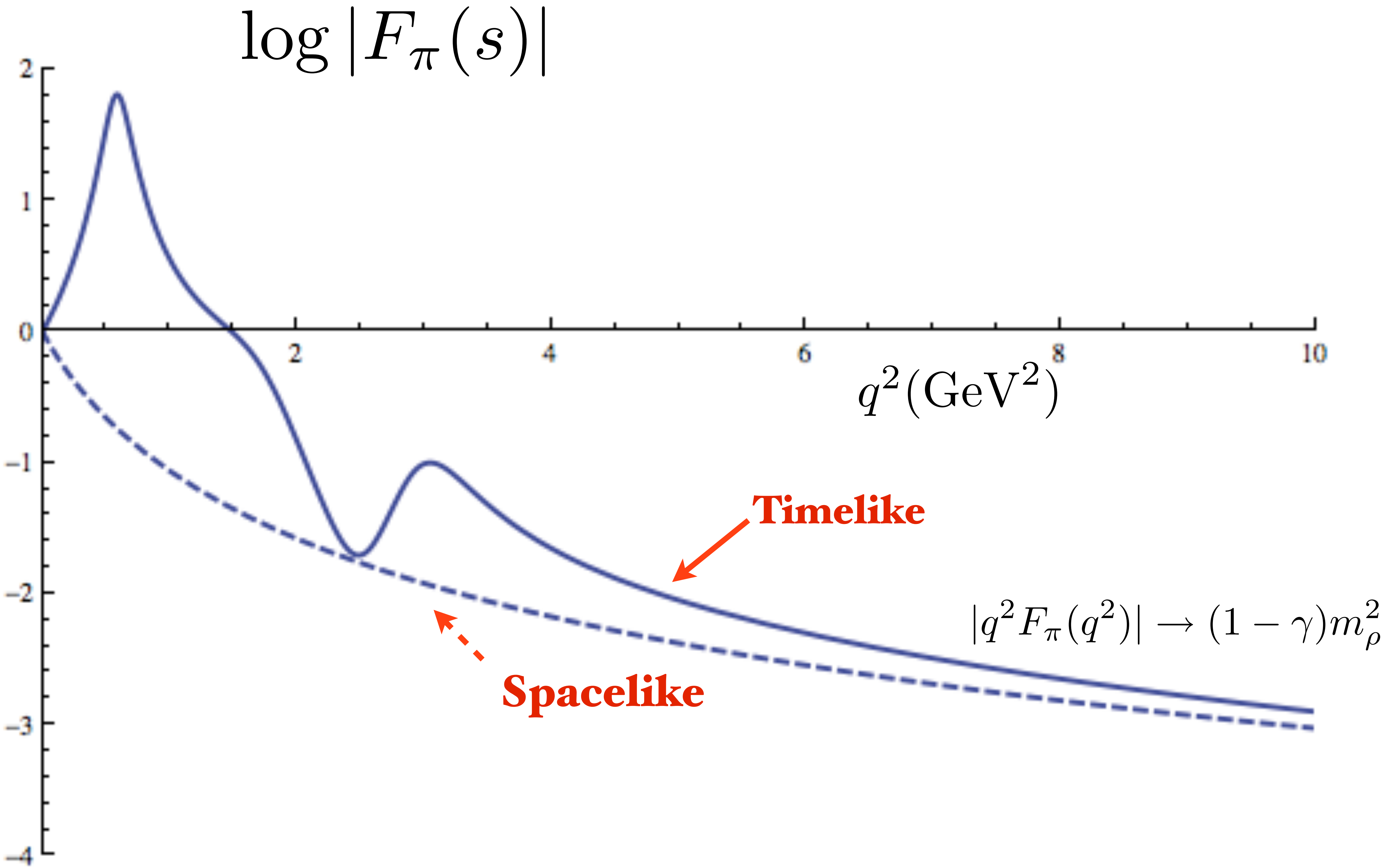
**G. de Teramond & sjb**

# Pion Timelike Form Factor (Includes Twist 2 to 5)



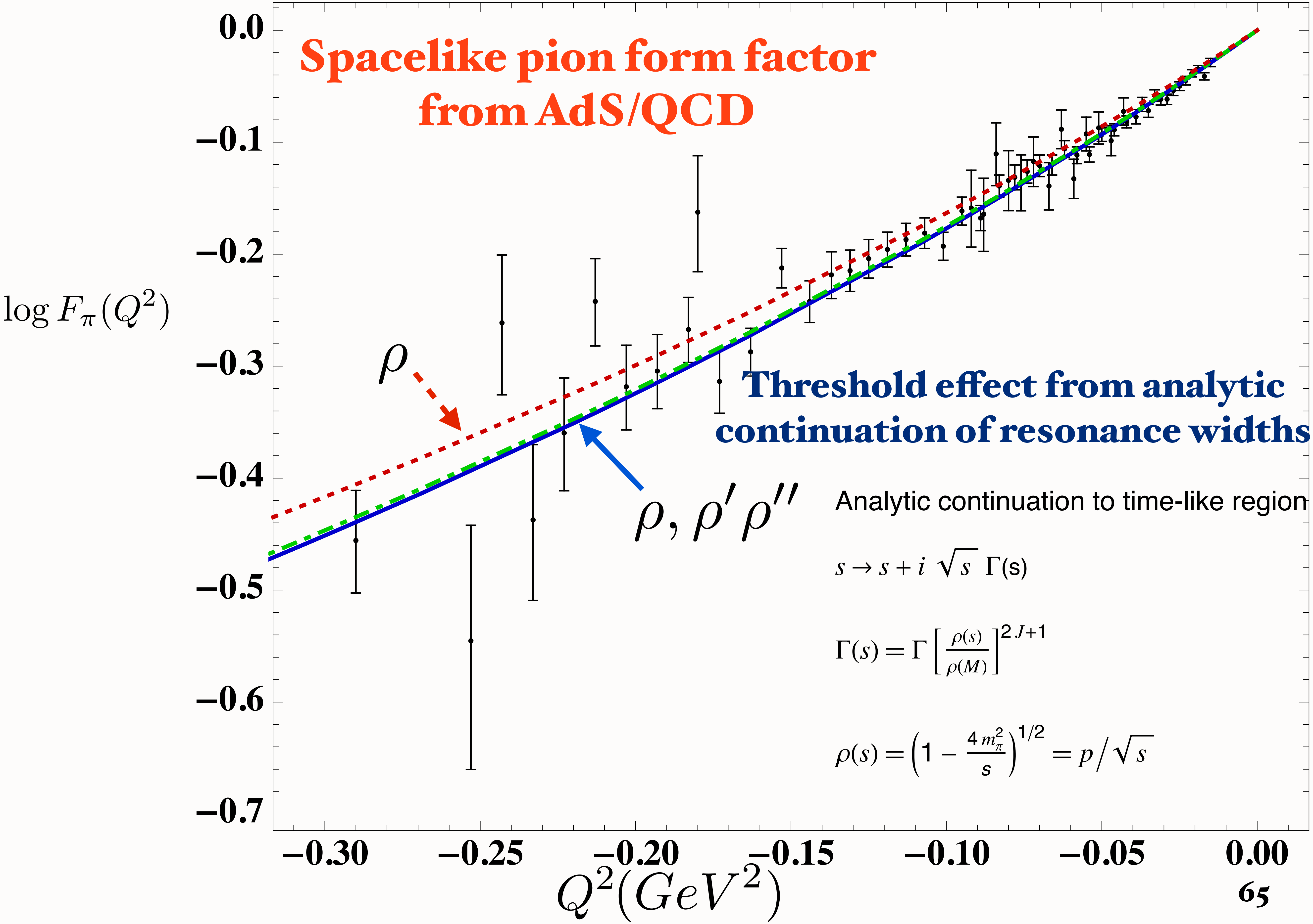
# Pion Form Factor from AdS/QCD and Light-Front Holography

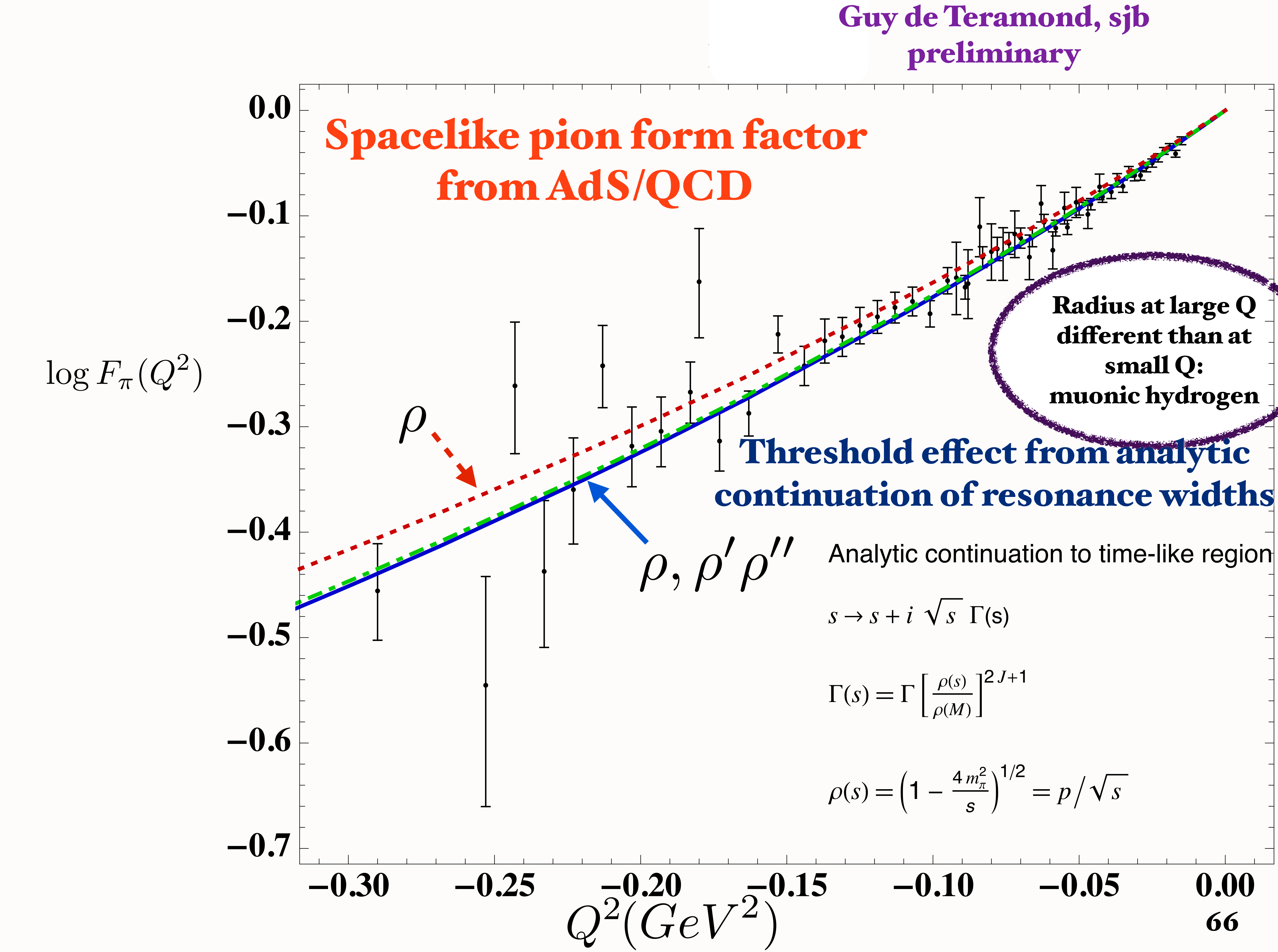




$\log|F_\pi(s)|$
2
1
0
−1
−2
−3
−4
2
4
6
8
10
$q^2(\mathrm{GeV}^2)$
Timelike
Spacelike
$|q^2 F_\pi(q^2)| \to (1-\gamma)m_\rho^2$

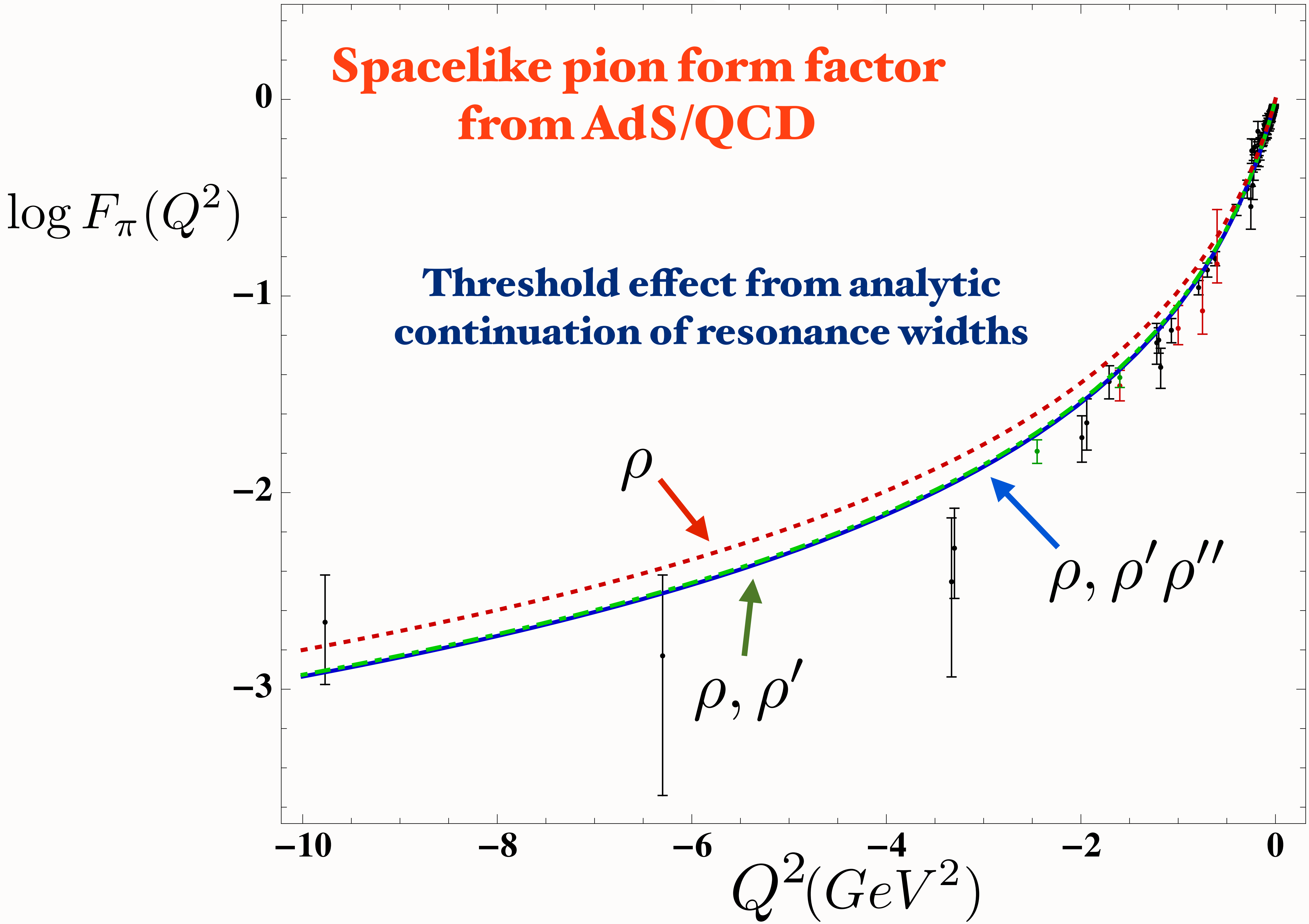
Guy de Teramond, sjb
preliminary





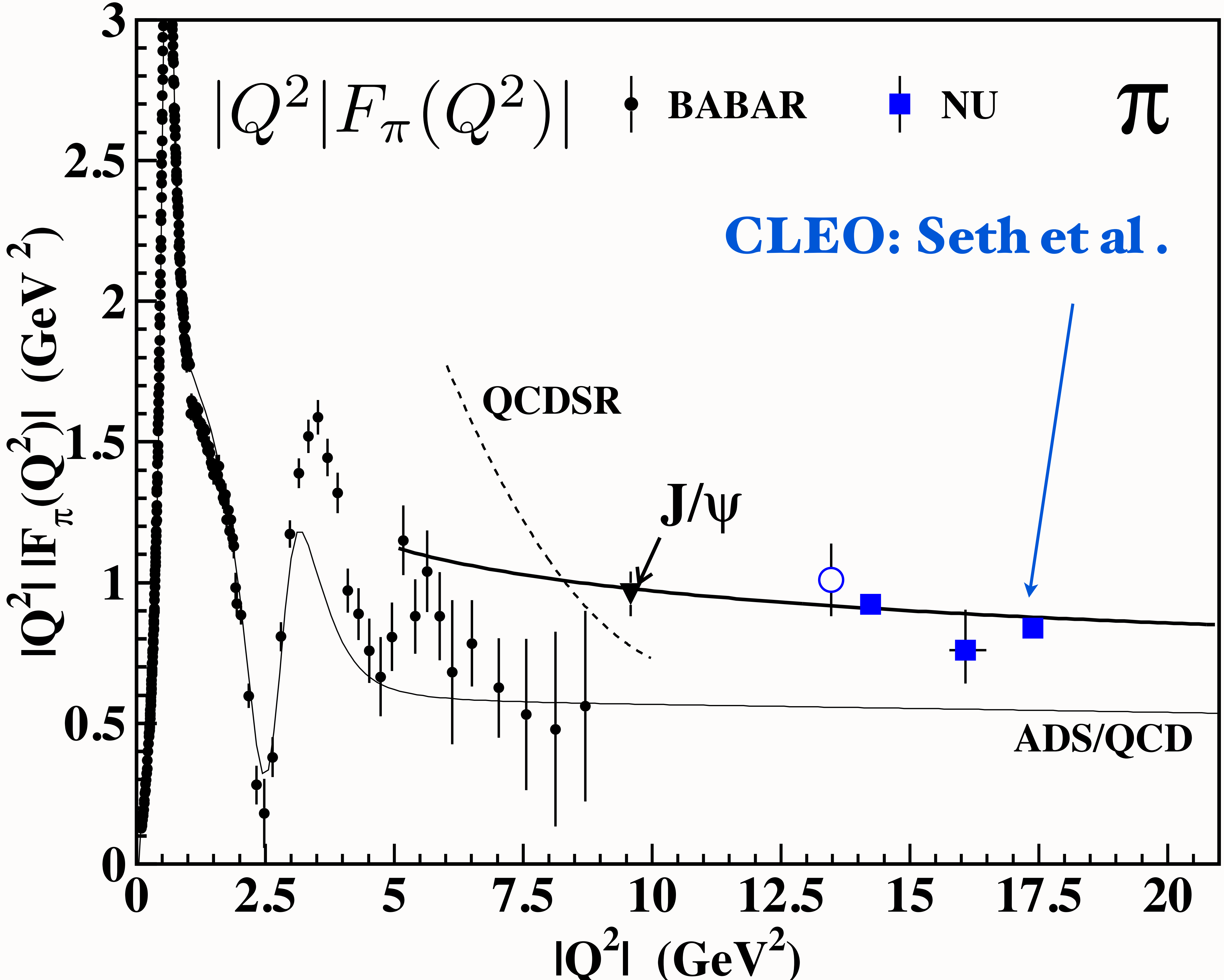
Guy de Teramond, sjb
preliminary
Spacelike pion form factor
from AdS/QCD
Radius at large Q
different than at
small Q:
muonic hydrogen
Threshold effect from analytic
continuation of resonance widths
Analytic continuation to time-like region
$s \to s + i\sqrt{s}\,\Gamma(s)$
$\Gamma(s) = \Gamma\left[\frac{\rho(s)}{\rho(M)}\right]^{2J+1}$
$\rho(s) = \left(1 - \frac{4m_\pi^2}{s}\right)^{1/2} = p/\sqrt{s}$
$\rho$
$\rho, \rho' \rho''$
$\log F_\pi(Q^2)$
$Q^2(GeV^2)$
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.30
−0.25
−0.20
−0.15
−0.10
−0.05
0.00

Guy de Teramond, sjb
preliminary



# Consistent with log fall-off of pQCD

## *Timelike Pion Form Factor*

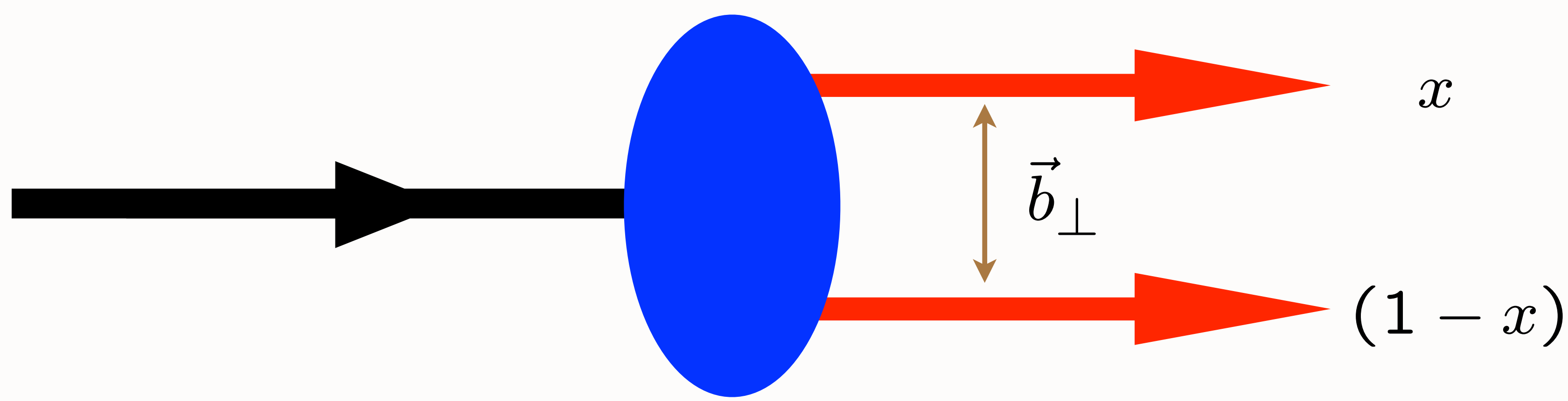


de Teramond, sjb

## LF(3+1) ⟷ $AdS_5$

$$\psi(x,\vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$




$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

*Light Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements*

- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

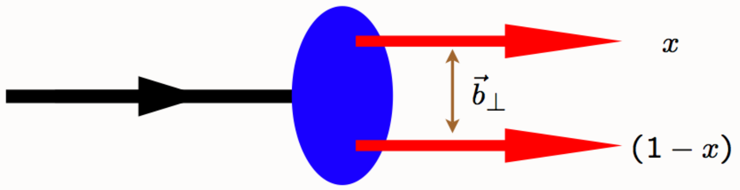
$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left( \frac{-\nabla^2_{\mathbf{b}_{\perp \ell}} + m_\ell^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Relevant variable conjugate to invariant mass in the limit of zero quark masses

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

  the $x$-weighted transverse impact coordinate of the spectator system (${x}$ active quark)

- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$




- To first approximation LF dynamics depend only on the invariant variable $\zeta$, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

  factoring angular $\varphi$, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

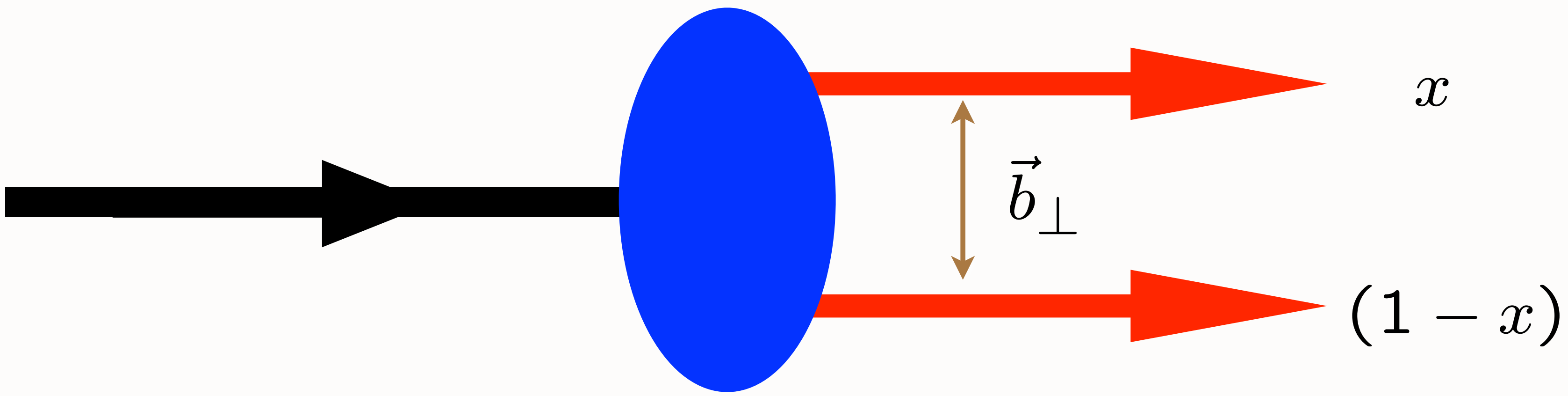
# Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$




$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

soft wall confining potential:

G. de Teramond, sjb

# Light-Front Schrödinger Equation

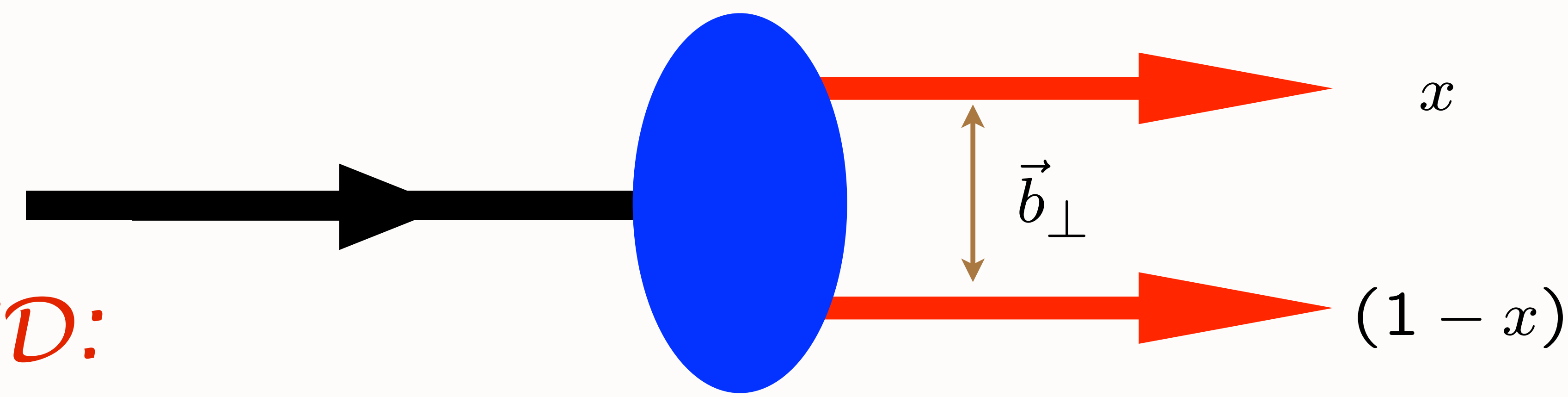
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \, \psi_{LF}(\zeta)$$

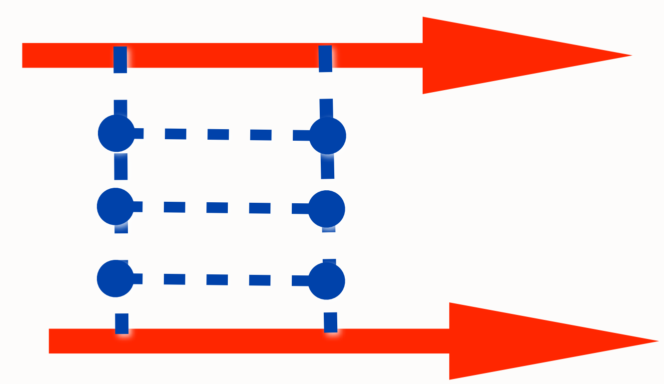
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



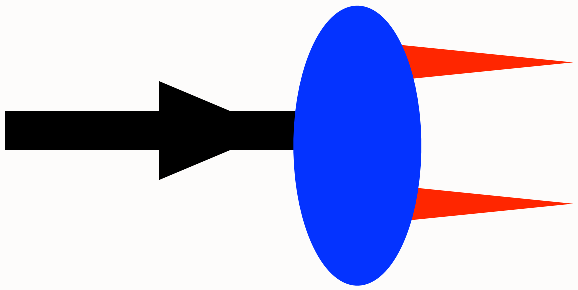

AdS/QCD:

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

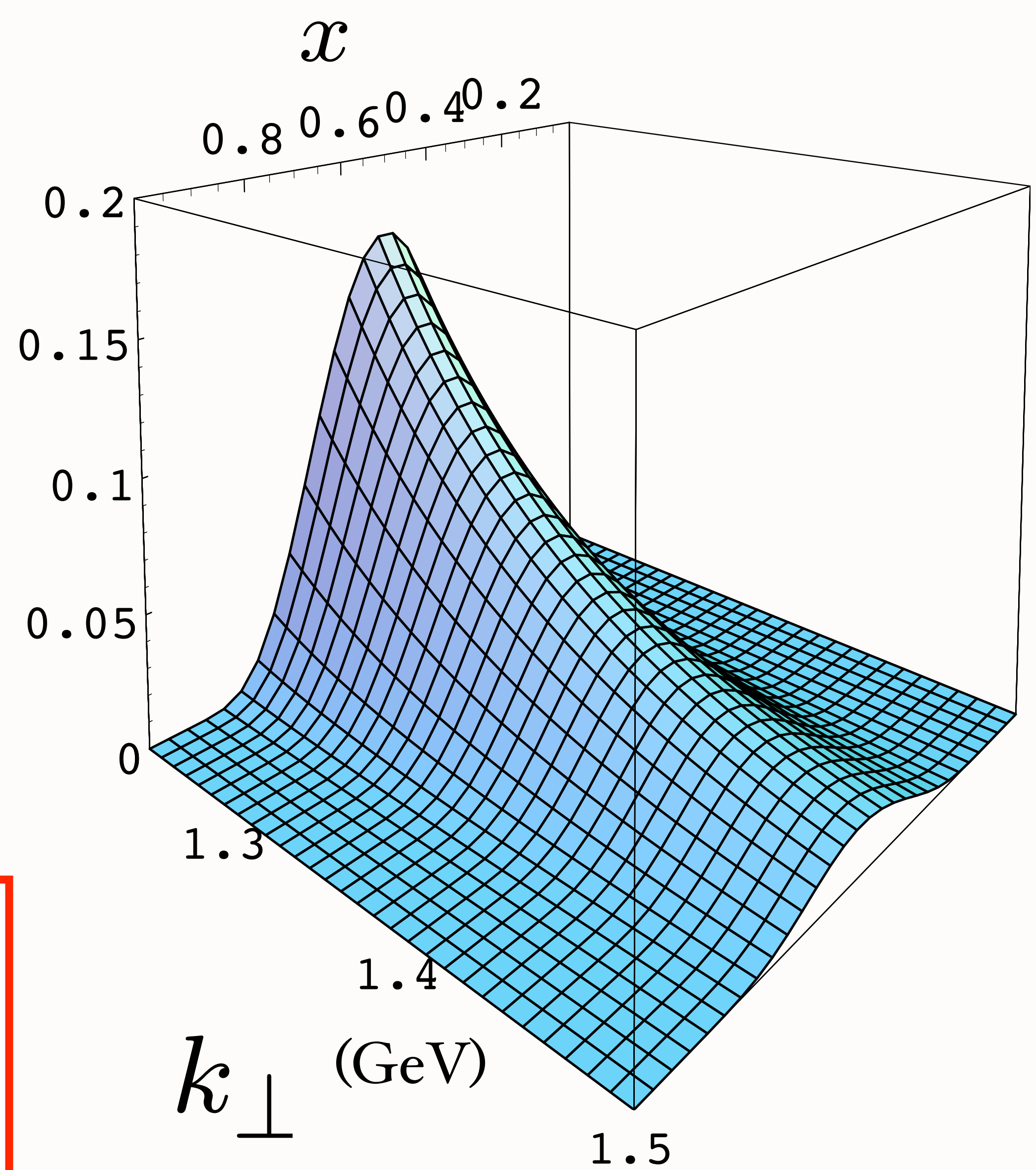
**U is the exact QCD potential**
**Conjecture: 'H'-diagrams generate**



# Prediction from AdS/CFT: Meson LFWF



$$\psi_M(x, k_\perp^2)$$




**de Teramond, sjb**

**"Soft Wall" model**

massless quarks

**Note coupling**

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

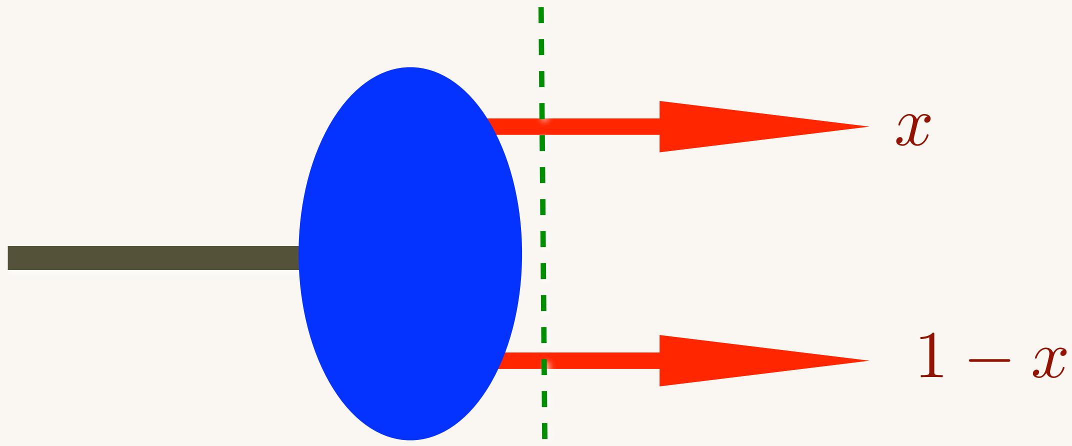
Connection of Confinement to TMDs

# Hadron Distribution Amplitudes

*Lepage, sjb*

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$




$k_\perp^2 < Q^2$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

*Lepage, sjb*
*Efremov, Radyushkin*

*Sachrajda, Frishman Lepage, sjb*

*Braun, Gardi*

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

# Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

# AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

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(Received 5 April 2012; published 20 August 2012)

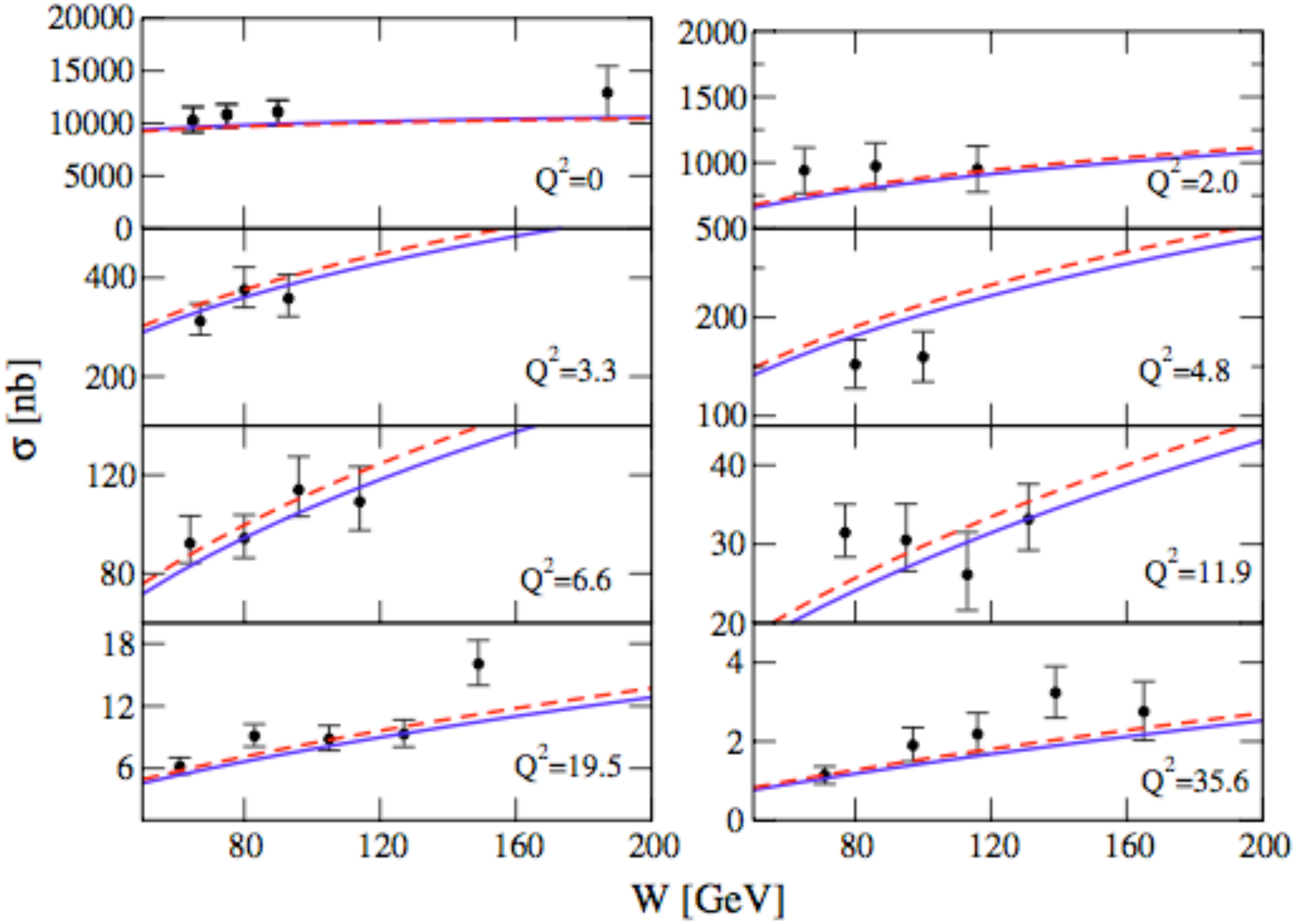
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive

$$\phi(x, \zeta) = \mathcal{N} \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)} \exp\left(-\frac{\kappa^2 \zeta^2}{2}\right),$$

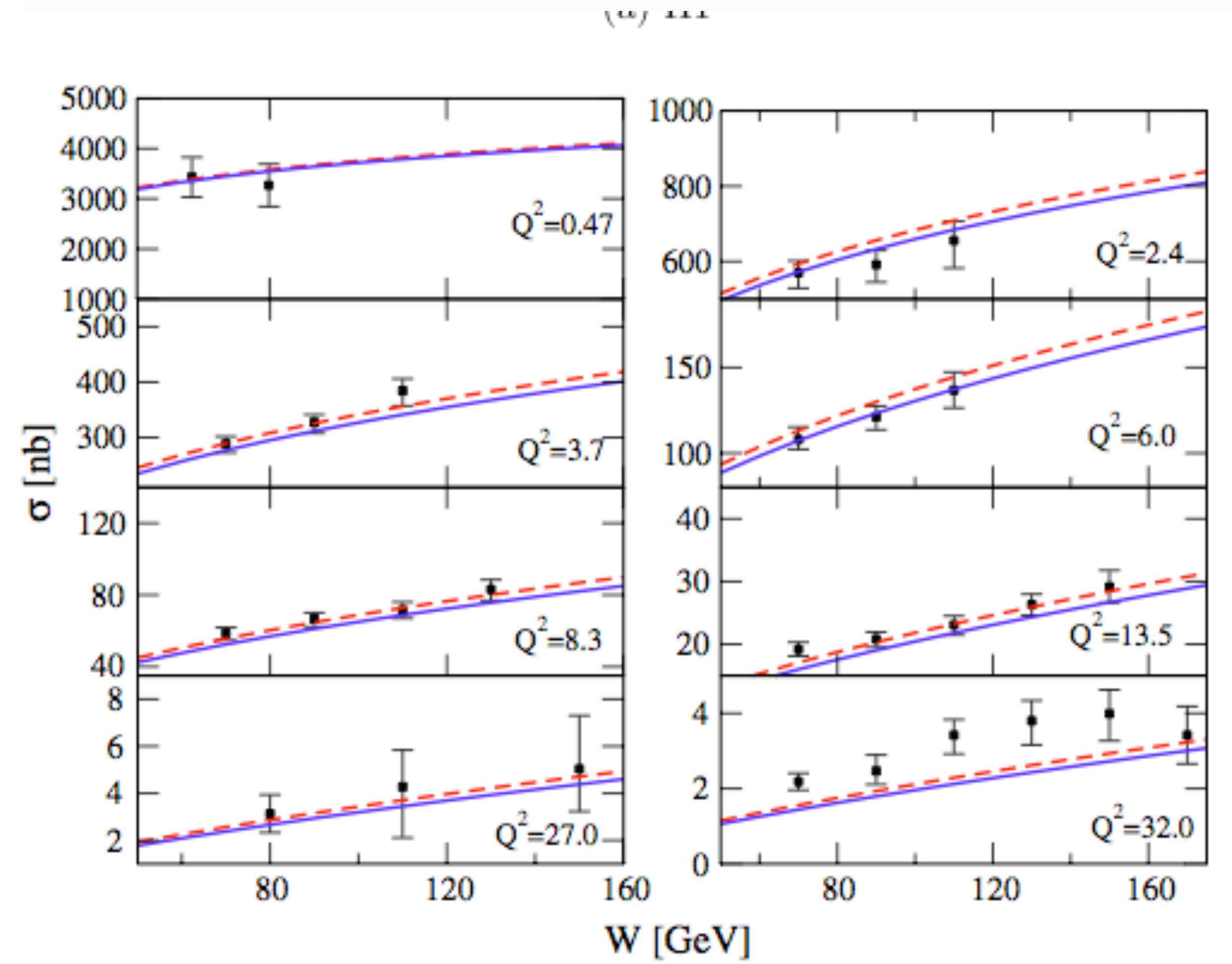
$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

# AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction



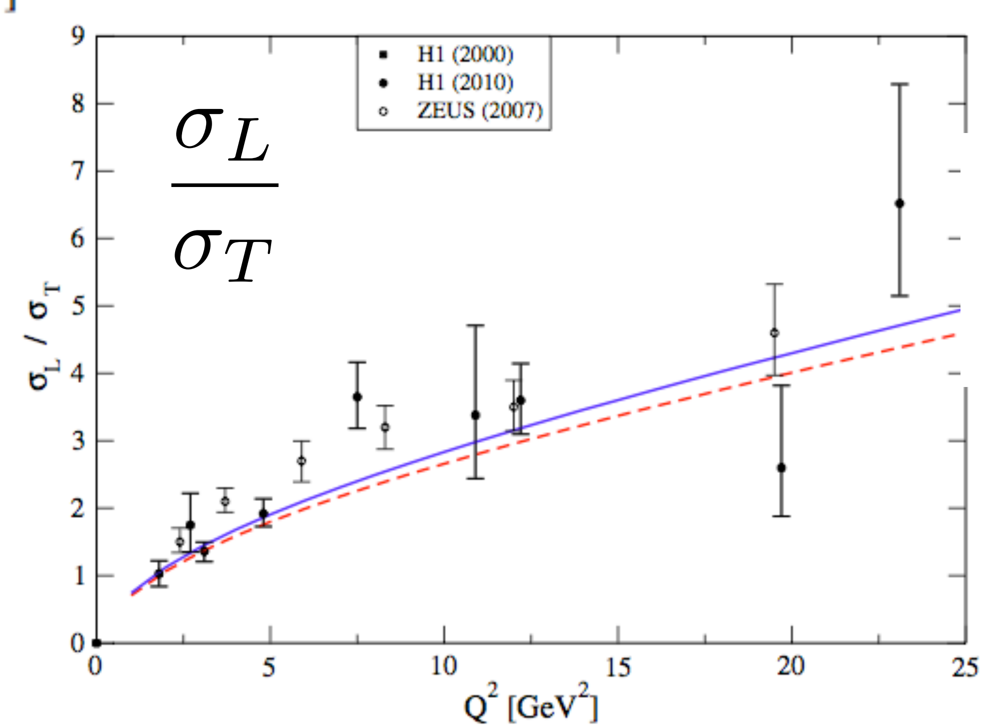

(a) H




(b) ZEUS

**J. R. Forshaw,**
**R. Sandapen**

$$\gamma^* p \to \rho^0 p'$$



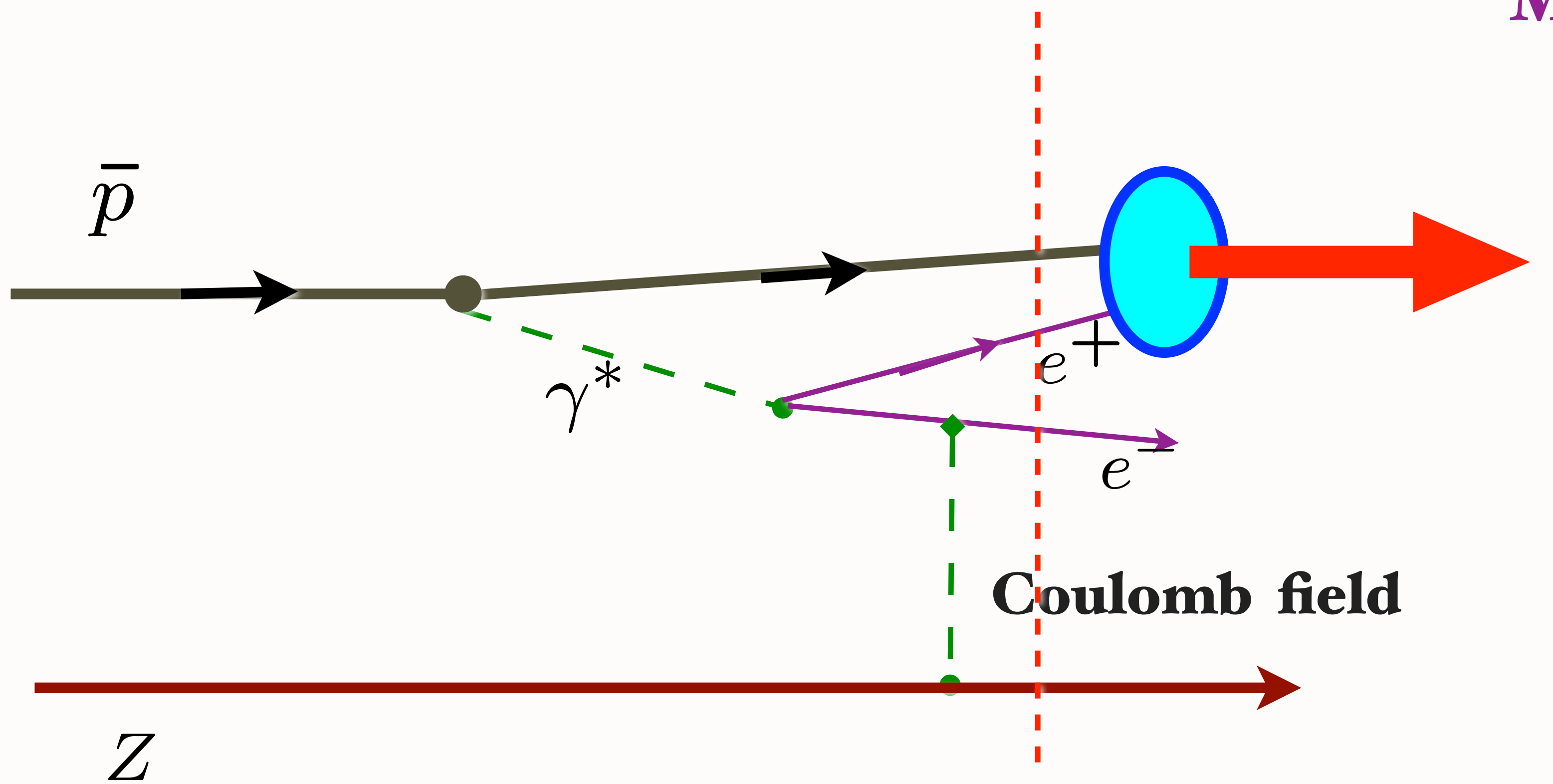

$$\frac{\sigma_L}{\sigma_T}$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

# Formation of Relativistic Anti-Hydrogen

**Measured at CERN-LEAR and FermiLab**

**Munger, Schmidt, sjb**




$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

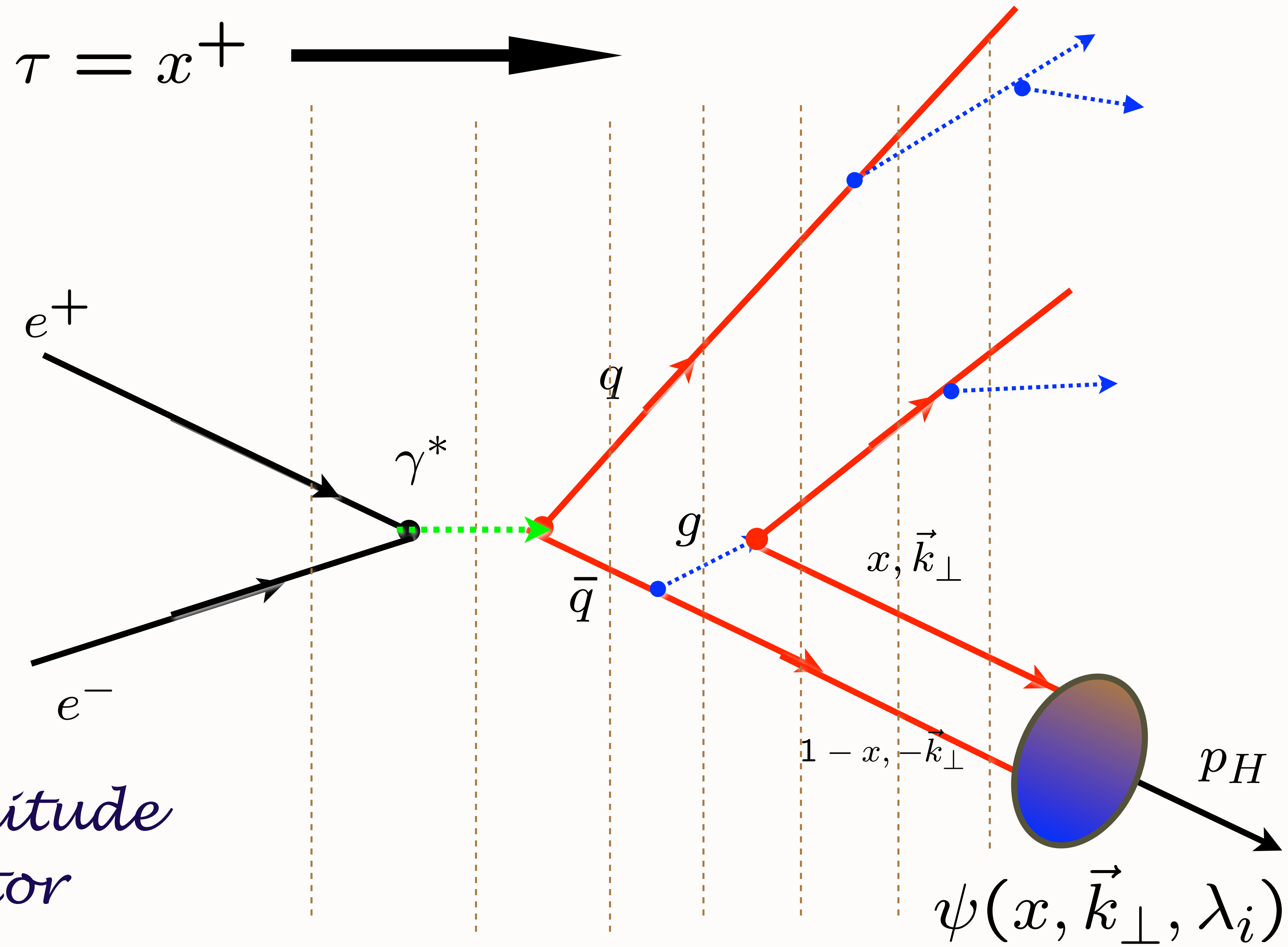
$$y_{\bar{p}} \simeq y_{e^+}$$

***Coalescence of** off-shell **co-moving positron and antiproton***

***Wavefunction maximal at small impact separation and equal rapidity***

"Hadronization" at the Amplitude Level

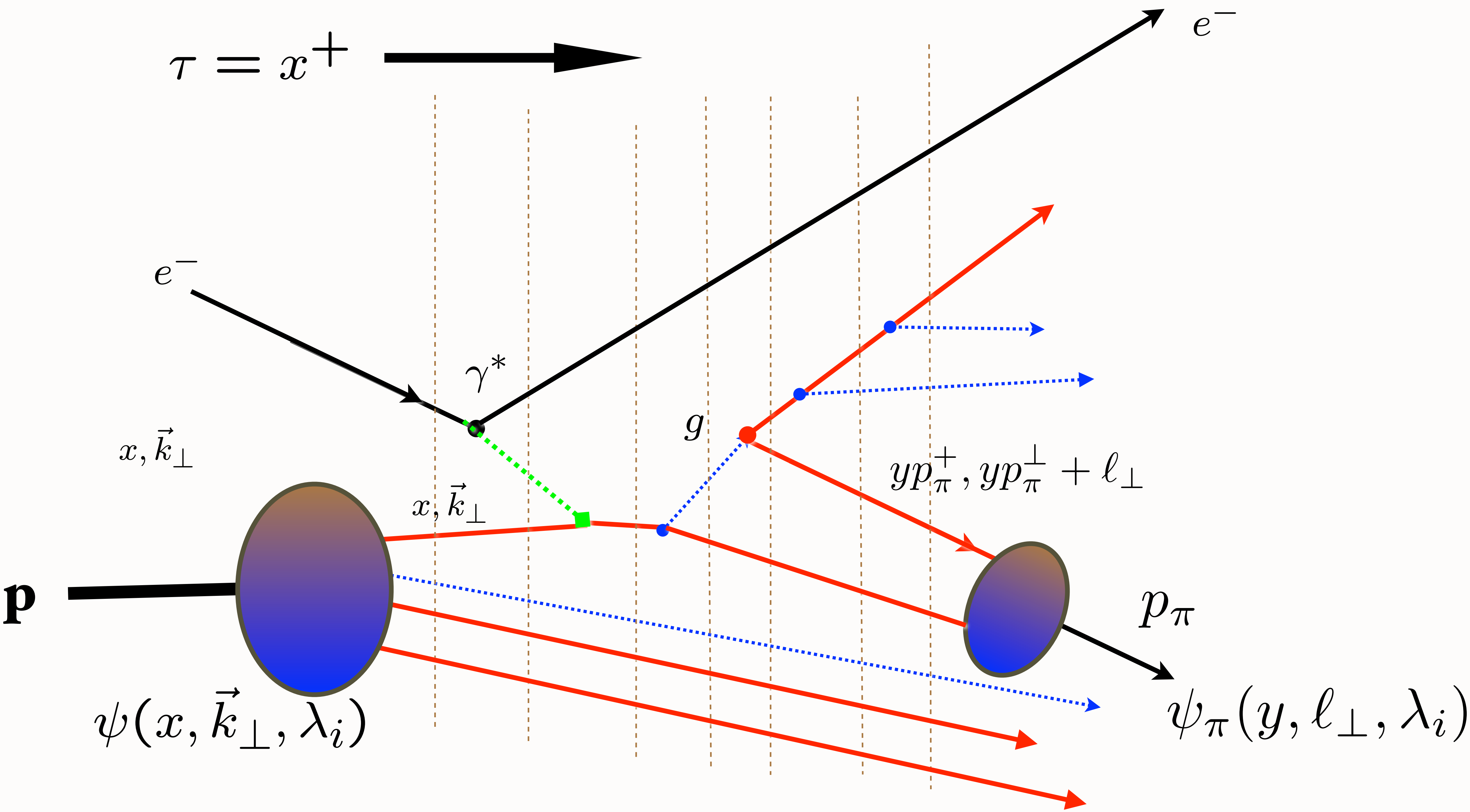
# Hadronization at the Amplitude Level




*Event amplitude generator*

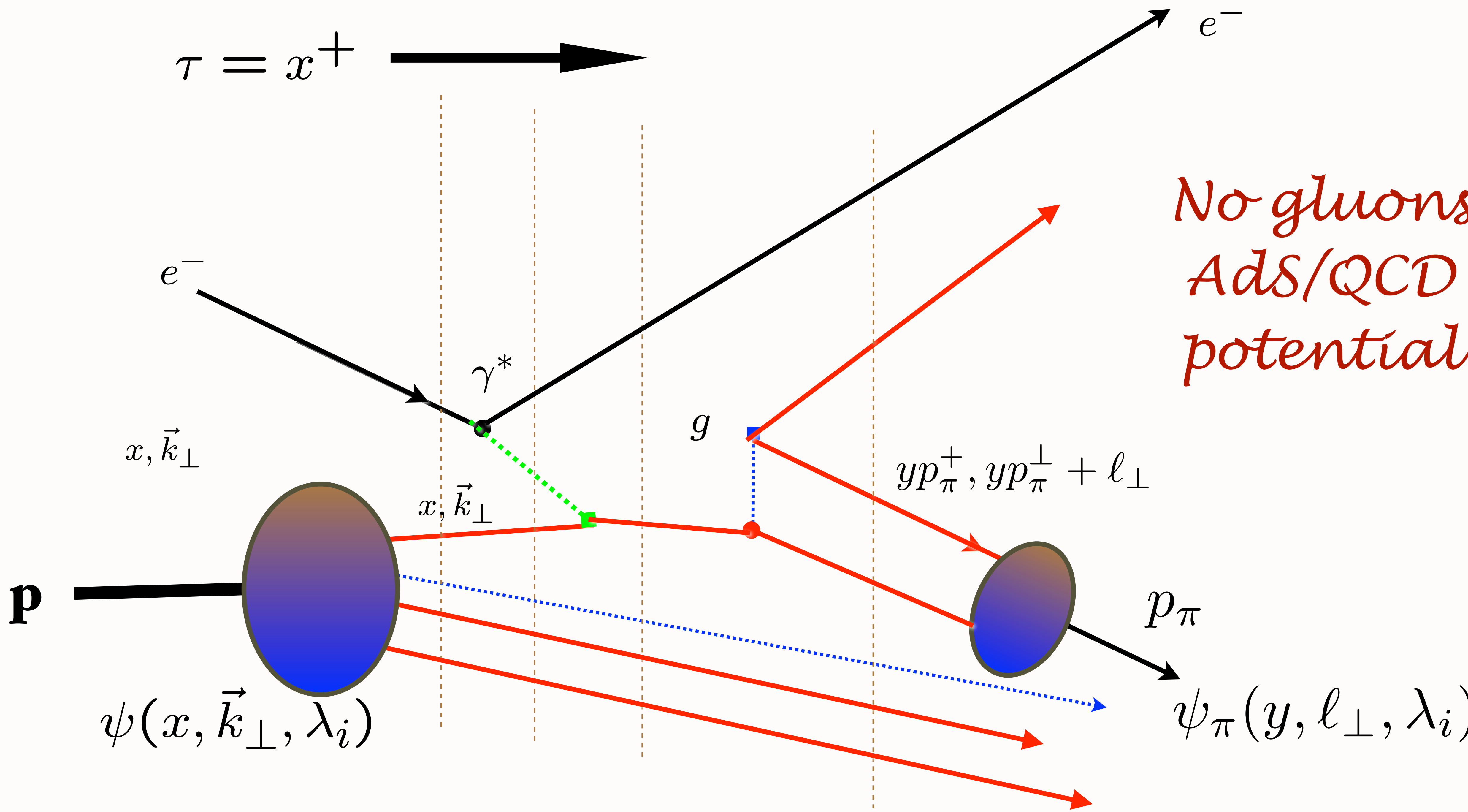
**Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs**

# Hadronization at the Amplitude Level




**Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs**

# Hadronization at the Amplitude Level



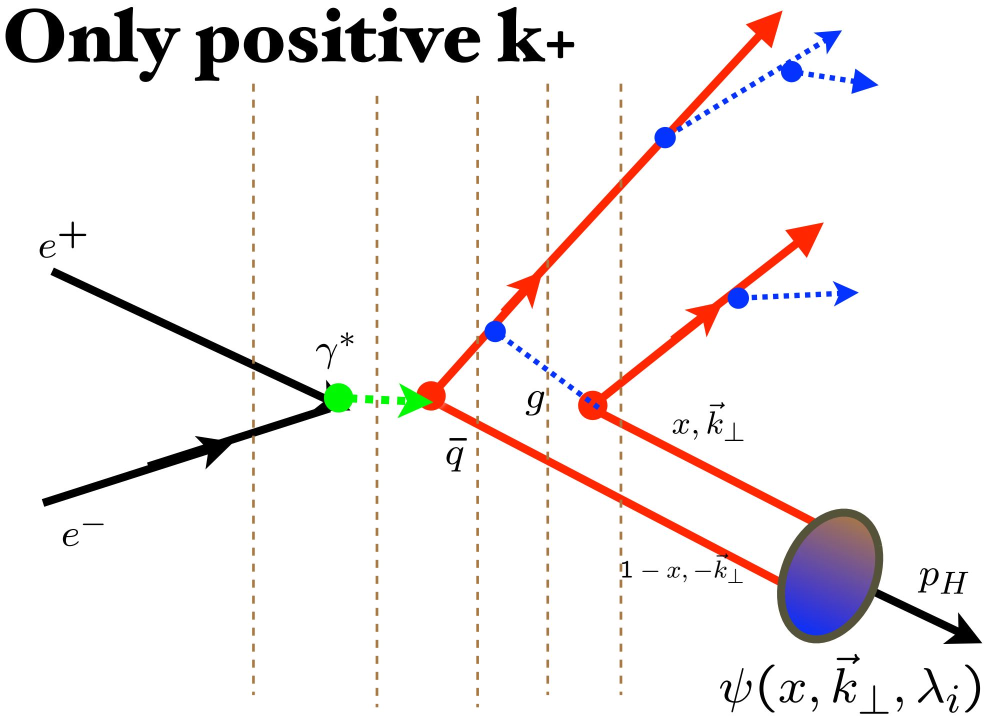

No gluons
AdS/QCD
potential

**Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs**

# Off -Shell T-Matrix

## *Event amplitude generator*

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k+**
- **$J^z$ Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** **Chueng Ji, sjb**
- **"History"-Numerator structure universal**
- **Renormalization- alternate denominators**
- **LFWF takes Off-shell to On-shell**
- **Tested in QED: g-2 to three loops**

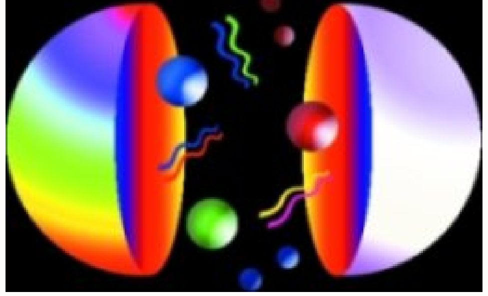



Roskies, Suaya, sjb

## Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g} e^{\varphi}(z) \left( i \overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z) \overline{\Psi} \Psi - \mu \overline{\Psi} \Psi \right)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x} \Psi(z)$

$$\left[ i \left( z \eta^{\ell m} \Gamma_\ell \partial_m + 2 \Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.$$

- Solution $(\nu = \mu R - \frac{1}{2},\ \nu = L + 1)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2 (n + L + 1)$$

positive parity

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from $\Psi$ by shifting dimensions
- Large $N_C$: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\, \Lambda_{\rm QCD}$

# Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator $\Pi$

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

Soft Wall

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

- Eigenvalues

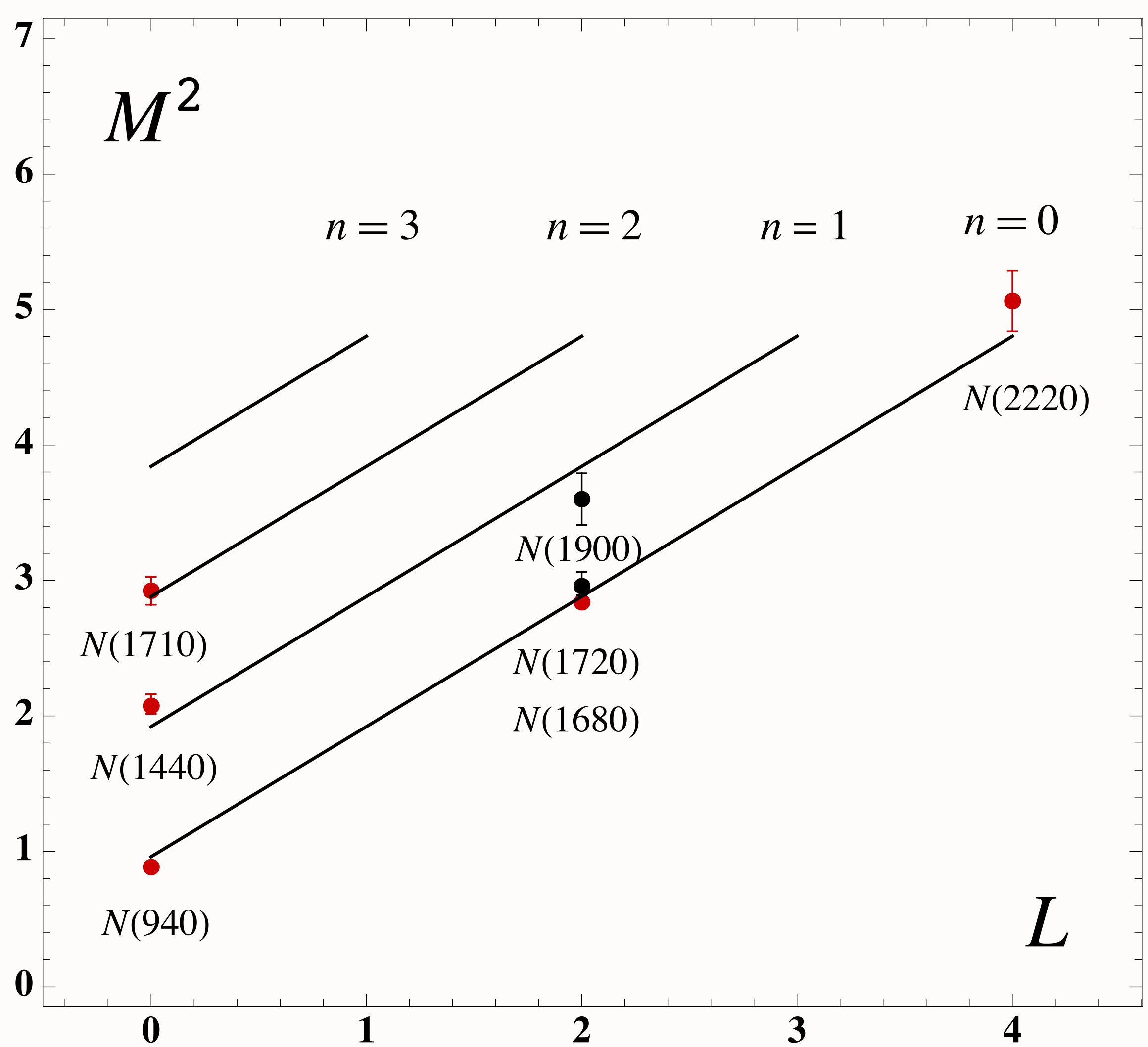
$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

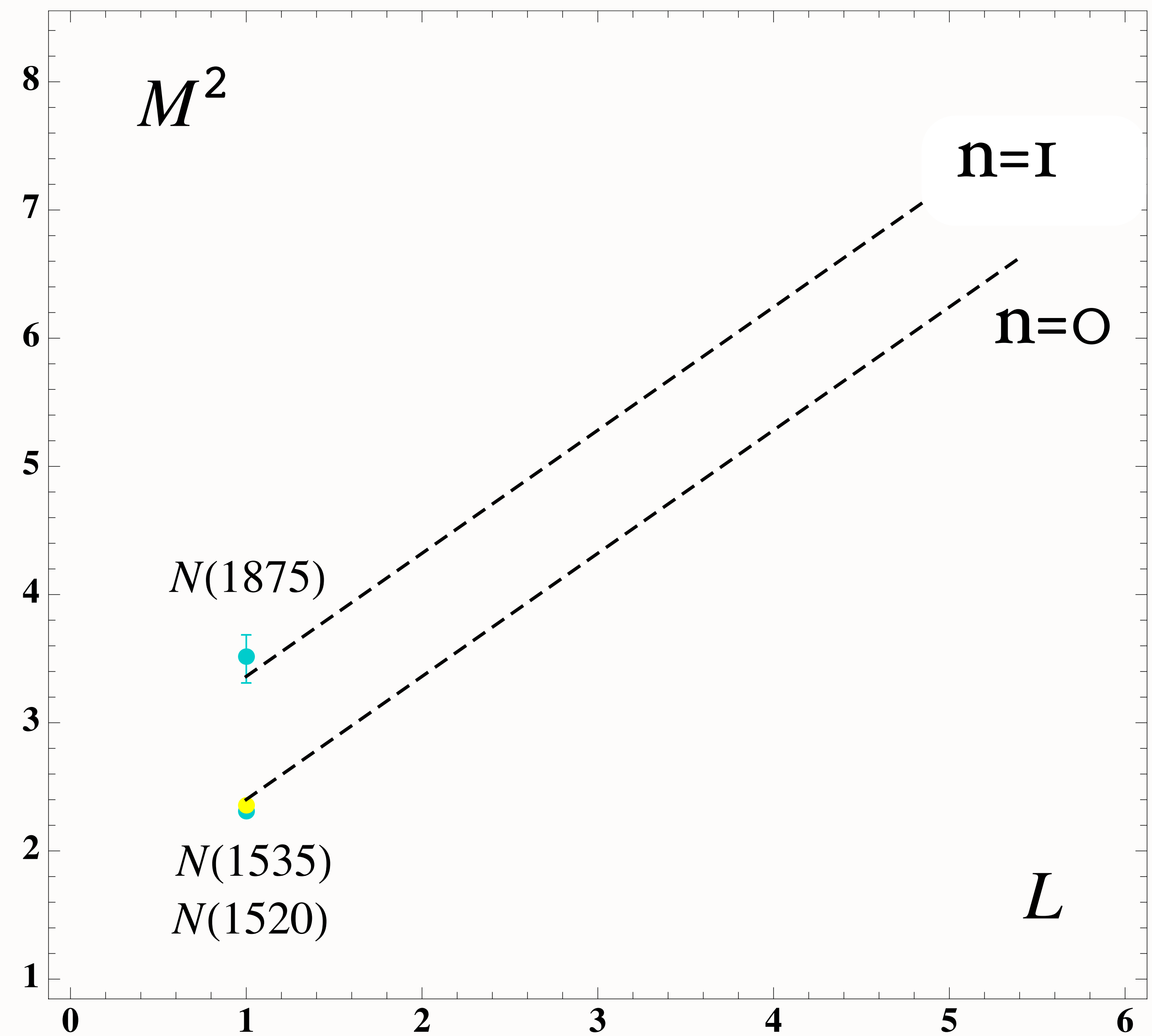
Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels $S$, $L$ and $n$ refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

| $SU(6)$ | $S$ | $L$ | $n$ | Baryon State |
|---|---|---|---|---|
| **56** | $\frac{1}{2}$ | 0 | 0 | $N\frac{1}{2}^+(940)$ |
| | $\frac{1}{2}$ | 0 | 1 | $N\frac{1}{2}^+(1440)$ |
| | $\frac{1}{2}$ | 0 | 2 | $N\frac{1}{2}^+(1710)$ |
| | $\frac{3}{2}$ | 0 | 0 | $\Delta\frac{3}{2}^+(1232)$ |
| | $\frac{3}{2}$ | 0 | 1 | $\Delta\frac{3}{2}^+(1600)$ |
| **70** | $\frac{1}{2}$ | 1 | 0 | $N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$ |
| | $\frac{3}{2}$ | 1 | 0 | $N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$ |
| | $\frac{3}{2}$ | 1 | 1 | $N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$ |
| | $\frac{1}{2}$ | 1 | 0 | $\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$ |
| **56** | $\frac{1}{2}$ | 2 | 0 | $N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$ |
| | $\frac{1}{2}$ | 2 | 1 | $N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$ |
| | $\frac{3}{2}$ | 2 | 0 | $\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$ |
| **70** | $\frac{1}{2}$ | 3 | 0 | $N\frac{5}{2}^-$ $N\frac{7}{2}^-$ |
| | $\frac{3}{2}$ | 3 | 0 | $N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$ |
| | $\frac{1}{2}$ | 3 | 0 | $\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$ |
| **56** | $\frac{1}{2}$ | 4 | 0 | $N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$ |
| | $\frac{3}{2}$ | 4 | 0 | $\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$ |
| **70** | $\frac{1}{2}$ | 5 | 0 | $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ |
| | $\frac{3}{2}$ | 5 | 0 | $N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$ |

**PDG 2012**

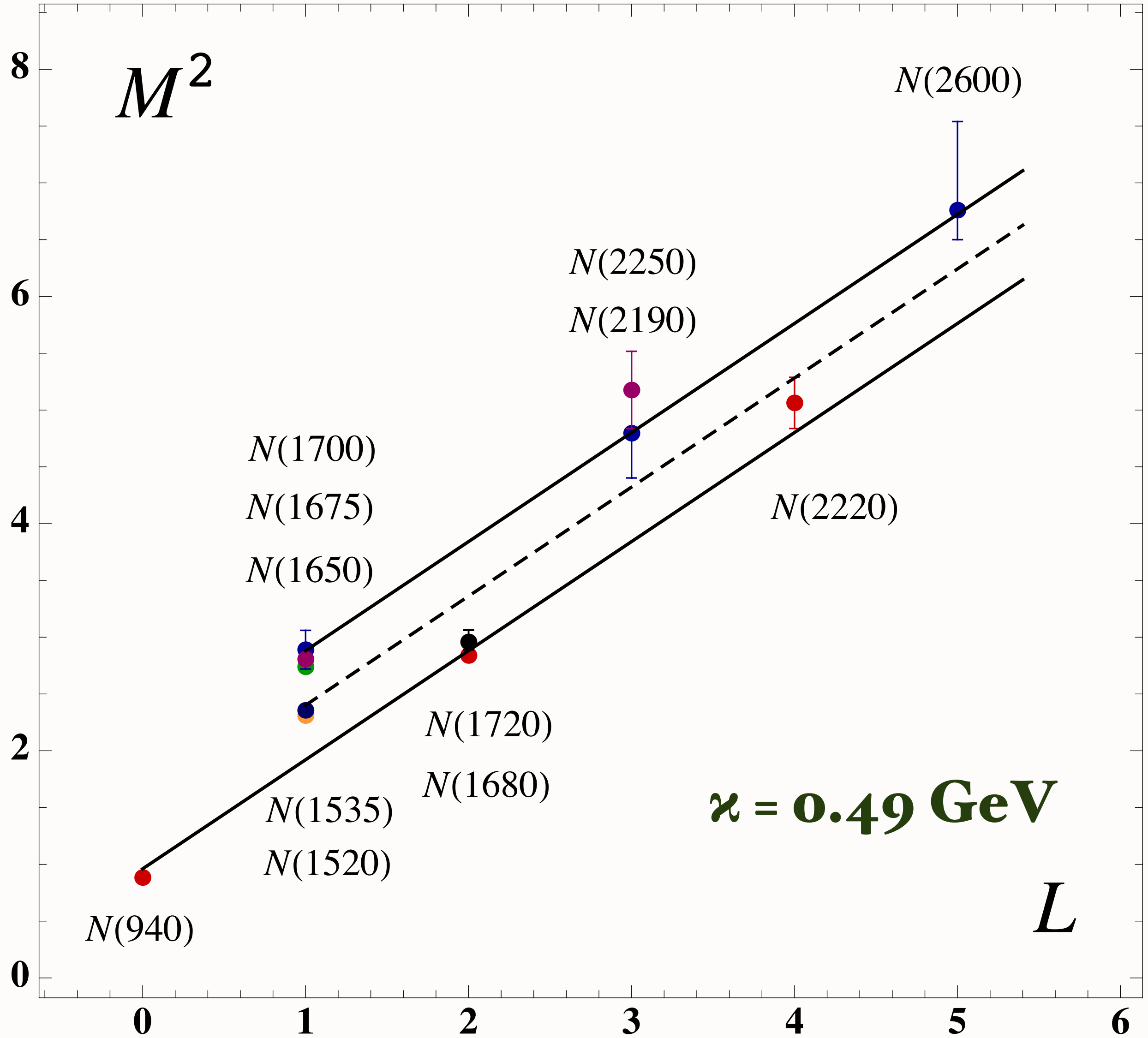
# PDG 2012

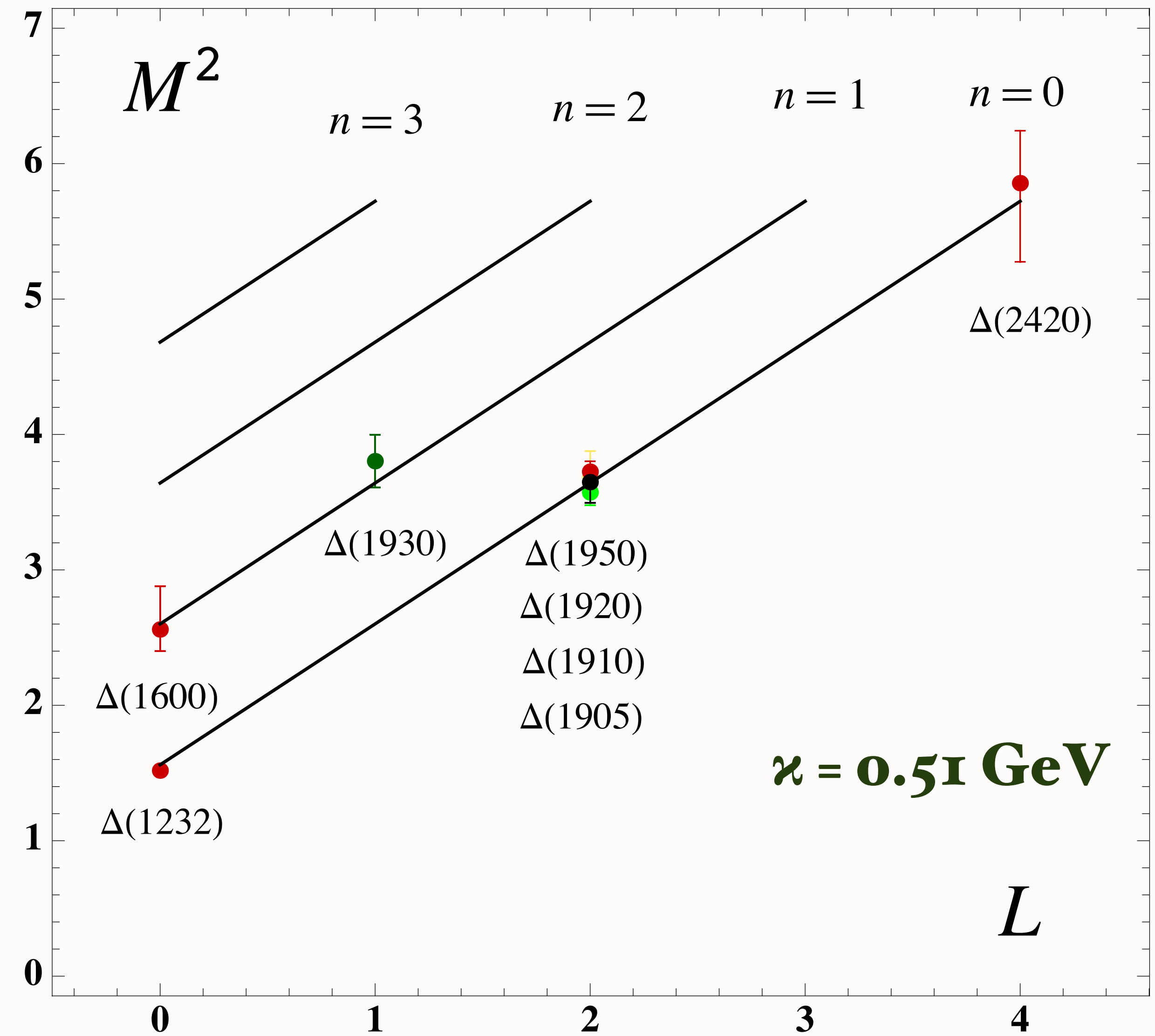



*LF Virial Theorem:*

*Nucleon Mass: 1/2 from LFKE*
*and 1/2 from Confinement Potential*



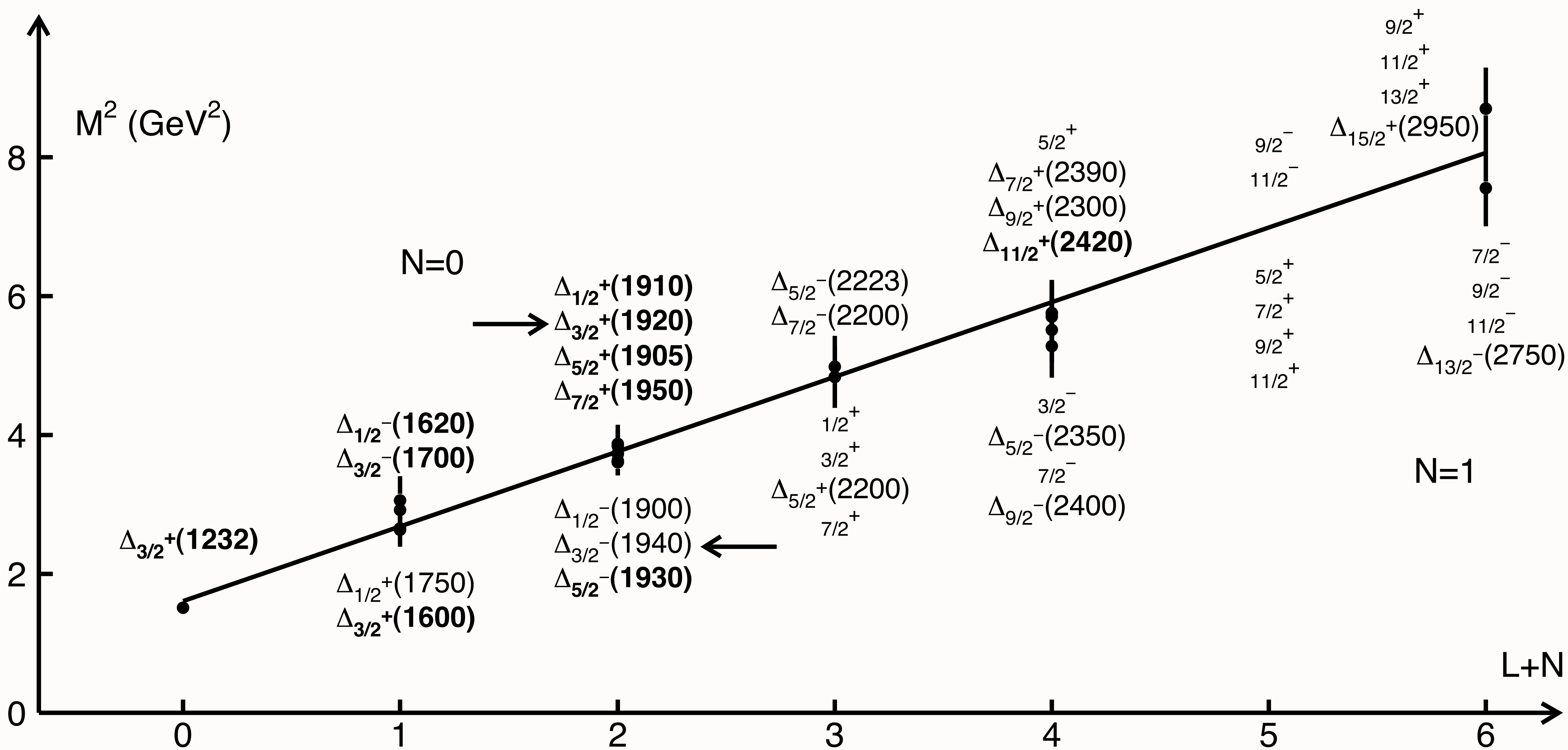

**de Teramond, sjb**

**All confirmed resonances from PDG 2012**

$$\mathcal{M}^{2\,(+)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{3}{4}\right),$$ **positive parity**

$$\mathcal{M}^{2\,(-)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{5}{4}\right),$$ **negative parity**

**See also Forkel, Beyer, Federico, Klempt**



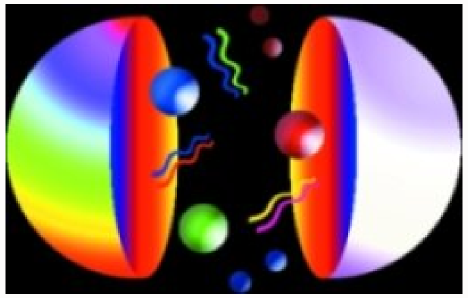

E. Klempt *et al.*: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

## Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

# Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum.**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different $L^z$**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

## Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges $g_+$ and $g_-$ are determined from the spin-flavor structure of the theory.

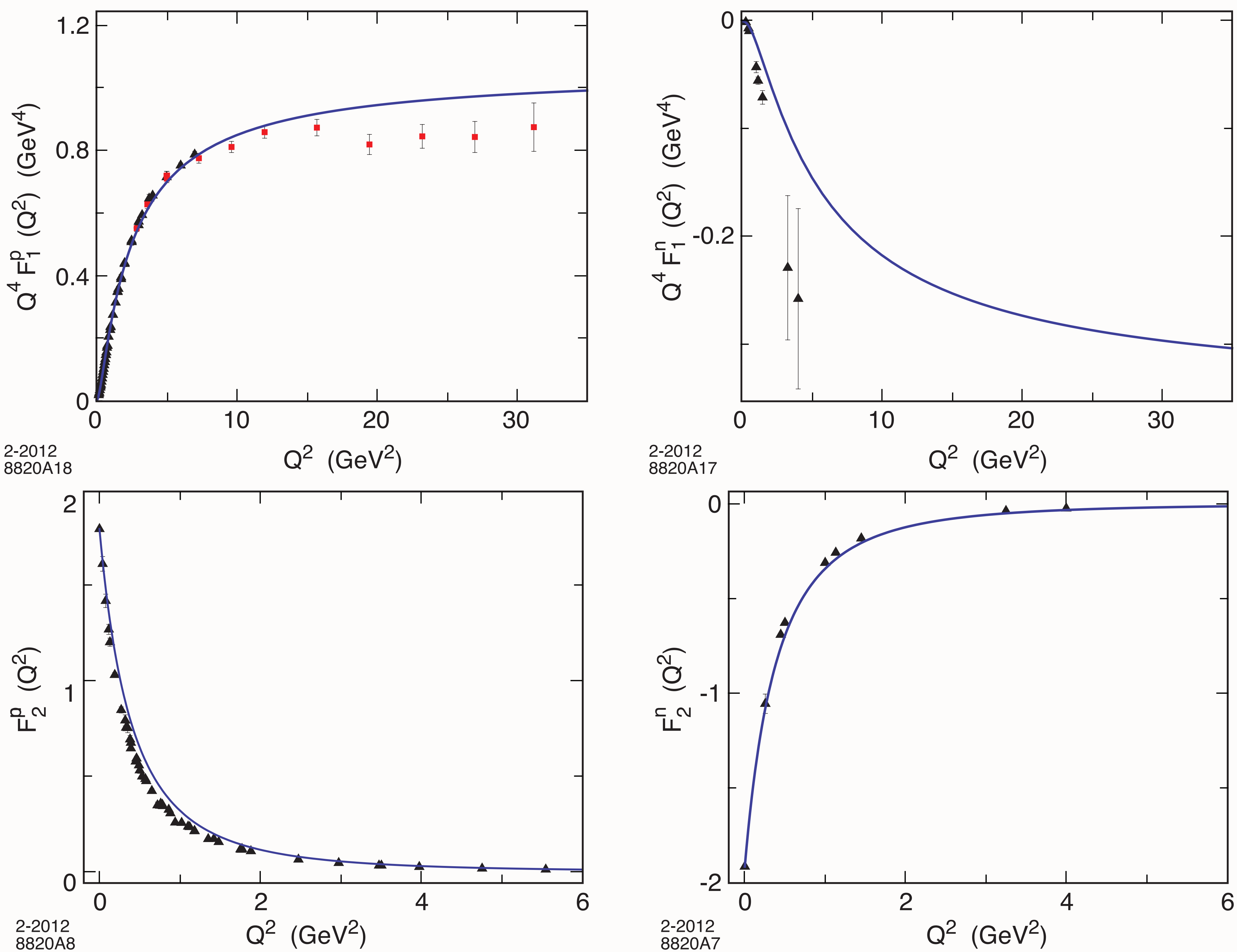
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

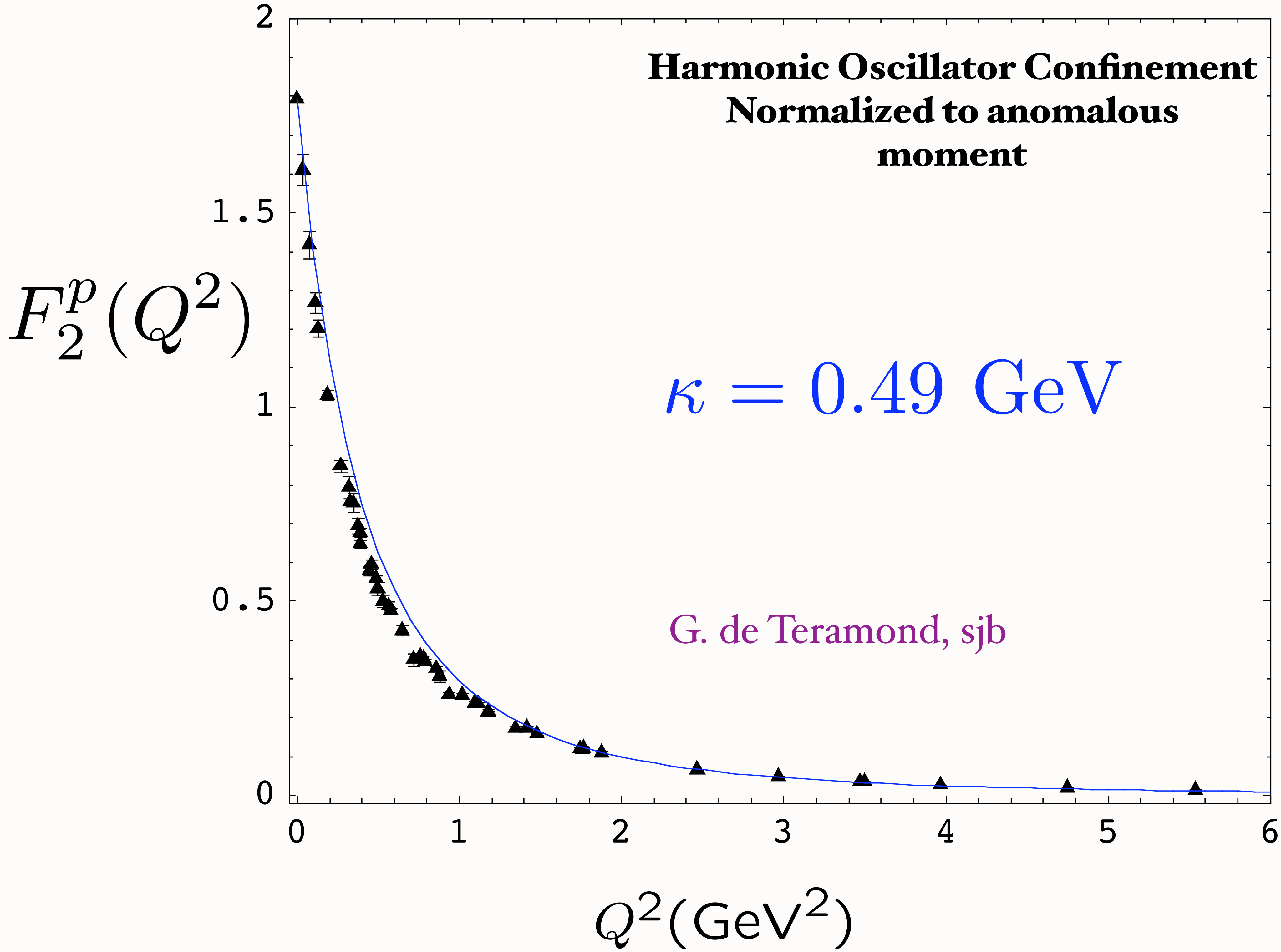
Using $SU(6)$ flavor symmetry and normalization to static quantities



# Spacelike Pauli Form Factor

*Preliminary*

From overlap of $L = 1$ and $L = 0$ LFWFs



# Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_1{}^p_{N \to N^*}(Q^2) = R^4 \int \frac{dz}{z^4} \, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\quad \left(F_1{}^p_{N \to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_1{}^p_{N \to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n + 1/2)$

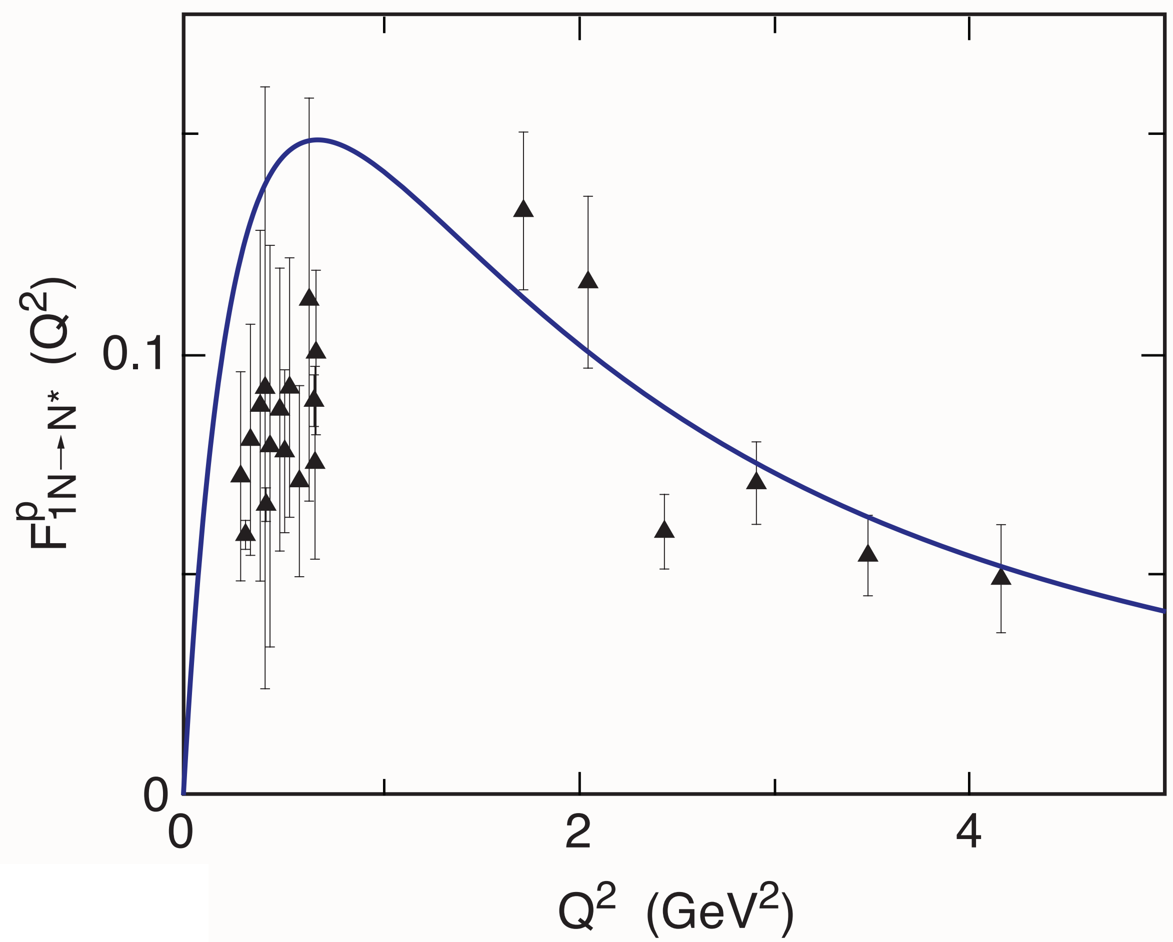
de Teramond, sjb

*Consistent with counting rule, twist 3*

# Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\,\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$

AdS\QCD
Light-Front
Holography

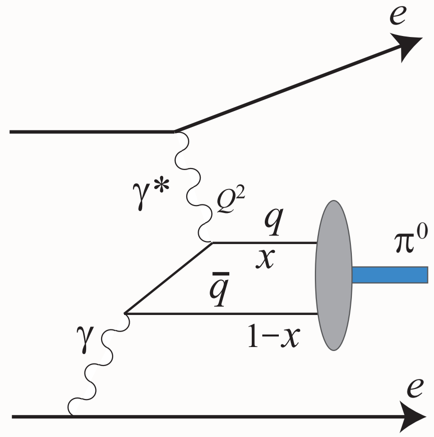



G. de Teramond, sjb

Proton transition form factor to the first radial excited state. Data from JLab

## Pion Transition Form-Factor

**Cao, de Teramond, sjb**




- Definition of $\pi - \gamma$ TFF from $\gamma^* \pi^0 \to \gamma$ vertex in the amplitude $e\pi \to e\gamma$

$$\Gamma^\mu = -ie^2 F_{\pi\gamma}(q^2) \epsilon_{\mu\nu\rho\sigma}(p_\pi)_\nu \epsilon_\rho(k) q_\sigma, \quad k^2 = 0$$

- Asymptotic value of pion TFF is determined by first principles in QCD:
  $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$ [Lepage and Brodsky (1980)]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)}\left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q)(p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Find for $A_z \propto \Phi_\pi(z)/z$

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\pi} \int_0^\infty \frac{dz}{z} \Phi_\pi(z) V\left(Q^2, z\right)$$

  with normalization fixed by asymptotic QCD prediction

- $V(Q^2, z)$ bulk-to-boundary propagator of $\gamma^*$

## Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)} \left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take $A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2P_{q\bar q}}\, \kappa\, z^2 e^{-\kappa^2 z^2/2}, \quad \langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar q}$
- Find $\left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \sqrt{P_{q\bar q}}\, \kappa/\sqrt{2}\pi\right)$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\bar q} Q^2 (1-x)/4\pi^2 f_\pi^2 \, x}\right]$$

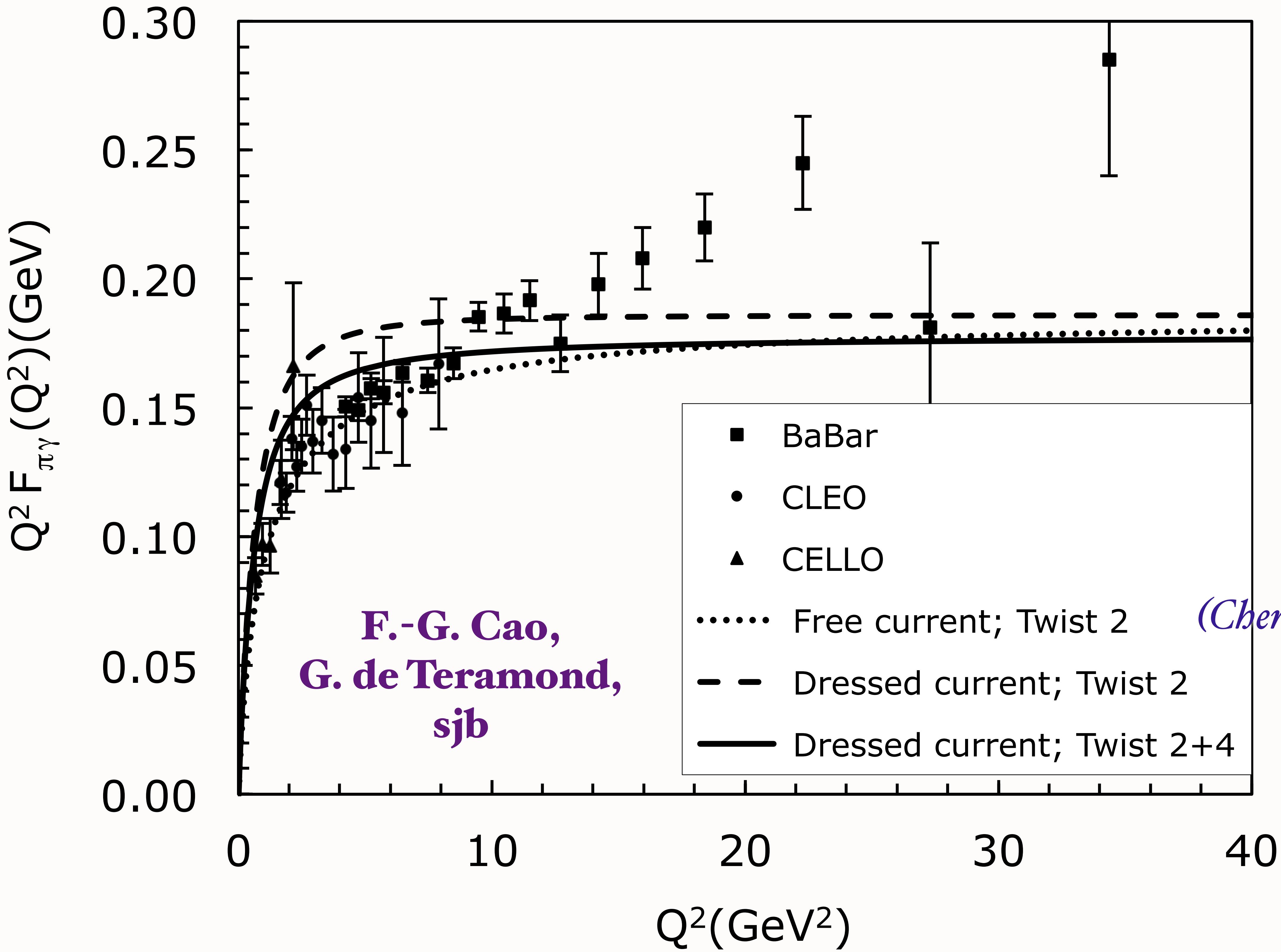
normalized to the asymptotic DA $[P_{q\bar q} = 1 \rightarrow$ Musatov and Radyushkin (1997)]

**G.P. Lepage, sjb**

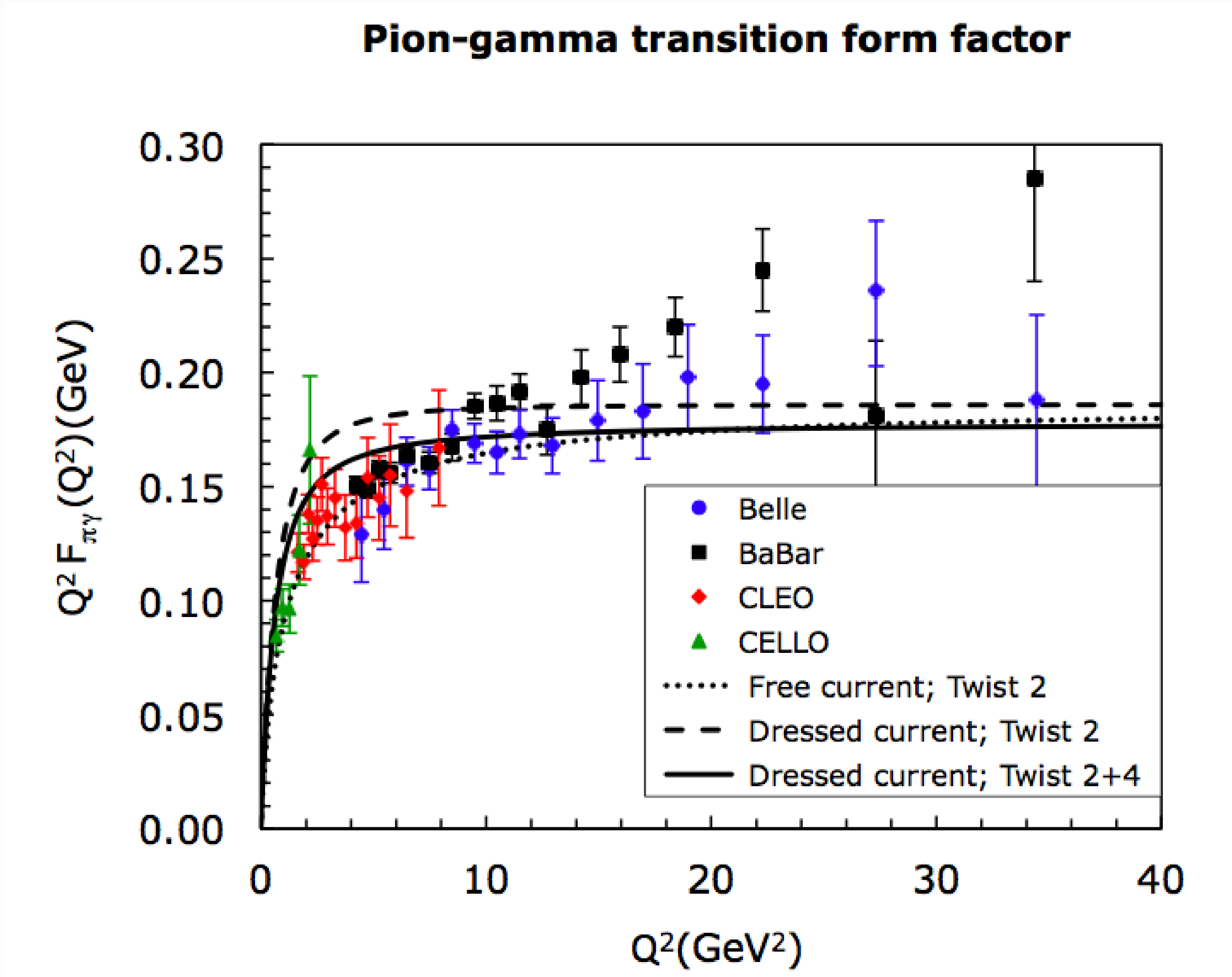
- Large $Q^2$ TFF is identical to first principles asymptotic QCD result $Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2 f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

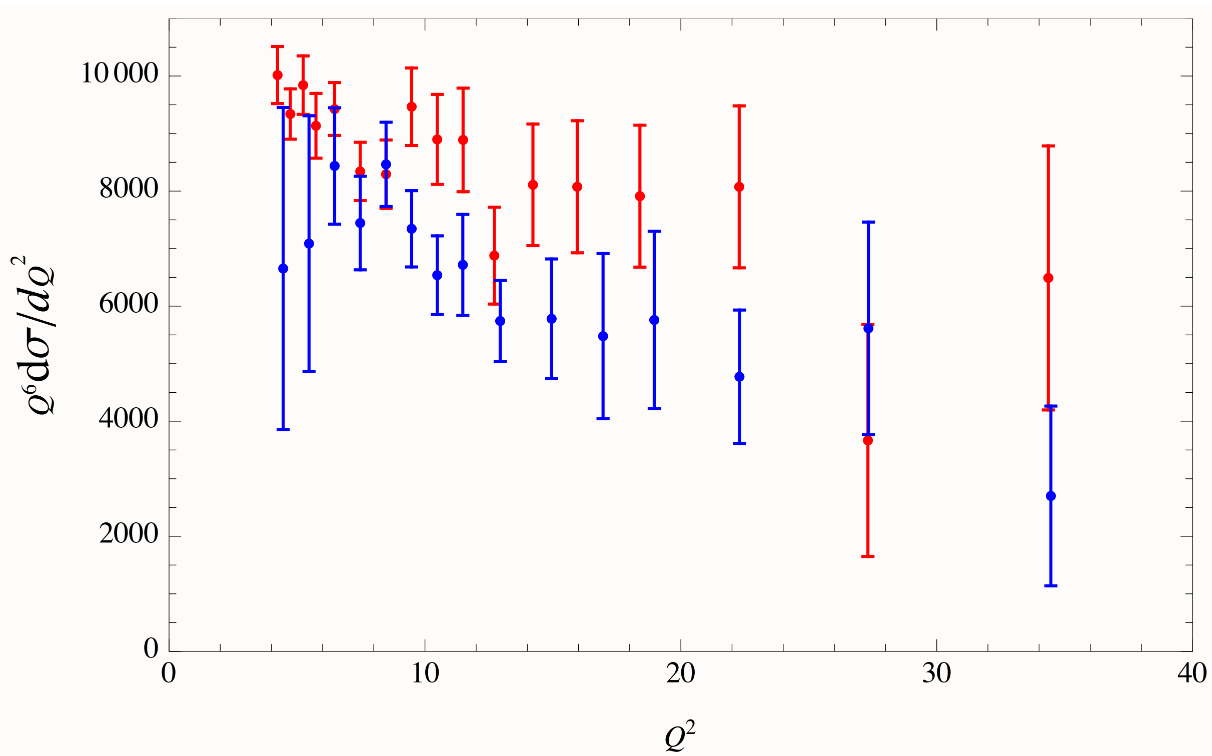
# *Photon-to-pion transition form factor*

Lepage, sjb $\quad Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$




*(Chern-Simons)*


Pion-gamma transition form factor
$Q^2 F_{\pi\gamma}(Q^2)$(GeV)
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40
$Q^2$(GeV$^2$)
Belle
BaBar
CLEO
CELLO
Free current; Twist 2
Dressed current; Twist 2
Dressed current; Twist 2+4


$Q^6 d\sigma/dQ^2$
10 000
8000
6000
4000
2000
0
0
10
20
30
40
$Q^2$

46

# Running Coupling from Modified AdS/QCD

**Deur, de Teramond, sjb**

- Consider five-dim gauge fields propagating in $AdS_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

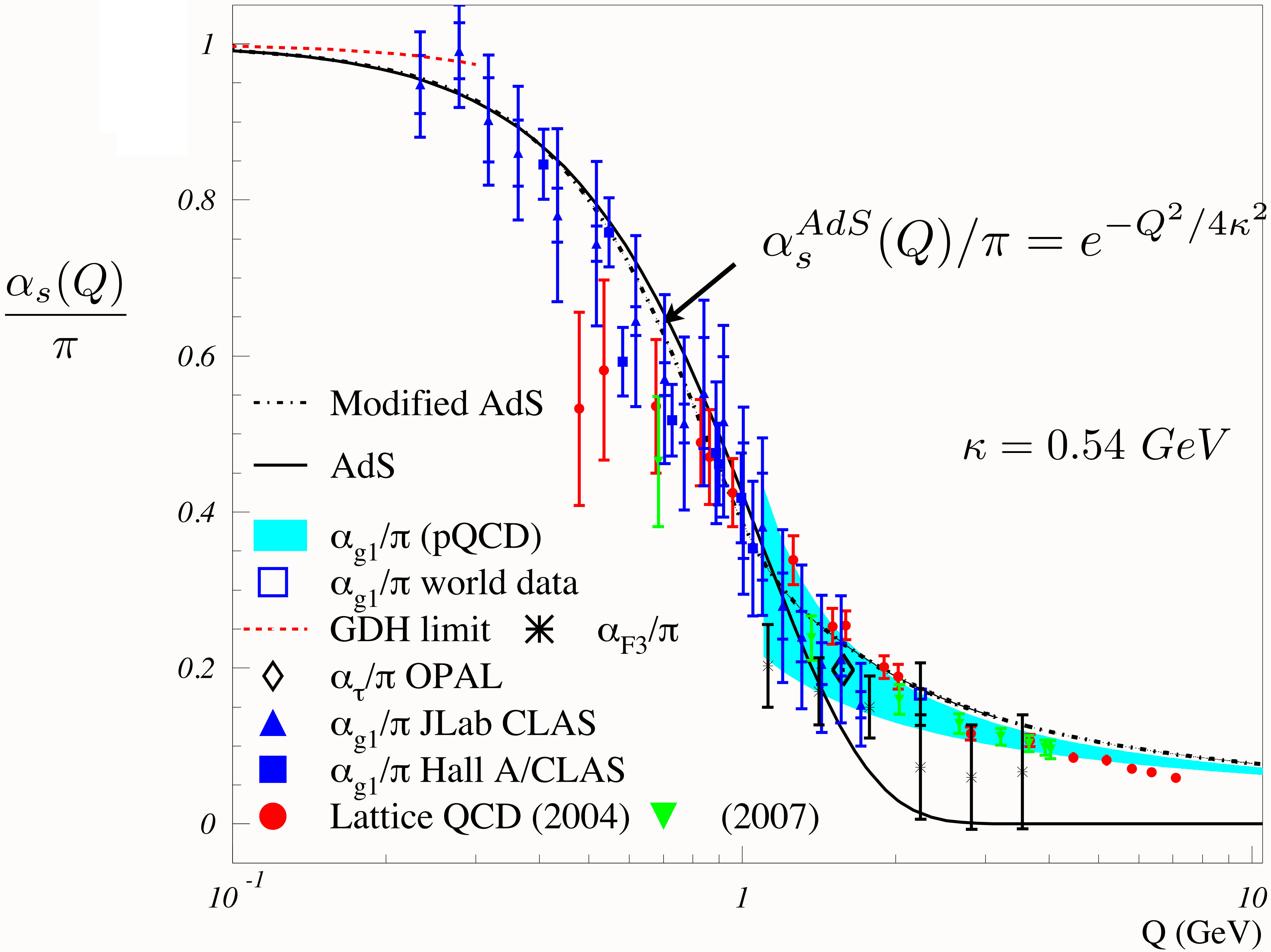
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling $\alpha_s^{AdS}$ incorporates the non-conformal dynamics of confinement

# Running Coupling from Light-Front Holography and AdS/QCD

## Analytic, defined at all scales, IR Fixed Point




**Deur, de Teramond, sjb**

# AdS/QCD and Light-Front Holography

- AdS/QCD: Incorporates scale transformations characteristic of QCD with a single scale -- RGE
- Light-Front Holography; unique connection of $AdS_5$ to Front-Form
- Profound connection between gravity in 5th dimension and physical 3+1 space time at fixed LF time $\tau$
- Gives unique interpretation of $z$ in AdS to physical variable $\zeta$ in 3+1 space-time

# An analytic first approximation to QCD

## *AdS/QCD + Light-Front Holography*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable $\zeta$ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter $\kappa$**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**

String Theory

↓

Goal: First Approximant to QCD

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Counting rules for Hard Exclusive Scattering
Regge Trajectories

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

QCD at the Amplitude Level

↓

Semi-Classical QCD / Wave Equations

↓

Holography

Boost Invariant 3+1 Light-Front Wave Equations

↓

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

# Applications of Light-Front holography

- **Diagonalize the LF Hamiltonian on the AdS/QCD basis**
- **Analytic form for two-photon reactions - analytic connection to DVCS - light-by-light contribution to g-2**
- **Set the factorization scale using AdS/QCD LFWFs**
- **Hadronization at the Amplitude Level**
- **Compute QCD amplitudes at the soft scale: e.g. Sivers SSA Asymmetry and Diffractive DDIS**
- **Sublimated gluons: Interplay of confinement and gluon exchange**
- **QCD puzzles: dominance of quark interchange in hard hadron-hadron scattering;** $J/\psi \to \rho\pi$

# *Gell-Mann Oakes Renner Formula in QCD*

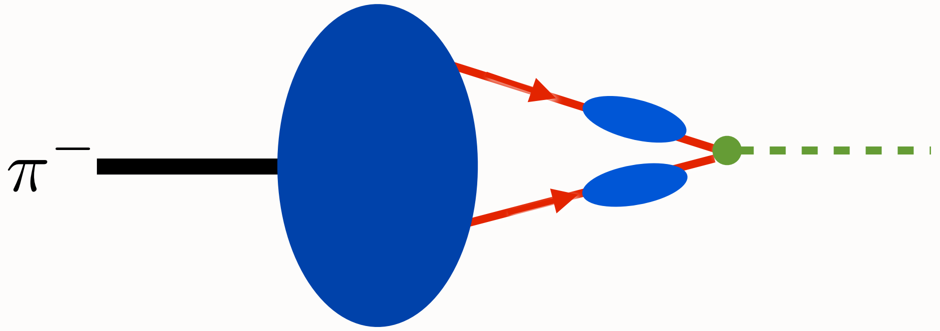
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

**current algebra:**
**effective pion field**

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

**QCD: composite pion**
**Bethe-Salpeter, LF**

*vacuum condensate actually is an "in-hadron condensate"*

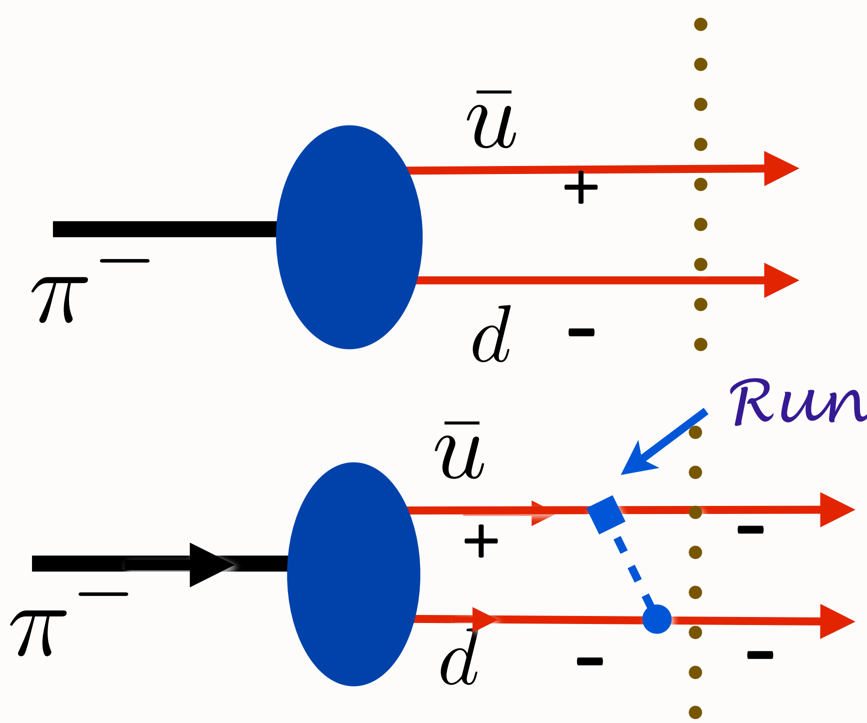



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

# Light-Front Pion Valence Wavefunctions

$S^z_{\bar{u}} + S^z_d = +1/2 - 1/2 = 0$




*Couples to*

$$L^z = 0, S^z = 0 \quad < \pi|\bar{q}\gamma^\mu\gamma_5 q|0 > \sim f_\pi$$

*Running constituent mass at vertex*

$$L^z = +1, S^z = -1 \quad < \pi|\bar{q}\gamma_5 q|0 > \ \sim \rho_\pi$$

$S^z_{\bar{u}} + S^z_d = -1/2 - 1/2 = -1$

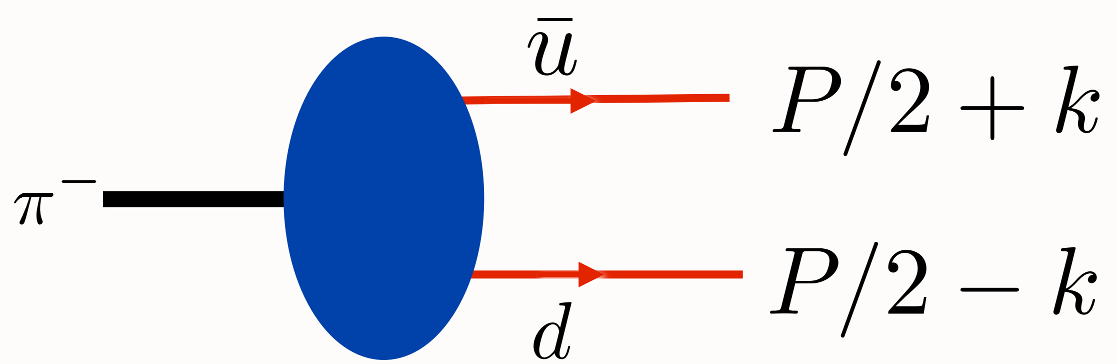
**Angular Momentum Conservation**

$$J^z = \sum_i^n S^z_i + \sum_i^{n-1} L^z_i$$

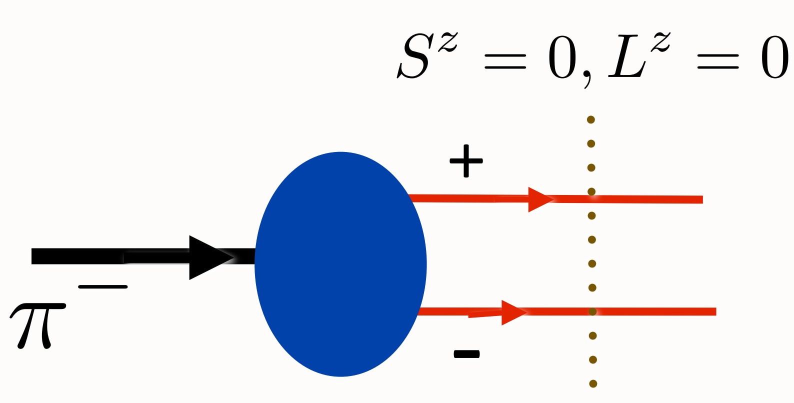
# General Form of Bethe-Salpeter Wavefunction

$$\Gamma_\pi(k;P) = i\gamma_5 E_\pi(k,P) + \gamma_5 \gamma\cdot P F_\pi(k;P)$$
$$+\gamma_5\gamma\cdot k G_\pi(k;P) - \gamma_5 \sigma_{\mu\nu} k^\mu P^\nu H_\pi(k;P)$$

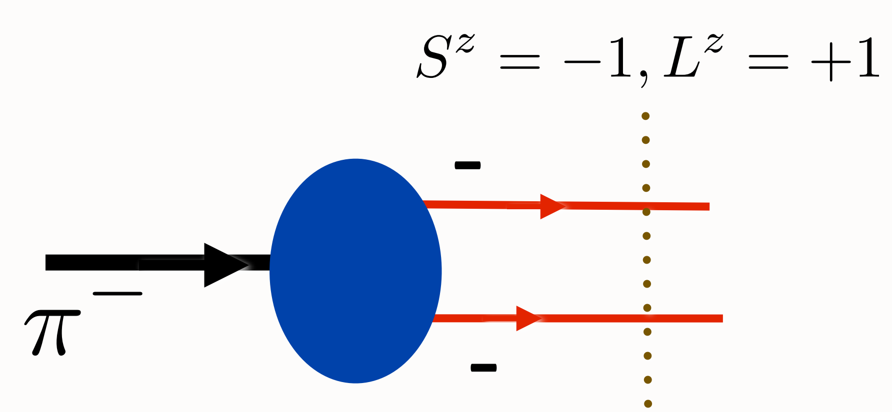
$\Gamma_\pi(k;P)$




Allows both $<0|\bar{q}\gamma_5\gamma_\mu q|\pi>$ and $<0|\bar{q}\gamma_5 q|\pi>$




LFWFs



# New perspective on QCD `Condensates'

- Condensates do not exist as space-time-independent phenomena

- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: "In-Hadron Condensates"   **Maris, Roberts, Tandy**

- Find: $$\frac{< 0|\bar{q}q|0 >}{f_\pi} \rightarrow - < 0|i\bar{q}\gamma_5 q|\pi >= \rho_\pi$$

$$< 0|\bar{q}i\gamma_5 q|\pi > \text{ similar to } < 0|\bar{q}\gamma^\mu\gamma_5 q|\pi >$$

- Zero contribution to cosmological constant!   Included in hadron mass

- $\rho_\pi$ survives for small $m_q$ -- enhanced running mass from gluon loops / multiparton Fock states

PHYSICAL REVIEW C **82**, 022201(R) (2010)

# New perspectives on the quark condensate

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(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

# “One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE
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*Kavil Institute for Theoretical Physics, University of California,*
*Santa Barbara, CA 93106, USA*
*zee@kitp.ucsb.edu*

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

**R. Shrock, sjb**

**arXiv:0905.1151 [hep- th], Proc. Nat’l. Acad. Sci.,**
**``Condensates in Quantum Chromodynamics and the Cosmological Constant.”**

# Light-Front vacuum can simulate empty universe

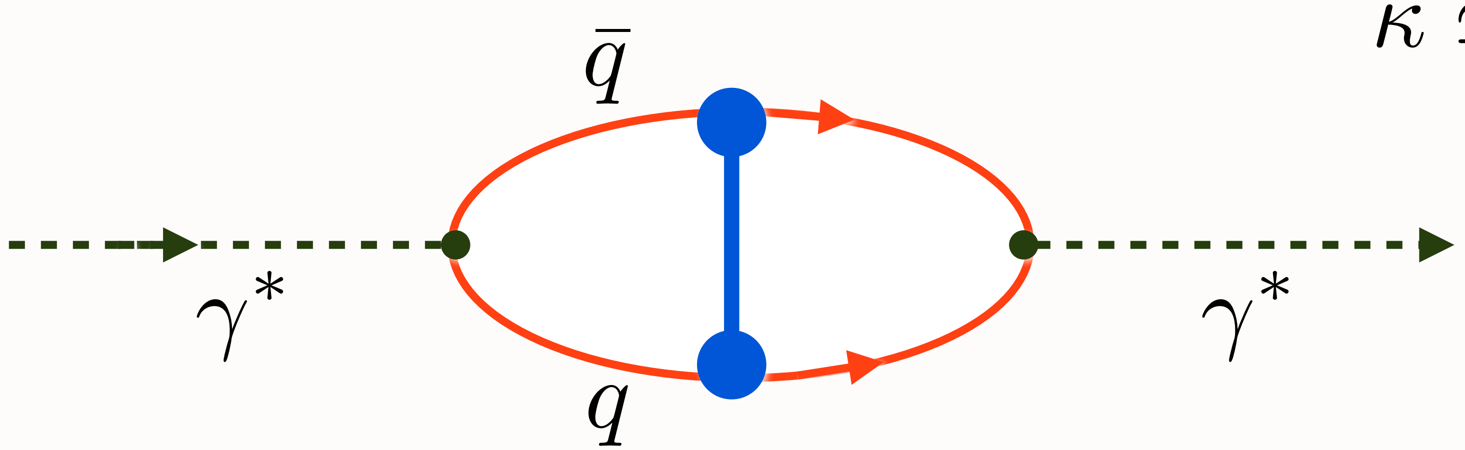
**Shrock, Tandy, Roberts, sjb**

- Independent of observer frame
- Causal
- Lowest invariant mass state M= 0.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: In hadron condensates (Maris, Tandy Roberts)
- QED vacuum; no loops
- Zero cosmological constant

# Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

$$M^2 = 4\kappa^2(n + L + S/2)$$ light-quark meson spectra

$\kappa \simeq 0.5$ GeV




$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

*mimics dimension-4 gluon condensate* $< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$ *in*

$e^+e^- \to X$, $\tau$ decay, $Q\bar{Q}$ phenomenology

# QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- heavy quarks only from gluon splitting
- renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary

# Novel QCD Phenomena and Perspectives

- Hadroproduction at large transverse momentum does not derive exclusively from 2 to 2 scattering subprocesses: Baryon Anomaly at RHIC Sickles, sjb

- Color Transparency Mueller, sjb; Diffractive Di-Jets and Tri-jets Strikman et al

- Heavy quark distributions do not derive exclusively from DGLAP or gluon splitting -- component intrinsic to hadron wavefunction. Hoyer, et al

- Higgs production at large $x_F$ from intrinsic heavy quarks Kopeliovitch, Goldhaber, Schmidt, Soffer, sjb

- Initial and final-state interactions are not always power suppressed in a hard QCD reaction: Sivers Effect, Diffractive DIS, Breakdown of Lam Tung PQCD Relation Schmidt, Hwang, Hoyer, Boer, sjb; Collins

- LFWFS are universal, but measured nuclear parton distributions are not universal -- antishadowing is flavor dependent Schmidt, Yang, sjb

- Renormalization scale is not arbitrary; multiple scales, unambiguous at given order. Disentangle running coupling and conformal effects, Skeleton expansion: Gardi, Grunberg, Rathsman, sjb

- Quark and Gluon condensates reside within hadrons: Shrock, sjb

# Goals

- Test QCD to maximum precision
- High precision determination of $\alpha_s(Q^2)$ at all scales
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Relate renormalization schemes without ambiguity
- Maximize sensitivity to new physics at the colliders

*"Principle of Maximum Conformality"*

# The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds -- number of active leptons set
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion
- Results are scheme independent!
- Predictions for physical observables cannot be scheme dependent

# Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

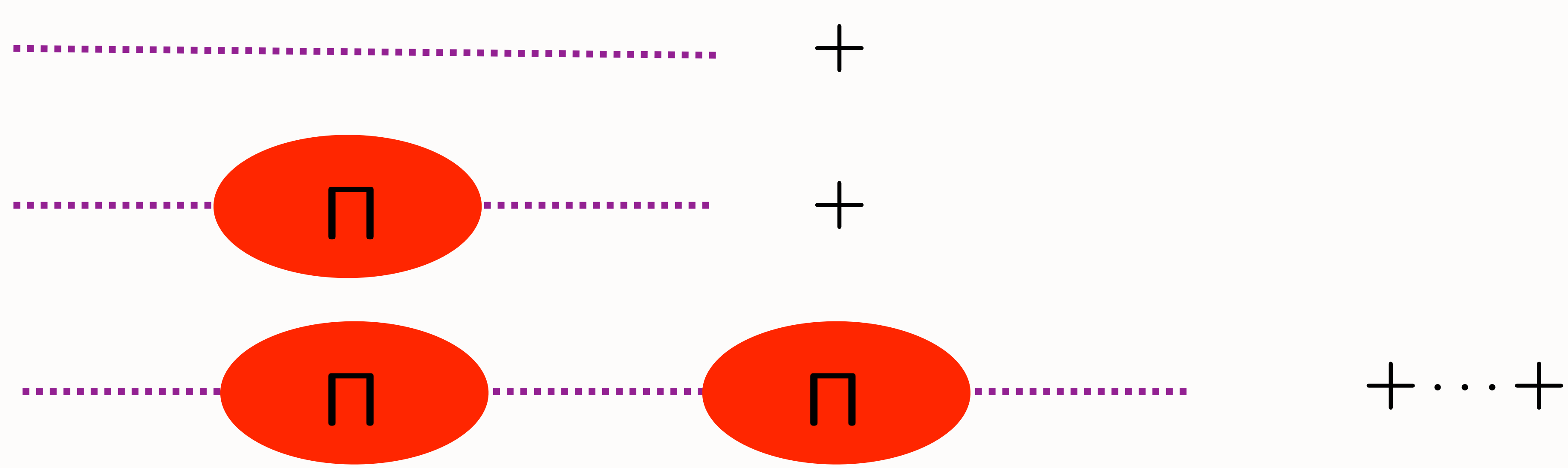

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

**Gell-Mann--Low Effective Charge**

# QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

*All-orders lepton-loop corrections to dressed photon propagator*



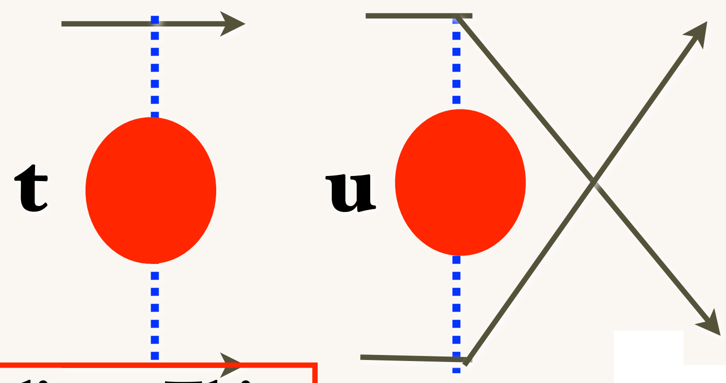

$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

***Initial*** **scale $t_0$ is arbitrary -- Variation gives RGE Equations**

**Physical renormalization scale t not arbitrary!**

# Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$




- Two separate physical scales: t, u = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!
- If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- No renormalization scale ambiguity!

# Features of BLM/PMC Scale Setting

**On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.**

**Lepage, Mackenzie, sjb** **Phys.Rev.D28:228,1983**

- **"Principle of Maximum Conformality"** **Di Giustino, Wu, Mojaza, sjb**
- **All terms associated with nonzero beta function summed into running coupling**
- **Standard procedure in QED**
- **Resulting series identical to conformal series**
- **Renormalon n! growth of PQCD coefficients from beta function eliminated!**
- *Scheme Independent* **!!!**
- **In general, BLM/PMC scales depend on all invariants**
- **Single Effective PMC scale at NLO**

# QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Scale-Free Conformal Series

Running Coupling Effects

Higher Twist from Hadron Dynamics

Intrinsic Heavy Quarks

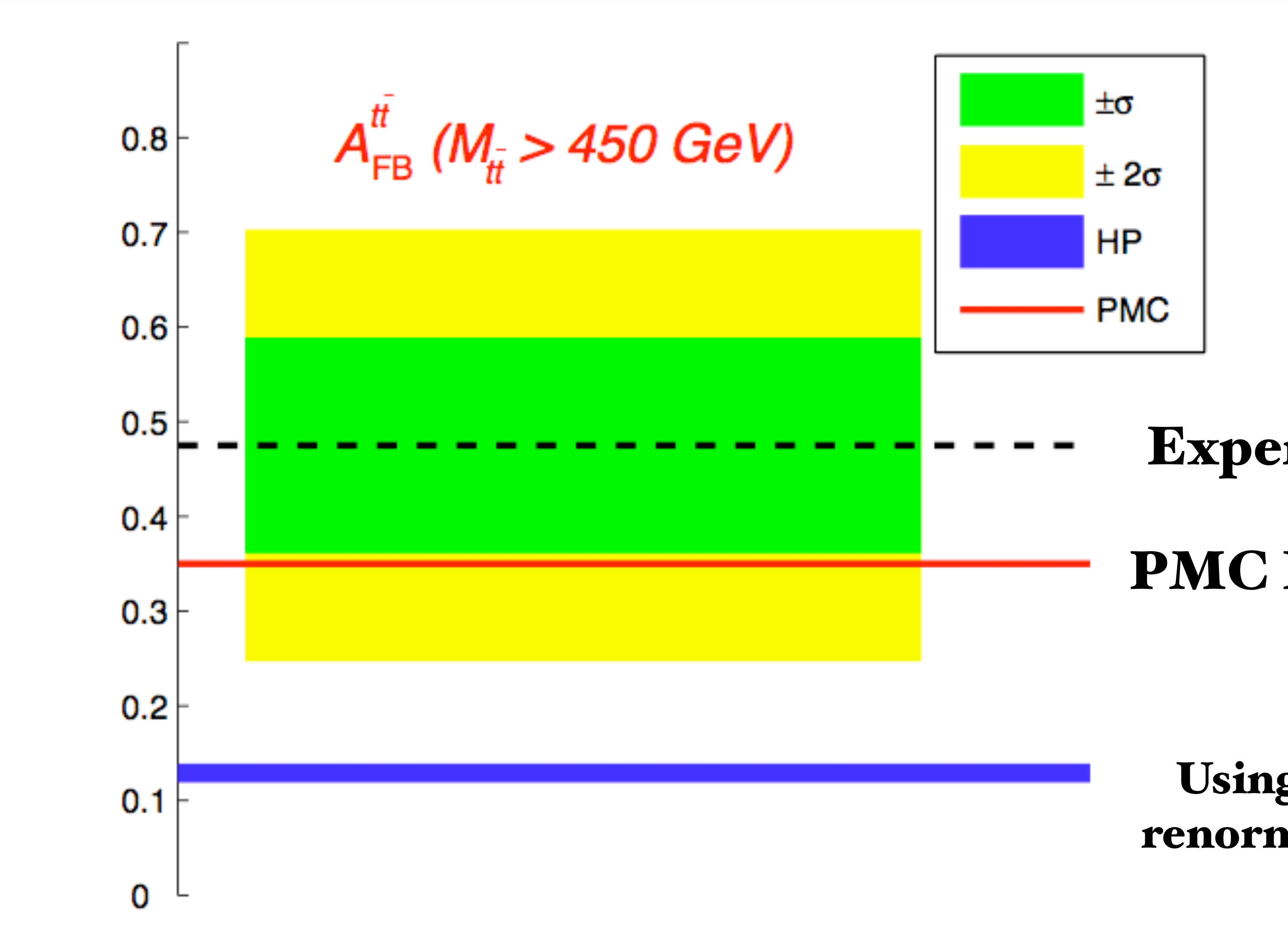
Light by Light Loops

***BLM/PMC: Absorb β-terms into running coupling***

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

*Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the 'Principle of Maximum Conformality' (PMC)*

Xing-Gang Wu
SJB




**Experimental asymmetry**

**PMC Prediction**

**Using conventional guess for renormalization scale and range**

$t\bar{t}$ asymmetry predicted by pQCD NNLO within 1 $\sigma$ of CDF/D0 measurements using PMC/BLM scale setting

# Need to set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

*Choose renormalization scheme; e.g.* $\alpha_s^R(\mu_R^{\rm init})$

↓

*Choose* $\mu_R^{init}$*; arbitrary initial renormalization scale*

↓

*Identify* $\{\beta_i^R\} - terms$ *using* $n_f - terms$ *through the* $PMC - BLM$ *correspondence principle*

↓

*Shift scale of* $\alpha_s$ *to* $\mu_R^{\rm PMC}$ *to eliminate* $\{\beta_i^R\} - terms$

↓

*Conformal Series*

*Result is independent of* $\mu_R^{\rm init}$ *and scheme at fixed order*

## PMC/BLM

**No renormalization scale ambiguity!**

**Result is independent of Renormalization scheme and initial scale!**

**Same as QED Scale Setting**

**Apply to Evolution kernels, hard subprocesses**

**Eliminates unnecessary systematic uncertainty**

**Xing-Gang Wu**
**Leonardo di Giustino, SJB**

## Principle of Maximum Conformality

# Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

Lu, Kataev, Gabadadze, Sjb

# Generalized Crewther Relation

$$[1+\frac{\alpha_R(s^*)}{\pi}][1-\frac{\alpha_{g_1}(q^2)}{\pi}]=1$$

$$\sqrt{s^*}\simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

*Nonconformal terms set relative scales (BLM)*

*No renormalization scale ambiguity!*

**Both observables go through new quark thresholds at commensurate scales!**

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

QCD $\to$ Abelian Gauge Theory

*Analytic Feature of SU(Nc) Gauge Theory*

*Scale-Setting procedure for QCD must be applicable to QED*

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# New perspectives on the quark condensate

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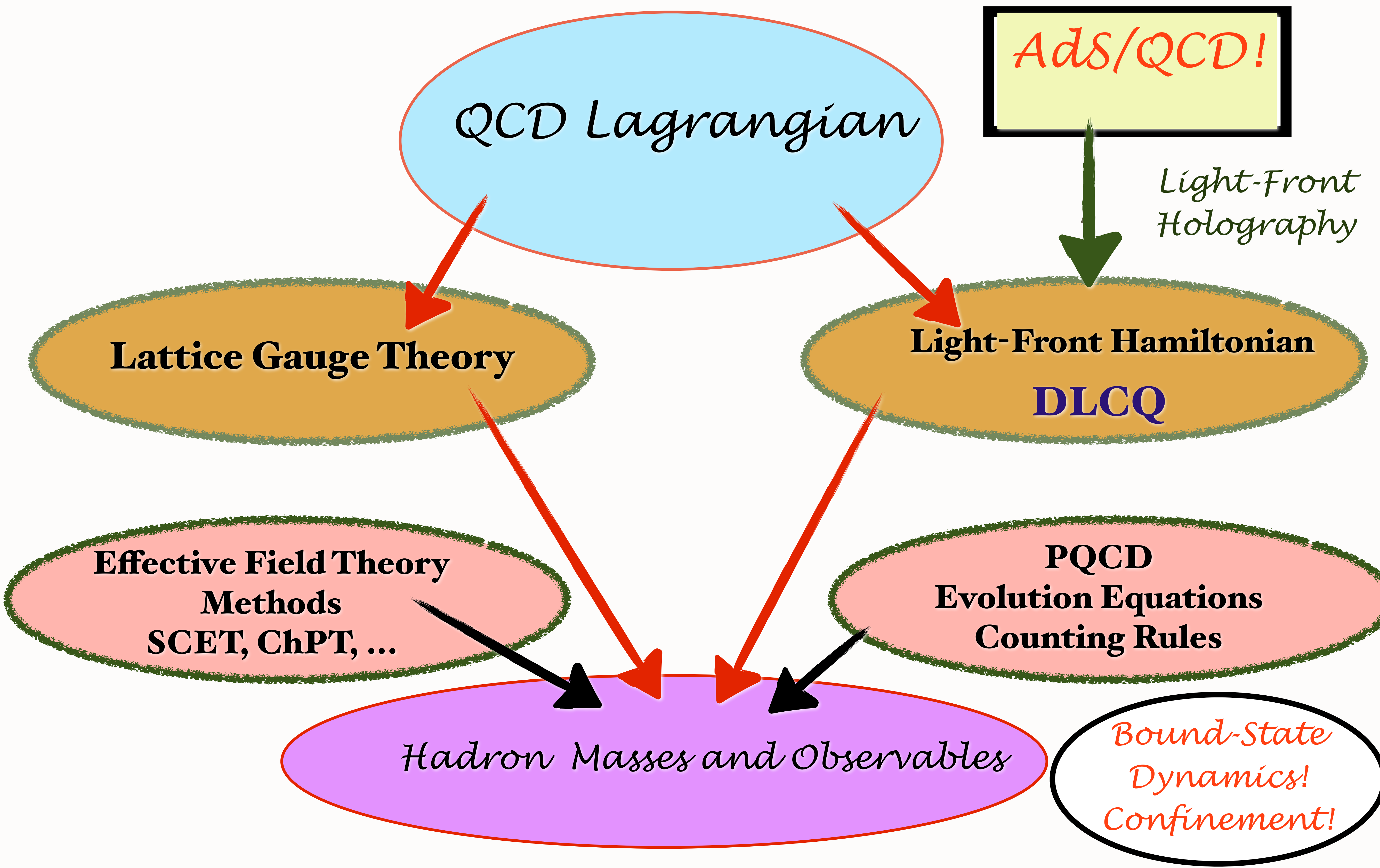
We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

# Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind    Maris, Roberts, Tandy    Shrock and sjb

- **Light-Front Quantization**
- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

# *Predict Hadron Properties from First Principles!*



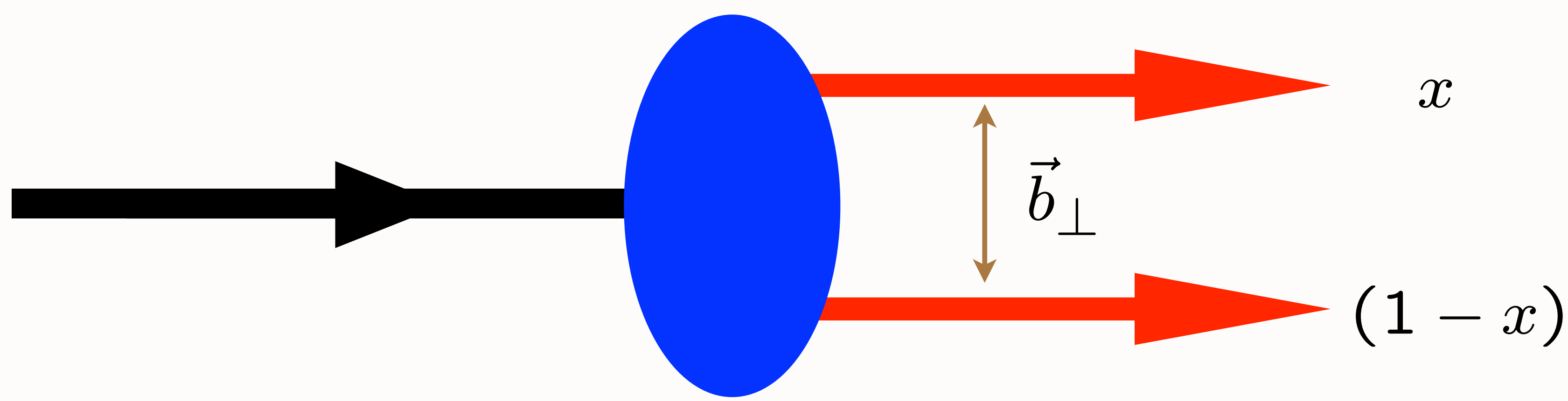
de Teramond, sjb

LF(3+1) ⟷ $AdS_5$

# Light-Front Holography

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$




$$\psi(x,\zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

*Light Front Holography: Unique mapping derived from equality of LF and AdS formulae for bound-states and form factors*

# Light-Front Schrödinger Equation

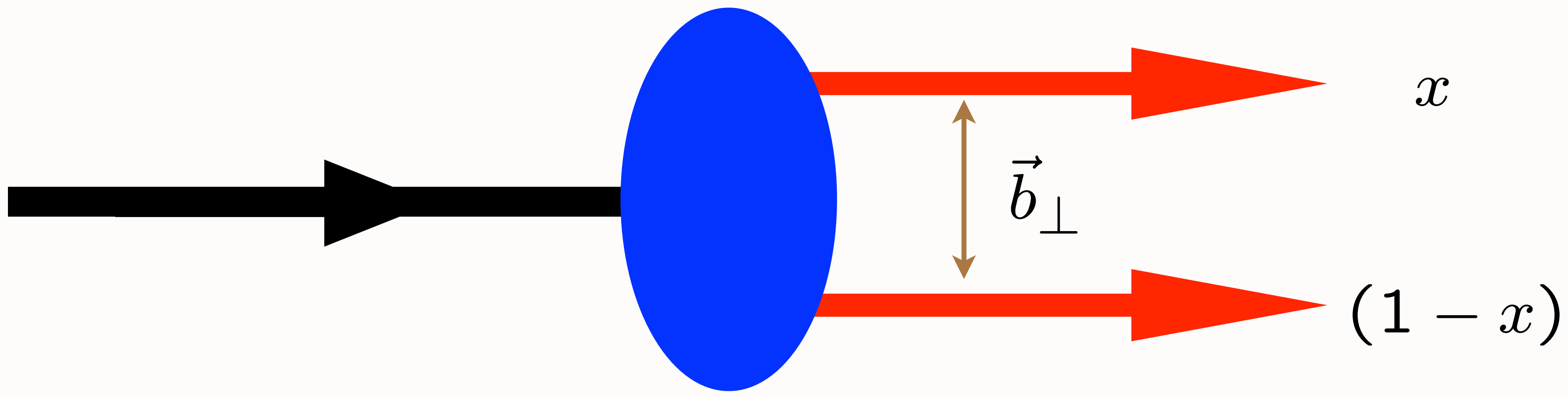
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

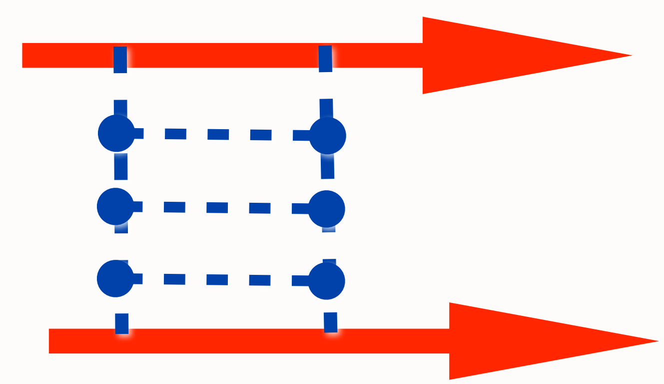
Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta^2,J,L,M^2)\right]\Psi_{J,L}(\zeta^2)=M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$




**U is the exact QCD potential**
**Conjecture: 'H'-diagrams generate**



$$U(\zeta,S,L)=\kappa^2\zeta^2+\kappa^2(L+S-1/2)$$

## LF Quantization

Bjorken, Kogut, Soper, Susskind

## LFWFs and Exclusive QCD:

Lepage and SJB, Efremov, Radyushkin

## RGE and LF Hamiltonians:

Glazek & Wilson

## DLCQ:

Hornbostel, Pauli, & SJB
Pinsky, Hiller

## Renormalization of $H_{LF}$

Hiller, Chabysheva, Pauli, Pinsky, McCartor, Suaya, sjb

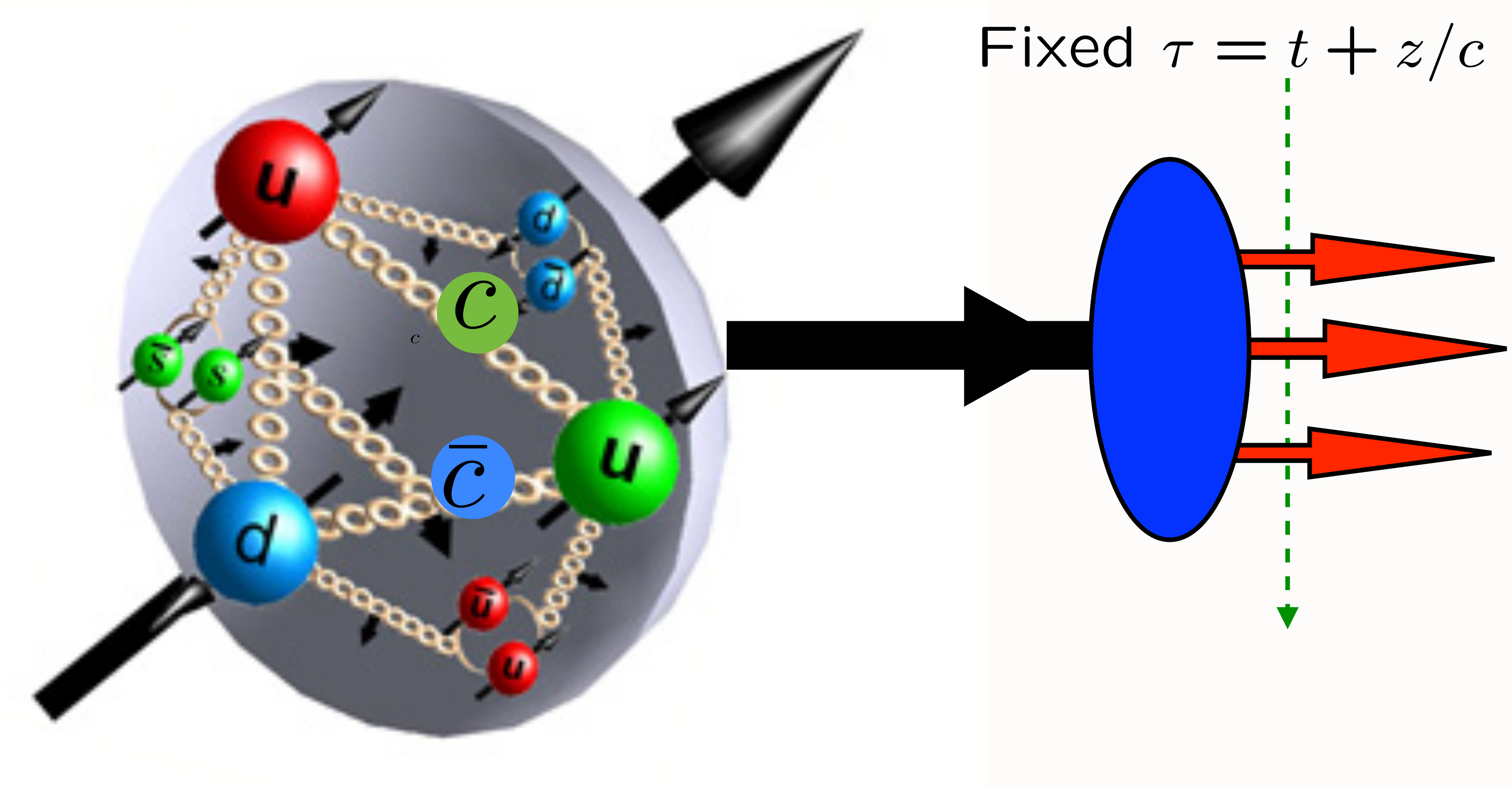
## Rotation Invariance, Regularization

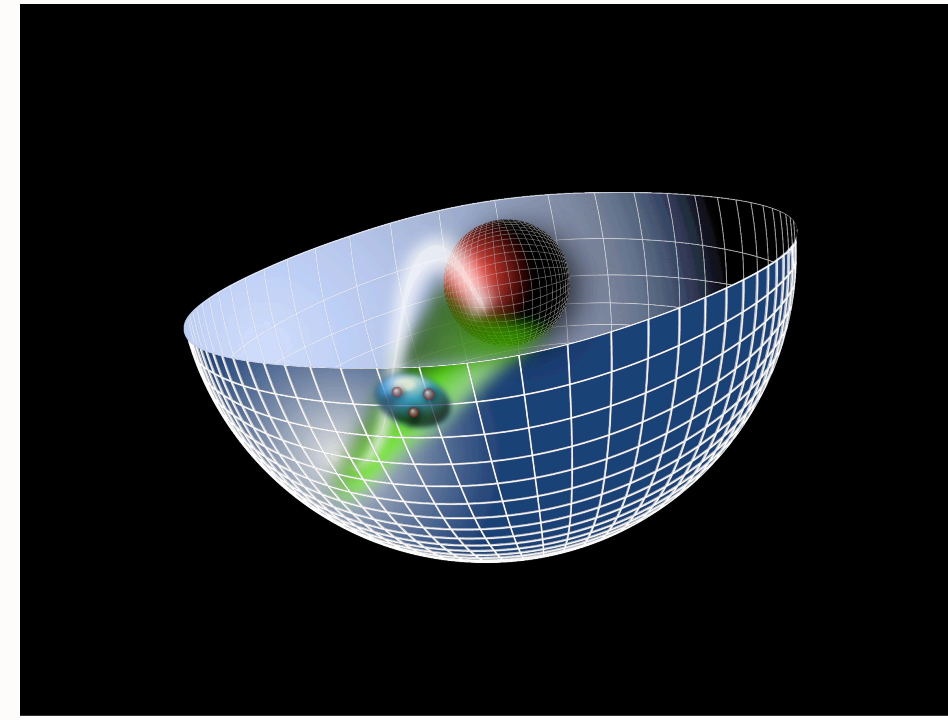
Karmanov, Mathiot

## Zero-Modes: Standard Model

Srivastava, sjb

# AdS/QCD, Light-Front Holography, and Color Confinement



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