Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 3

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30.10.2013

Exercise 10

Calculate the expectation value of $\langle \vec{\sigma} \cdot \vec{a} \otimes \vec{\sigma} \cdot \vec{b} \rangle_{\psi^-}$ for the Bell state $|\psi^-\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle \right)$.

Exercise 11

Calculate the Eigenstates of $\vec{n} \cdot \vec{\sigma}$, where $\vec{n}$ is a normalized vector.

$$\vec{n} \cdot \vec{\sigma} |\pm\vec{n}\rangle = \pm |\pm\vec{n}\rangle$$

Remark:
Use spherical coordinates for vector $\vec{n}$:

$$\vec{n} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

Exercise 12

Calculate the probability for measuring at Alice's side spin up along direction $\vec{a}$ and at Bob's side spin up along direction $\vec{b}$

$$P(\Uparrow \vec{a}, \Uparrow \vec{b}) = \left| \left(\langle +\vec{a}|_A \otimes \langle \vec{b}|_B \right) |\psi^-\rangle \right|^2$$

for the given Bell state $|\psi^-\rangle$.