Exercises for decoherence and open quantum systems
Sheet 1

Prof. Reinhold A. Bertlmann, Philipp Köhler & Veronika Baumann

13.03.2013

Exercise 1

Show that for mixed states the following properties are fulfilled:

$$\rho^2 \neq \rho$$
$$\mathrm{Tr}\left(\rho^2\right) < 1$$

Exercise 2

Consider the Hamiltonian

$$H = -\vec{\mu} \cdot \vec{B}$$

with

$$\vec{\mu} = \frac{g\mu_B}{2}\vec{\sigma} = \frac{g\mu_B}{2}\sigma_z \quad (\text{if } \vec{B} \parallel \text{z-axis})$$

What is the time evolution of a general density matrix $\rho(t)$?

Calculate the expectation values $\langle\vec{\sigma}\rangle(t)$.

Exercise 3

Calculate the Eigenstates of $\vec{n} \cdot \vec{\sigma}$, where $\vec{n}$ is a normalized vector.

$$\vec{n} \cdot \vec{\sigma}|\pm\vec{n}\rangle = \pm|\pm\vec{n}\rangle$$

Remark:
Use spherical coordinates for vector $\vec{n}$:

$$\vec{n} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

Exercise 4

Calculate the following expectation values:

$$\langle\sigma_i\rangle_{+\vec{n}} = \langle+\vec{n}|\sigma_i|+\vec{n}\rangle \qquad i = 1, 2, 3$$