

15. Standard model of particle physics

spontaneously broken gauge theory with gauge group

$$\underbrace{\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y}_{\begin{matrix} \text{strong} \\ \text{electroweak} \end{matrix}} \xrightarrow{\text{SSB}} \text{SU}(3)_c \times \text{U}(1)_{\text{em}}$$

$\text{SU}(3)_c$ (QCD) already discussed

→ electroweak interaction $G_{\text{ew}} = \text{SU}(2)_L \times \text{U}(1)_Y$

Glashow - Weinberg - Salam (Nobel prize 1979)

a) gauge group and multiplets

fundamental fields:

spin 1 four generators of $\text{SU}(2) \times \text{U}(1) \rightarrow W^\pm, Z^0, g$

spin 1/2 leptons:

$$\begin{array}{cccc} (\nu_e) & (\nu_\mu) & (\nu_\tau) & 0 \\ (e^-) & (\mu^-) & (\tau^-) & -1 \end{array}$$

3 generations

quarks:

$$\begin{array}{ccc} (u) & (c) & (t) \\ (d) & (s) & (b) \end{array}$$

$$\begin{array}{c} 2/3 \\ -1/3 \end{array}$$

of quarks and leptons

V-A theory \rightarrow lefthanded SU(2) doublets
righthanded SU(2) singlets

\rightarrow mesons $q_1 \bar{q}_2$, baryons $q_1 q_2 q_3$

SSB $\xrightarrow{?}$ Higgs field

$$G_{ew} = SU(2)_L \times U(1)_Y \xrightarrow{\text{weak hypercharge } Y}$$

W_μ^α B_μ gauge bosons

$$\alpha = 1, 2, 3$$

L_j e_{Rj} leptons

$$j = 1, 2, 3 \quad (\text{generation index}) \quad f_{L,R} = \frac{1 + \gamma_5}{2} f_{L,R}$$

q_{Lj} $u_{R,j}, d_{R,j}$ quarks

$$T_a = \tau_a / 2 \quad 0$$

ϕ Higgs doublet

gauge transformations:

SU(2): $\psi \rightarrow U\psi$ for doublets

$\psi \rightarrow \psi$ for singlets

$\phi \rightarrow U\phi \quad U \in SU(2)$

$$\vec{W}_\mu \cdot \frac{\vec{\tau}}{2} \rightarrow U \vec{W}_\mu \cdot \frac{\vec{\tau}}{2} U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1}$$

$$B_\mu \rightarrow B_\mu$$

U(1)_y: $\psi \rightarrow e^{-i\alpha \frac{Y_\psi}{2}} \psi$

$$\phi \rightarrow e^{-i\alpha \frac{Y_\phi}{2}} \phi$$

$$\vec{W}_\mu \rightarrow \vec{W}_\mu$$

$$B_\mu \rightarrow B_\mu + \frac{1}{g'} \partial_\mu \alpha$$

Lagrange density:

$$\mathcal{L}_W = -\frac{1}{4} \sum_{a=1}^3 F_a^{\mu\nu} F_{a\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu$$

$$\mathcal{L}_B = -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu)$$

$$\mathcal{L}_\psi = i \bar{\psi} \gamma^\mu D_\mu \psi , \quad D_\mu = \partial_\mu + ig \vec{T} \cdot \vec{N}_\mu + ig' \frac{Y}{2} B_\mu$$

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger D^\mu \phi \quad \Rightarrow \quad \begin{cases} \vec{\tau}/2 & \text{doublets} \\ \vec{\sigma} & \text{singlets} \end{cases}$$

$$-\mathcal{L}_Y = \bar{q}_{Li} \not{+} d_{Rj} \Gamma_{ij} + \bar{q}_{Li} \not{+} u_{Rj} \Delta_{ij} \\ + \bar{l}_i \not{+} l_{Rj} \gamma_{ij} + h.c.$$

Yukawa couplings

$$\tilde{\phi} = i \tau^2 \phi^* \Rightarrow Y_{\tilde{\phi}} = -Y_\phi$$

$$\tilde{\phi} \rightarrow U \tilde{\phi} \quad \text{as } \tau_2 U^* = U \tau_2$$

(representations are equivalent)

$$V(\phi) = -r\phi^\dagger \phi + \alpha (\phi^\dagger \phi)^2, r > 0 \rightarrow SSB$$

Higgs potential

$$\mathcal{L}_{GWS} = \mathcal{L}_W + \mathcal{L}_B + \sum_\psi \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_Y - V$$

remark: neutrinos massless in this minimal version of the SM

experimental evidence for massive neutrinos (neutrino oscillations) \rightarrow extension of \mathcal{L}_{GWS} necessary (exercise: find correct extension of SM \rightarrow receive Nobel prize)

b) SSB

$$r > 0 \Rightarrow SSB$$

$$SU(2) \times U(1) \text{ invariance} \rightarrow \langle 0 | \phi | 0 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

($v > 0$) without loss of generality

$U(1)_{\text{em}}$ unbroken $\Rightarrow Q_{\text{em}} = \text{group generator}$

which annihilates $\frac{\sigma}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \langle 0 | \phi | 1 \rangle$

$$\underbrace{\left(\vec{c} \cdot \frac{\vec{\tau}}{2} + d \frac{Y_+}{2} \right)}_{Q_{\text{em}}^+} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0, \quad \vec{c} \in \mathbb{R}^3, d \in \mathbb{R}$$

$$\Rightarrow c_1 = c_2 = 0, \quad c_3 = d Y_+$$

$$\Rightarrow Q_{\text{em}}^+ = c_3 \left(\frac{\tau_3}{2} + \frac{1}{2} \right) = c_3 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

elm. charge matrix
for Higgs doublet

general form (arbitrary representation):

$$Q_{\text{em}} = c_3 T_3 + d \frac{Y}{2}$$

charge difference within lepton or quark
doublets $\begin{pmatrix} e^- \\ e^- \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow c_3 = +1$

$$\Rightarrow Q_{em}^{\phi} = \frac{T_3}{2} + \frac{1}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

choice of "normalization" $d=1$ (fixes Y)

$$\Rightarrow Q = T_3 + \frac{Y}{2}$$

Known charges of fundamental particles

$$\rightarrow Y(L) = -1, \quad Y(l_R) = -2$$

$$\rightarrow Y(q_L) = \frac{1}{3}, \quad Y(u_R) = \frac{4}{3}, \quad Y(d_R) = -\frac{2}{3}$$

$$Y(\phi) = 1$$

c) bosonic sector

mass term of vector bosons:

$$D_\mu = \partial_\mu + ig \frac{\vec{\Sigma}}{2} \vec{W}_\mu + ig' \frac{1}{2} B_\mu \quad \text{for Higgs doublet}$$

adjoint representation of $SU(2)$

$$T_3^{\text{ad}} = (-i \epsilon_{3ab}) = -i \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix} \rightarrow y=0 \text{ for all gauge bosons}$$

$$\rightarrow Q_{W,B} = -i \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

eigenvalues $\pm 1, 0^{(2)}$

$$\begin{matrix} \uparrow & \uparrow \\ W_{1,2} & W_3 - B \\ \text{subspace} \end{matrix}$$

$$Q_{W,B} \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} \quad \text{eigenvector (eigenvalue + 1)}$$

$$Q_{W,B} \begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix} = - \begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix} \quad -1/- \quad (-1/- \quad -1)$$

$$\Rightarrow W_1 = \frac{W^+ + W^-}{\sqrt{2}}, \quad W_2 = i \frac{W^+ - iW^-}{\sqrt{2}}$$



$$W^+ = \frac{W_1 - iW_2}{\sqrt{2}}, \quad W^- = \frac{W_1 + iW_2}{\sqrt{2}}$$

$$\frac{1}{2} (\tau_1 W_1 + \tau_2 W_2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}$$

$$D_\mu \underbrace{\langle 0 | \phi | 10 \rangle}_{\frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \frac{iv}{\sqrt{2}} \left[\frac{g}{\sqrt{2}} W_\mu^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \right. \\ \left. + \frac{1}{2} (-g W_\mu^3 + g' B_\mu) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$$

mass term:

$$(D_\mu \langle 0 | \phi | 10 \rangle)^T D^\mu \langle 0 | \phi | 10 \rangle =$$

$$= \frac{g^2 v^2}{4} W_\mu^- W^{+\mu}$$

$$+ \frac{v^2}{8} \underbrace{(-g W_\mu^3 + g' B_\mu) (-g W_3^\mu + g' B^\mu)}_{\text{linear combination of neutral VBs with mass}}$$

diagonalization of mass matrix in the neutral sector \rightarrow mass eigenfields (Z^0, g)

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}$$

$$\cos \Theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \Theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

Θ_W = Weinberg angle

mass term : $M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$

$$M_W^2 = \frac{g^2 v^2}{4}, \quad M_Z^2 = \frac{(g^2 + g'^2)v^2}{4}$$

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2} = \cos^2 \Theta_W$$

$$\frac{M_W}{M_Z} = \cos \Theta_W$$

general form of the covariant derivative
expressed in terms of mass eigenfields:

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) +$$

$$+ ig T_3 (\cos \Theta_W Z_\mu + \sin \Theta_W A_\mu)$$

$$+ ig' \frac{Y}{2} (-\sin \Theta_W Z_\mu + \cos \Theta_W A_\mu)$$

$$T_\pm = T_1 \pm i T_2 = \begin{cases} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} & \text{for SU(2) doublets} \\ 0 & \text{-1- -1- singlets} \end{cases}$$

coefficient of A_μ :

$$g T_3 \sin \Theta_W + g' \cos \Theta_W \frac{1}{2} Y$$

$$= g \sin \Theta_W T_3 + g \frac{\sin \Theta_W}{\cos \Theta_W} \cancel{\cos \Theta_W} \frac{Y}{2}$$

$$= g \sin \Theta_W \left(T_3 + \frac{Y}{2} \right) = g \sin \Theta_W Q_{em}$$

$$g \sin \Theta_W = e$$

coefficient of Z_μ :

$$g \cos \Theta_W T_3 - g' \sin \Theta_W \frac{Y}{2}$$

$$= g \cos \Theta_W T_3 - g \frac{\sin^2 \Theta_W}{\cos \Theta_W} \frac{Y}{2}$$

$$= g \cos \Theta_W T_3 - g \frac{\sin^2 \Theta_W}{\cos \Theta_W} (Q_{em} - T_3)$$

$$= \frac{g}{\cos \Theta_W} (T_3 - \sin^2 \Theta_W Q_{em})$$

$$\Rightarrow D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-)$$

$$+ \frac{ig}{\cos \Theta_W} (T_3 - \sin^2 \Theta_W Q_{em}) Z_\mu$$

$$+ ie Q_{em} A_\mu$$

unitary gauge: $\phi(x) \rightarrow \frac{1}{\sqrt{2}} (v + h(x)) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\langle 0 | h(x) | 0 \rangle = 0$$

$h(x)$ = Higgs boson of SM

Higgs - vector boson coupling:

$$D_\mu \phi \xrightarrow{\text{unit. gauge}} \frac{1}{\sqrt{2}} D_\mu (v + h) \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{\sqrt{2}} \left[\partial_\mu h \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{ig}{\sqrt{2}} W_\mu^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} (v + h) \right.$$

$$\left. - \frac{ig}{2 \cos \Theta_W} Z_\mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} (v + h) \right]$$

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger D^\mu \phi \rightarrow$$

$$\rightarrow \frac{1}{2} \partial_\mu h \partial^\mu h + M_W^2 W_\mu^+ W^\mu_- \left(1 + \frac{h}{v}\right)^2$$

$$+ \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{v}\right)^2$$



Higgs potential:

$$V(\phi) = -r \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad r, \lambda > 0$$

minima determined by $-r + 2\phi^\dagger \phi = 0$

$$\langle 0 | \phi | 0 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow v^2 = \frac{r}{\lambda}$$

$$V(\phi) = \lambda \phi^\dagger \phi (-v^2 + \phi^\dagger \phi)$$

$$V\left(\frac{v+h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \lambda \frac{(v+h)^2}{2} \left(-v^2 + \frac{(v+h)^2}{2}\right)$$

$$= \frac{\lambda}{4} (v^2 + 2vh + h^2) \left(-v^2 + \frac{(v+h)^2}{2}\right)$$

$$= -\underbrace{\frac{\lambda v^4}{4}}_{V_0} + \frac{\lambda}{4} (2vh + h^2)^2$$

$$= V_0 + \underbrace{\frac{\lambda v^2 h^2}{4}}_{M_h^2/2} \left(1 + \frac{h}{2v}\right)^2$$

$$M_h^2 = 2\lambda v^2$$

mass term and cubic and quartic self couplings of the Higgs field

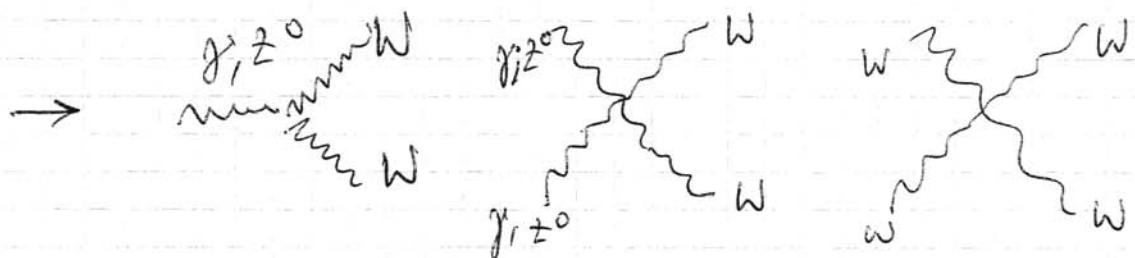
self couplings of the vector bosons (W^\pm, Z^0, γ)
from

$$-\frac{1}{4} \overline{F_{\alpha\mu\nu}} F_\alpha^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad \overline{F_\alpha}^{\mu\nu} = \partial^\mu W_\alpha^\nu - \partial^\nu W_\alpha^\mu$$

$$- g \epsilon_{abc} W_b^\mu W_c^\nu$$

substitutions : $W_1 = \frac{W^+ + W^-}{\sqrt{2}}, \quad W_2 = i \frac{W^+ - W^-}{\sqrt{2}},$

$$W_3 = \cos \Theta_W Z + \sin \Theta_W A, \quad B = -\sin \Theta_W Z + \cos \Theta_W A$$



d) fermion masses

Yukawa couplings :

$$- \mathcal{L}_y = \bar{q}_{Li} \phi d_{Rj} \Gamma_{ij} + \bar{q}_{Li} \tilde{\Phi} u_{Rj} \Delta_{ij}$$

$$+ \bar{l}_i \phi l_{Rj} g_{ij} + h.c.$$

$$\tilde{\Phi} = i \tau_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} (\phi^*)^* \\ (\phi^0)^* \end{pmatrix} = \begin{pmatrix} (\phi^0)^* \\ -\phi^- \end{pmatrix}$$

$$\Rightarrow \langle 0 | \tilde{\phi} | 0 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\tilde{\phi} = \frac{v+h}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{unitary gauge})$$

$$\bar{q}_{Li} \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} d_{Rj} \Gamma_{ij} = \frac{v}{\sqrt{2}} \Gamma_{ij} (\bar{u}_{Li}, \bar{d}_{Li}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} d_{Rj} =$$

$$= \bar{d}_{Li} \underbrace{\frac{v}{\sqrt{2}} \Gamma_{ij}}_{(M_d)_{ij}} d_{Rj}$$

$M_d = \frac{v}{\sqrt{2}} \Gamma$ mass matrix of down-type quarks
 (d, s, b)

$$\bar{q}_{Li} \frac{v}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_{Rj} \Delta_{ij} = (\bar{u}_{Li}, \bar{d}_{Li}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_{Rj} \frac{v}{\sqrt{2}} \Delta_{ij}$$

$$= \bar{u}_{Li} \underbrace{\frac{v}{\sqrt{2}} \Delta_{ij}}_{(M_u)_{ij}} u_{Rj}$$

$M_u = \frac{v}{\sqrt{2}} \Delta$ mass matrix of up-type quarks
 (u, c, t)

$$\overline{L}_i \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} l_{Rj} g_{ij} = (\overline{\ell}_{Li}, \overline{\ell}_{Li}) \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} l_{Rj} g_{ij}$$

$$= \overline{\ell}_{Li} \underbrace{\frac{v}{\sqrt{2}} g_{ij} l_{Rj}}_{(M_\ell)_{ij}}$$

$$M_\ell = \frac{v}{\sqrt{2}} g \quad \text{mass matrix of (charged) leptons}$$

M arbitrary complex $n \times n$ matrix $\Rightarrow \exists$ unitary $n \times n$ matrices U_L, U_R such that $U_L^\dagger M U_R = \hat{M}$
diagonal and nonnegative (proof: $M = R U$,

$$R^\dagger = R, R \geq 0, U^\dagger = U^{-1} \Rightarrow R = U_L^\dagger \hat{M} U_L$$

$$\Rightarrow M = U_L^\dagger \hat{M} \underbrace{U_L U}_{U_R})$$

$$u_L = \underbrace{U_L^u}_{R} u_L' , d_L = \underbrace{U_L^d}_{R} d_L' , l_L = \underbrace{U_L^l}_{R} l_L'$$

$$-\mathcal{L}_Y = \overline{d}_L' \hat{M}_d d_R' \left(1 + \frac{h}{v}\right) + \overline{u}_R' \hat{M}_u u_R' \left(1 + \frac{h}{v}\right) \\ + \overline{\ell}_L \hat{M}_\ell \ell_R \left(1 + \frac{h}{v}\right) + \text{h.c.}$$

$$\bar{f}_L \hat{M} f_R + h.c. = \bar{f}_L \hat{M} f_R + \bar{f}_R \hat{M} f_L =$$

$$= \overline{(f_L + f_R)} \hat{M} \underbrace{(f_L + f_R)}_{=: f} = \bar{f} \hat{M} f$$

$$(\bar{f}_L f_L = 0)$$

$$\Rightarrow -\mathcal{L}_y = -\sum_F m_F \bar{f} f \left(1 + \frac{\hbar}{v} \right)$$

$f = u_i', d_i', \ell_i'$ ($i=1,2,3$) physical fields

remark: chiral fields = fundamental fermionic building blocks (left- and right-handed fields in different multiplets); but: after SSB and diagonalization of mass matrices \rightarrow \rightarrow Dirac fields for physical particles

remark: neutrinos massless in minimal version of standard model \rightarrow clearly incomplete description of the real world \rightarrow extension of SM

e) charged current

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} \sum_{\psi} \bar{\psi} \gamma^{\mu} (T_+ W_{\mu}^+ + T_- W_{\mu}^-) \psi$$

sum runs over all fermion multiplets (singlets do not contribute)

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (\bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_L \gamma^{\mu} \ell_L) W_{\mu}^+ + \text{h.c.}$$

→ physical fields $u_L = U_L^u u'_L$, $d_L = U_L^d d'_L$

$$\rightarrow \bar{u}_L \gamma^{\mu} d_L = \bar{u}'_L \gamma^{\mu} \underbrace{U_L^{u\dagger} U_L^d}_{V} d'_L$$

$V = U_L^{u\dagger} U_L^d$ Cabibbo - Kobayashi - Maskawa matrix

(quark mixing matrix)

leptons: $\ell_L = U_L^l \ell'_L$

for massless neutrinos one defines $\nu_L = U_L^l \nu'_L$

$\Rightarrow \bar{\nu}_L \gamma^{\mu} \ell_L = \bar{\nu}'_L \gamma^{\mu} \ell'_L$ no mixing matrix for massless ν_s

experiment: ν -oscillations observed $\rightarrow \nu_s$ have different masses

$$\rightarrow -\mathcal{L}_{cc} = \frac{g}{2F^2} [\bar{u}' \gamma^\mu (1 - \gamma_5) V d' + \\ + \bar{\nu}' \gamma^\mu (1 - \gamma_5) \ell'] W_\mu^+ + \text{h.c.}$$

f) neutral current

$$-\mathcal{L}_{NC} = \frac{g}{\cos \Theta_W} \sum_{\psi} \bar{\psi} \gamma^\mu (T_3 - \sin^2 \Theta_W Q_{em}) \psi Z_\mu$$

(now also contributions from SU(2) singlets)

$$-\mathcal{L}_{NC} = \frac{g}{\cos \Theta_W} [\bar{u}_L \gamma^\mu u_L (\frac{1}{2} - \sin^2 \Theta_W \frac{2}{3}) + \\ + \bar{d}_L \gamma^\mu d_L (-\frac{1}{2} + \sin^2 \Theta_W \frac{1}{3}) + \bar{\ell}_L \gamma^\mu \ell_L (-\frac{1}{2} + \sin^2 \Theta_W) \\ + \bar{\nu}_L \gamma^\mu \nu_L \frac{1}{2} + \bar{u}_R \gamma^\mu u_R (-\sin^2 \Theta_W \frac{2}{3}) \\ + \bar{d}_R \gamma^\mu d_R \sin^2 \Theta_W \frac{1}{3}) + \bar{\ell}_R \gamma^\mu \ell_R \sin^2 \Theta_W] Z_\mu$$

\rightarrow physical fields : $\bar{u}_L \gamma^\mu u_L = \bar{u}'_L \gamma^\mu u'_L$, etc.

GIM mechanism (Glashow - Iliopoulos - Maiani)

neutral currents do not change flavour

experiment: no flavour changing neutral currents (FCNC) observed (so far)

$$-\mathcal{L}_{NC} = \frac{g}{2\cos\Theta_W} \sum_f \bar{f} g^\mu (\alpha_f - \beta_f g_5) f \gamma_\mu$$

$$\alpha_f = t_3^L f - 2\sin^2\Theta_W Q_f$$

$$\beta_f = t_3^L f$$

$t_3^L f$ = eigenvalue of T_3 in $SU(2)$ doublet

Q_f = charge of f

	α_f	β_f
u	$\frac{1}{2} - \frac{4}{3}\sin^2\Theta_W$	$\frac{1}{2}$
d	$-\frac{1}{2} + \frac{2}{3}\sin^2\Theta_W$	$-\frac{1}{2}$
e	$\frac{1}{2}$	$\frac{1}{2}$
l	$-\frac{1}{2} + 2\sin^2\Theta_W$	$-\frac{1}{2}$

- g) electromagnetic current

$$-\mathcal{L}_{em} = e \sum_f Q_f \bar{f} \gamma^\mu f A_\mu$$

remark: $U_{L,R}^{u,d,l}$ cancel (like in \mathcal{L}_{NC})

h) V-A theory

$$-\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} (\bar{u}' \gamma^\mu (1 - \gamma_5) V d' + \bar{\nu}' \gamma^\mu (1 - \gamma_5) l') W_\mu^+$$

+ h.c.

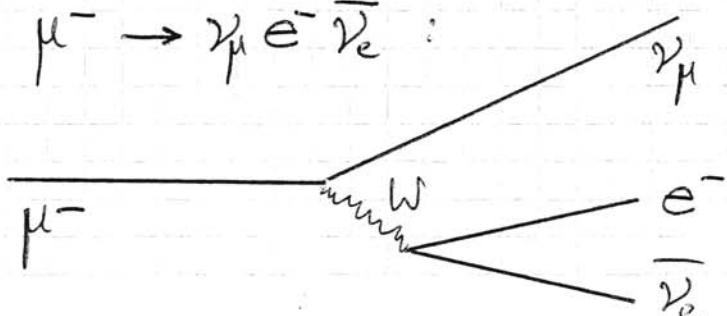
$$V^\mu := \bar{u}' \gamma^\mu V d' + \bar{\nu}' \gamma^\mu l' \quad (\text{charged}) \quad \underline{\text{vector current}}$$

$$A^\mu := \bar{u}' \gamma^\mu \gamma_5 V d' + \bar{\nu}' \gamma^\mu \gamma_5 l' \quad (\text{charged}) \quad \underline{\text{axial vector current}}$$

$$\Rightarrow -\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} [(V^\mu - A^\mu) W_\mu^+ + (V^\mu - A^\mu)^+ W_\mu^-]$$

describes for instance $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$:

muon decay



low-energy effective theory (energies $\ll M_W$)

"integrate out" W (heavy degree of freedom)

W propagator $\int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} i \frac{-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2}$

$\xrightarrow{|k| \ll M_W} \frac{i g_{\mu\nu}}{M_W^2} S^{(4)}(x-y)$ point interaction

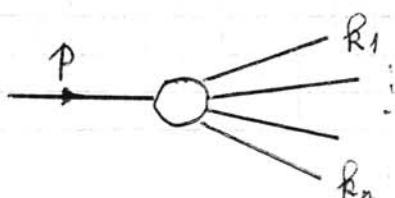
$\rightarrow \mathcal{L}_{V-A} = -\frac{G_F}{\sqrt{2}} (V^\mu - A^\mu)(V_\mu - A_\mu)^t$

Fermi coupling constant $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$

$G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ (PDG 2008)

remark: computation of decay width

$$d\Gamma = \frac{S}{2M} |N_f|^2 \frac{d^3 k_1}{(2\pi)^3 2E_1} \dots \frac{d^3 k_n}{(2\pi)^3 2E_n} (2\pi)^4 S^{(4)}(p - \sum_{i=1}^n k_i)$$

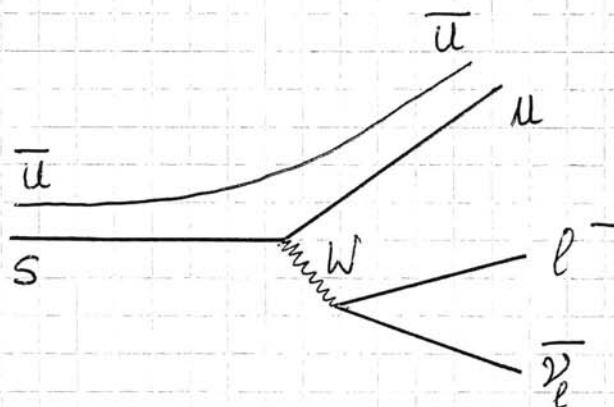


$p^2 = M^2, S = \text{statistical factor}$

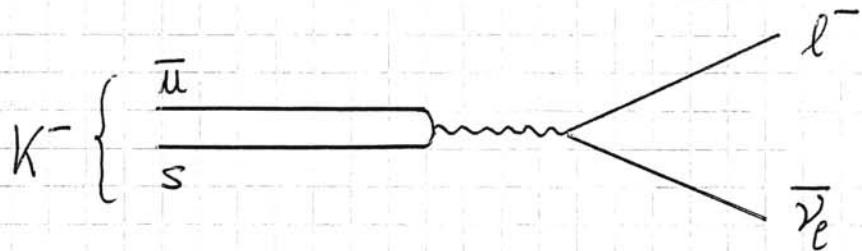
$\mathcal{L}_{V-A} \rightarrow$ weak decays of μ, τ , quarks

example: $s \rightarrow u \bar{l}^- \bar{\nu}_l$ ($l = e, \mu$)

amplitude $\sim V_{us}$



$K^- \rightarrow \pi^0 \bar{l}^- \bar{\nu}_l$ (K_{e3} decay)



$K^- \rightarrow \bar{l}^- \bar{\nu}_l$ (K_{e2} decay)

"semileptonic" decays

$$c \rightarrow \begin{matrix} d \\ s \end{matrix} l^+ \bar{\nu}_l \quad (u\bar{d}, u\bar{s})$$

$$l = e, \mu, \tau$$

$$b \rightarrow \begin{matrix} c \\ u \end{matrix} l^- \bar{\nu}_l \quad (\bar{u}d, \bar{u}s, \bar{c}d, \bar{c}s)$$

$$\text{amplitude } (b \rightarrow c l^- \bar{\nu}_l) \sim V_{cb}$$

$$\rightarrow \quad (b \rightarrow c \bar{u}s) \sim V_{cb} V_{us}^*$$

remark: $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} \stackrel{e=g \sin \Theta_W}{=} \frac{\pi \alpha}{2 \sin^2 \Theta_W M_W^2}$

$$\rightarrow M_W = \frac{37.3 \text{ GeV}}{\sin \Theta_W}$$

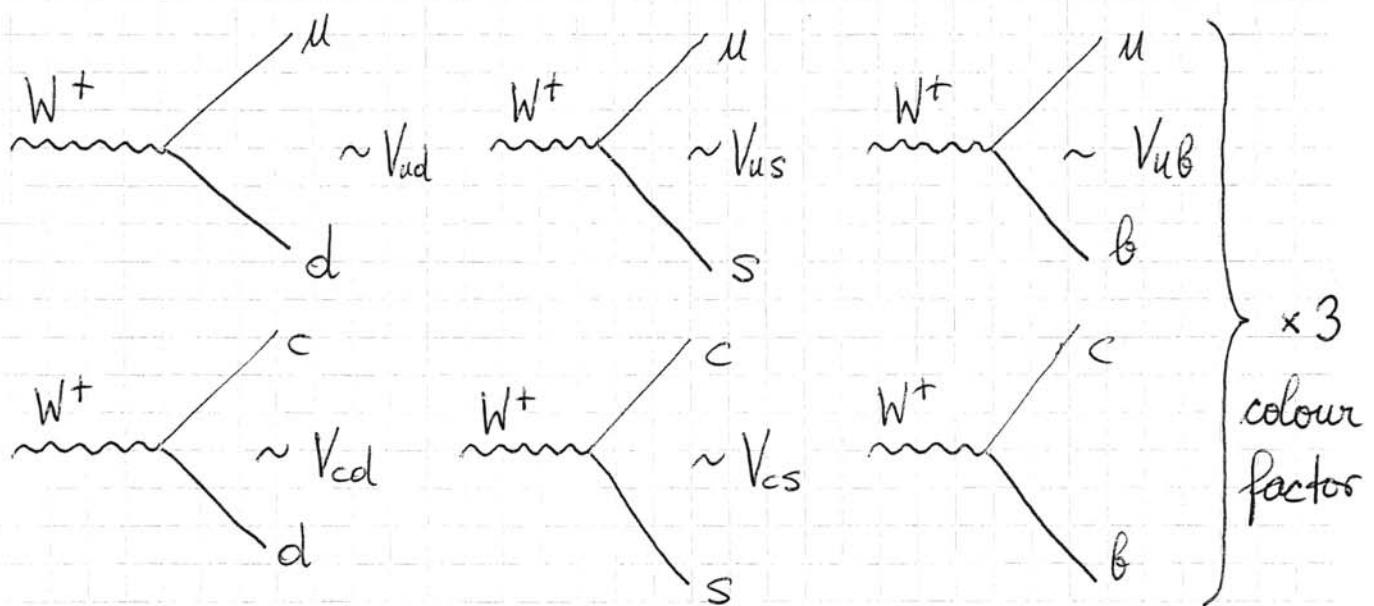
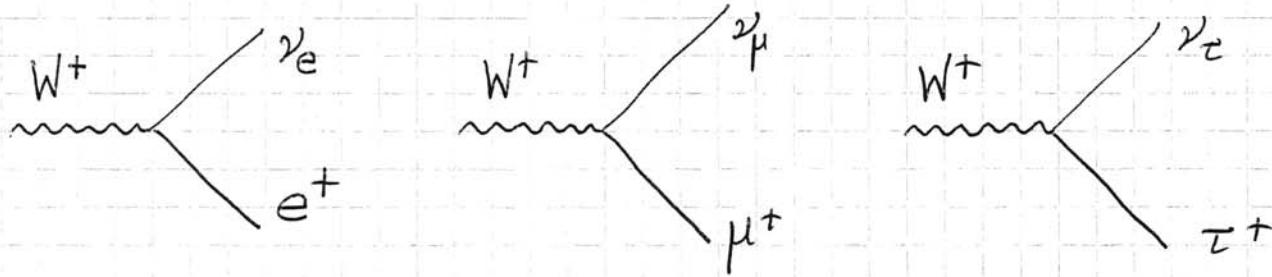
NC experiments before discovery of W^\pm : $\sin^2 \Theta_W \approx 0.23$

\rightarrow prediction $M_W \approx 78 \text{ GeV}$

current value (PDG 2008): $M_W = 80.398 \pm 0.025 \text{ GeV}$

(radiative corrections!)

exercise: compute the widths of the following decay modes of the W boson:



remark:

$$\sum_{\lambda=1}^3 \varepsilon^\mu(\vec{R}, \lambda) \varepsilon^\nu(\vec{R}, \lambda) = -g^{\mu\nu} + \frac{R^\mu R^\nu}{M^2}$$

for massive vector bosons

i) Cabibbo - Kobayashi - Maskawa mixing matrix

number of independent (observable) parameters in V :

n_G = number of generations

V : n_G -dimensional unitary matrix $\rightarrow n_G^2$ real parameters (write $V = \exp(iA)$, $A = A^\dagger \rightarrow A$ has $n_G + \frac{2(n_G^2 - n_G)}{2} = n_G^2$ real parameters)

$\binom{n_G}{2} = \frac{n_G(n_G-1)}{2}$ of these parameters are

angles (generalized Euler angles parametrizing $SO(n_G)$: $SO(n_G)$ matrix = $\mathbb{1} + \Omega + \dots$, $\Omega^T = -\Omega$ $\rightarrow \frac{n_G^2 - n_G}{2}$ real parameters)

\rightarrow remaining $n_G^2 - \frac{n_G(n_G-1)}{2} = \frac{n_G(n_G+1)}{2}$ parameters are phases

however: define diagonal matrices

$$e^{i\alpha} = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}, \quad e^{i\beta} = \begin{pmatrix} e^{i\beta_1} & & \\ & e^{i\beta_2} & \\ & & e^{i\beta_3} \end{pmatrix}$$

$$u' \rightarrow e^{i\alpha} u', \quad d' \rightarrow e^{i\beta} d'$$

(same transformation for L, R) $\rightarrow L_y, L_{NC}, L_{em}$
 remain unchanged, but

$$V \rightarrow e^{-i\alpha} V e^{i\beta} \quad \text{in } L_{CC}$$

$\rightarrow 2n_g - 1$ phases can be absorbed by

this transformation \rightarrow

$$\rightarrow \frac{n_g(n_g+1)}{2} - (2n_g - 1) = \frac{(n_g-1)(n_g-2)}{2}$$

measurable phases

$$\underline{n_G = 2} : \quad V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}$$

$\theta_c = \underline{\text{Cabibbo angle}}$

no observable phase

$n_G = 3$: 3 angles, 1 phase \rightarrow only mechanism of (minimal) SM for CP violation

\rightarrow all CP violating observables $\sim \sin \delta_{KM}$
(Kobayashi - Maskawa 1973)

exercise: study the possible parametrizations of the CKM matrix shown in PDG 2008

remark: "hard" CP violation in the SM
(in contrast to spontaneous CP violation)

want to show: mixing matrix V can be source of CP-violation for $n_G \geq 3$:

$$\text{Particle Physics I: } \psi(x^0, \vec{x}) \xrightarrow{\mathcal{P}} \gamma^0 \psi(x^0, -\vec{x})$$

$$\psi(x^0, \vec{x}) \xrightarrow{C} C \gamma^0 \psi^*(x^0, \vec{x})$$

$$C = -i \gamma^2 \gamma^0$$

properties of charge-conjugation matrix C :

$$C \gamma_\mu^T C^{-1} = -\gamma_\mu, \quad C^T = -C, \quad C^\dagger = C^{-1}$$

$$(\Rightarrow C \gamma_5^T C^{-1} = \gamma_5)$$

$$\Rightarrow \psi(x^0, \vec{x}) \xrightarrow{\mathcal{CP}} \underbrace{C \gamma^0 \psi^*(x^0, -\vec{x})}_{\text{II}} = C \psi^*(x^0, -\vec{x})$$

(up to a phase factor)

$$\rightarrow \text{we define: } u'(x) \xrightarrow{\mathcal{CP}} e^{ix_u} C u^*(\tilde{x})$$

$$d'(x) \xrightarrow{\mathcal{CP}} e^{ix_d} C d^*(\tilde{x})$$

$\tilde{x} = (x^0, -\vec{x}), \quad e^{ix_u}, e^{ix_d}$ diagonal phase matrices

\rightarrow mass terms and kinetic terms invariant

$$u' \gamma^\mu (1 - \gamma_5) V d' \xrightarrow{CP}$$

$$(e^{i\alpha_u} c u'^*)^+ \gamma^0 \gamma^\mu (1 - \gamma_5) V e^{i\alpha_d} c d'^*$$

$$= u'^T C \gamma^0 \gamma^\mu (1 - \gamma_5) V e^{i\alpha_d} c d'^*$$

$$= u'^T \gamma^0 \gamma^\mu (1 - \gamma_5^T) e^{-i\alpha_u} V e^{i\alpha_d} d'^*$$

$$= u'_\alpha [(1 - \gamma_5^T) \gamma^\mu \gamma^0]_{\beta\alpha} (e^{-i\alpha_u} V e^{i\alpha_d})_{ij} d'_{j\beta}^*$$

$$= - d'_{j\beta}^* \underbrace{[\gamma^\mu \gamma^0 (1 - \gamma_5^T)]_{\beta\alpha}}_{\begin{matrix} \uparrow \\ \text{anti-comm. fields!} \end{matrix}} (e^{-i\alpha_u} V e^{i\alpha_d})_{ji}^T u'_\alpha$$

$\varepsilon(\mu) \gamma^0 \gamma^\mu \quad \varepsilon(0) = 1, \quad \varepsilon(i) = -1$

$$= - \varepsilon(\mu) \bar{d}' \gamma^\mu (1 - \gamma_5) (e^{-i\alpha_u} V e^{i\alpha_d})^T u'$$

remember:

$$-\mathcal{L}_{V-A}^{\text{quarks}} = -\frac{G_F}{12} [\bar{u}' \gamma^\mu (1 - \gamma_5) V d'] [\bar{d}' \gamma^\mu (1 - \gamma_5) V^T \bar{u}']$$

CP invariance of $\mathcal{L}_{V-A}^{\text{quarks}} \Leftrightarrow \exists \alpha_u, \alpha_d$

$$\text{with } (e^{-i\alpha_u} V e^{i\alpha_d})^T = V^+$$

$$\Leftrightarrow e^{-i\alpha_u} V e^{i\alpha_d} = V^*$$

$\Rightarrow \exists$ phase-convention in which

CKM matrix real:

$$e^{-i\frac{\alpha_u}{2}} V e^{i\frac{\alpha_d}{2}} = e^{i\frac{\alpha_u}{2}} V^* e^{-i\frac{\alpha_d}{2}}$$

$$= (e^{-i\frac{\alpha_u}{2}} V e^{i\frac{\alpha_d}{2}})^*$$

however, for arbitrary complex Yukawa

couplings $\Gamma, \Delta \rightarrow \exists$ physical phase

for $n_G \geq 3 \Rightarrow \cancel{\text{CP}}$ for $n_G \geq 3$

j) effective four-Fermi-interaction for neutral currents

$$-\mathcal{L}_{NC} = \frac{g}{2\cos\theta_W} \underbrace{\sum_F \bar{f} g^\mu (\alpha_F - \beta_F g_5) f}_{\mathcal{J}_NC^\mu} \not{Z}_\mu$$

"integrate" out Z^0 :

$$\rightarrow \mathcal{L}_{NC}^{eff} = -\frac{g^2}{8M_Z^2 \cos^2\theta_W} \mathcal{J}_{NC}^\mu \mathcal{J}_\mu^{NC}$$

$$M_W = M_Z \cos\theta_W$$

$$\rightarrow \mathcal{L}_{NC}^{eff} = -\frac{g^2}{8M_W^2} \underbrace{\mathcal{J}_{NC}^\mu \mathcal{J}_\mu^{NC}}_{\frac{G_F}{12}}$$

$$M_Z = \frac{M_W}{\cos\theta_W} = \frac{37.3 \text{ GeV}}{\sin\theta_W \cos\theta_W} \simeq 89 \text{ GeV}$$

current experimental value (PDG 2008): $M_Z = 91.1876 \pm 0.0021 \text{ GeV}$

(loop corrections!)

decay modes of the Z^0 :

visible : $Z^0 \rightarrow l^+ l^-$, $q\bar{q}$ (jets)

invisible : $Z^0 \rightarrow \nu_e \bar{\nu}_e$ ($l = e, \mu, \tau$)

exercise : $\Gamma(Z^0 \rightarrow \nu_e \bar{\nu}_e) = \frac{G_F M_Z^3}{12 \sqrt{2} \pi} = 166 \text{ MeV}$

$$\Gamma_{\text{tot}} - \Gamma_{\text{visible}} = \Gamma_{\text{invisible}} = 499.0 \pm 1.5 \text{ MeV} \quad (\text{PDG 2008})$$

$\rightarrow N_\nu = 2.984 \pm 0.008$ light neutrino types
 (PDG 2008) (with SM couplings to Z^0)

R) number of parameters of the (minimal) SM

$\hat{M}_{u,d,l}$ 9 fermion masses (electroweak sector only)

V 3 angles, 1 phase

g, g' 2 gauge couplings (alternatively: α, Θ_W)

v VEV

M_h Higgs mass

→ 17 parameters in the electroweak sector
of the minimal SM

extension of the SM: additional (free?)

parameters: neutrino masses, neutrino mixing angles,

CP-violation in the lepton sector (?), ... (?)

2. Quantum Chromodynamics (QCD)

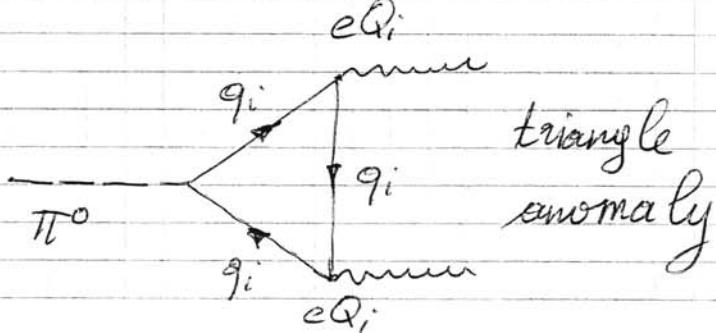
a few arguments in favour of three colour degrees of freedom

$$\text{a) } R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \rightarrow N_c = 3$$

(see p. 9/11)

$$\text{b) } \pi^0 \rightarrow 2\gamma$$

chiral anomaly



$$\Gamma(\bar{\pi}^0 \rightarrow 2\gamma) = \frac{\alpha^2 M_\pi^3}{16\pi^3 F_\pi^3} N_c^2 S$$

F_π = pion decay constant = 92.2 MeV

from $\pi^\pm \rightarrow e^\pm \bar{\nu}_e$

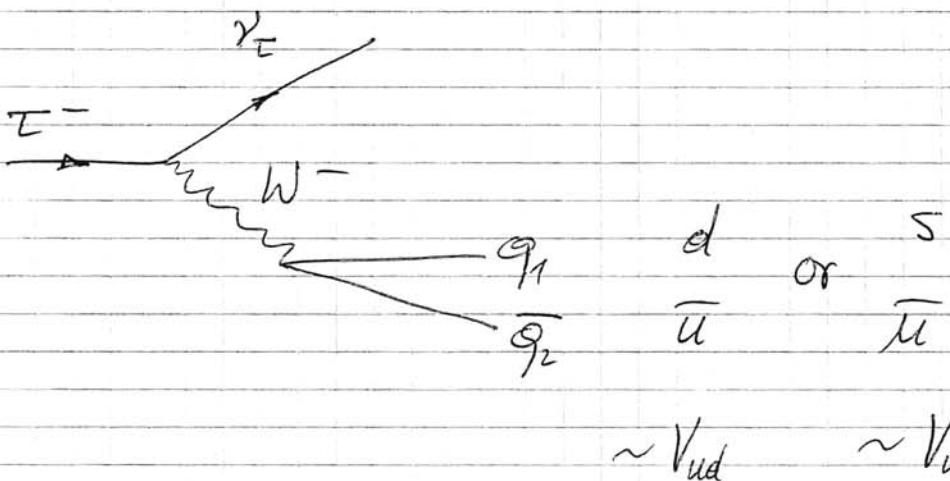
$$S = \sum_{q \in \pi^0} (I_3)_q Q_q^2 = \frac{1}{2} \left(\frac{2}{3}\right)^2 - \frac{1}{2} \left(-\frac{1}{3}\right)^2 = \frac{1}{6}$$

$$\Gamma(\bar{\pi}^0 \rightarrow 2\gamma) = 0.859 N_c^2 \text{ eV}$$

experiment : $\Gamma(\bar{\pi}^0 \rightarrow 2\gamma) = (7.74 \pm 0.60) \text{ eV}$

$$\rightarrow N_c^2 = 9.0 \pm 0.7$$

c) $\tau \rightarrow \nu_\tau + \text{hadrons}$



remark: c cannot be produced

$$(m_\tau = 1776.84 \text{ MeV}, m_{D^\pm} = 1869.62 \text{ MeV})$$

$$m_{D^0} = 1864.84 \text{ MeV})$$

$$D^+ = c\bar{d}, D^0 = c\bar{u}$$

$$R_\tau = \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \simeq N_c$$

$$\text{exp.: } R_\tau = 3.65 \pm 0.020 \quad (\text{PDG 2008})$$

$$\text{QCD corrections: } R_\tau = N_c (1 + \mathcal{O}(g_s^2))$$

→ allows determination of g_s

d) consistent quantization of SM

absence of anomalies $\rightarrow N_c = 3$

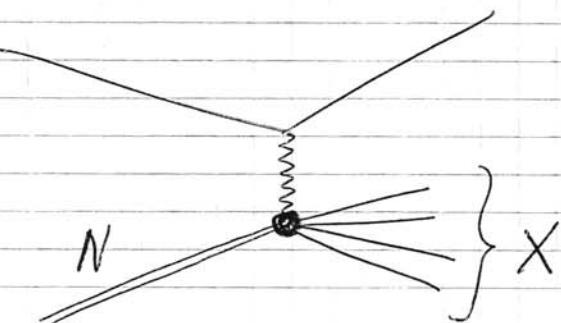
(cancellation of quark- and lepton-contributions
to the anomaly)

e) energy-momentum balance of "partons" in deep-inelastic scattering, at high energies

$$l + N \rightarrow l + X \quad \text{elm. / neutral} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} j, Z^0$$

$$\nu_e + N \rightarrow \ell + X \quad \text{charged weak} \quad \left. \begin{array}{l} \\ \text{current} \\ \end{array} \right\} W^\pm$$

$$\bar{\nu}_e + N \rightarrow \bar{\nu}_e + X \quad \text{neutral weak} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} Z^0$$



exp.: $\sim 1/2$ of partons in nucleon N does not feel electroweak interaction \rightarrow gluons

gauging of colour degree of freedom \rightarrow QCD

\rightarrow standard model $L_{SM} = L_{ew} + L_{QCD}$

in addition to the 17 parameters of L_{ew} \rightarrow

$\rightarrow g_s$ (strong coupling constant)

Θ_{QCD} (strong CP violation)

\rightarrow 19 parameters in the minimal SM