



# The Geometric SMEFT description of curved Higgs Field Space(s)



## Key collaborators and developments in geoSMEFT:

geoSMEFT talk online at All Things EFT



A. Helset



T. Corbett



A. Martin



C. Hays



J. Talbert

1803.08001 Helset, Paraskevas, Trott.

2001.01453 Helset, Martin, Trott.

2010.08451 Corbett, Trott

2010.15852 Corbett,

2107.03951 Talbert, Trott.

2106.10284 Corbett,

2110.03694 Corbett, Rasmussen

1909.08470

2007.00565

2102.02819

2106.13794

2107.07470

2109.05595

Corbett, Helset, Trott

Hays, Helset, Martin, Trott

Helset, Corbett, Martin, Trott

Trott

Corbett, Martin, Trott

Martin, Trott

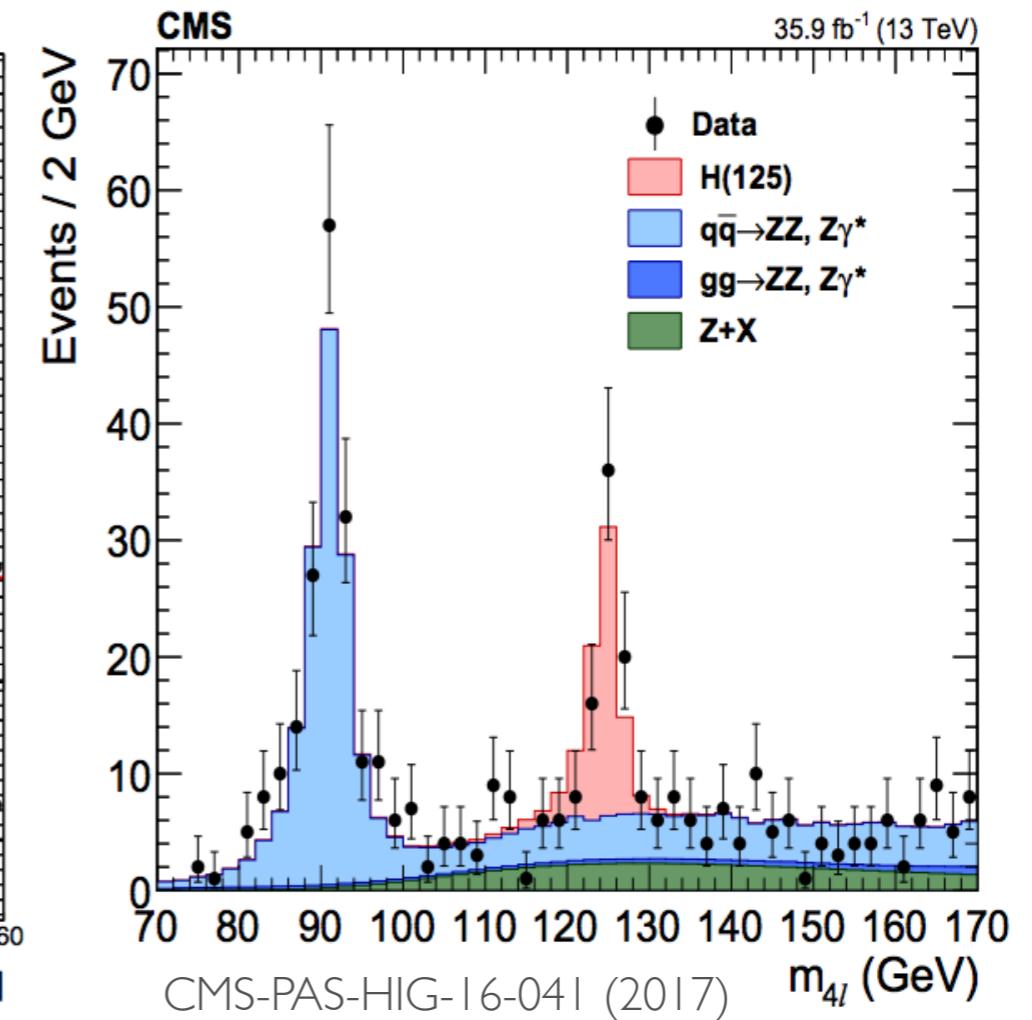
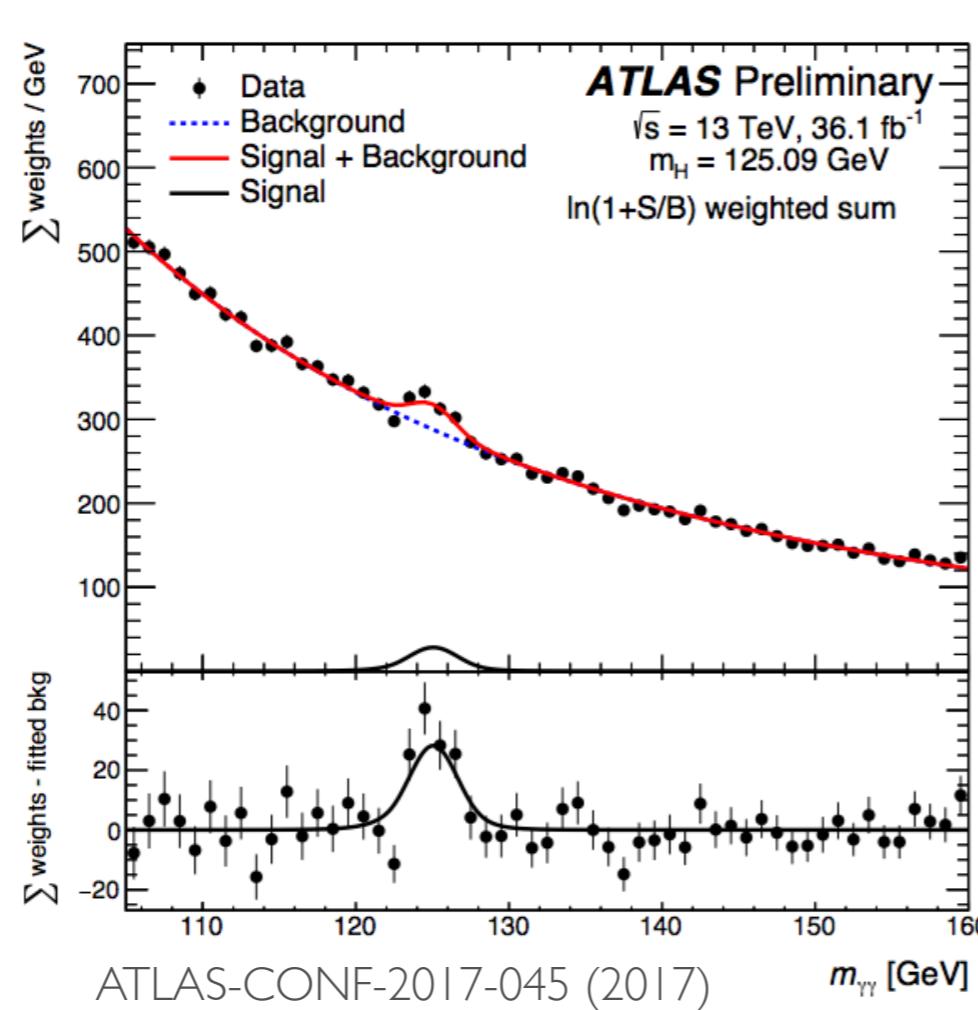
VILLUM FONDEN



Why all the SMEFT stuff.

# What was discovered at LHC, a particle

- Discovery of a (Higgs like)  $J^P \sim 0^+$  particle in 2012



# What wasn't discovered at LHC

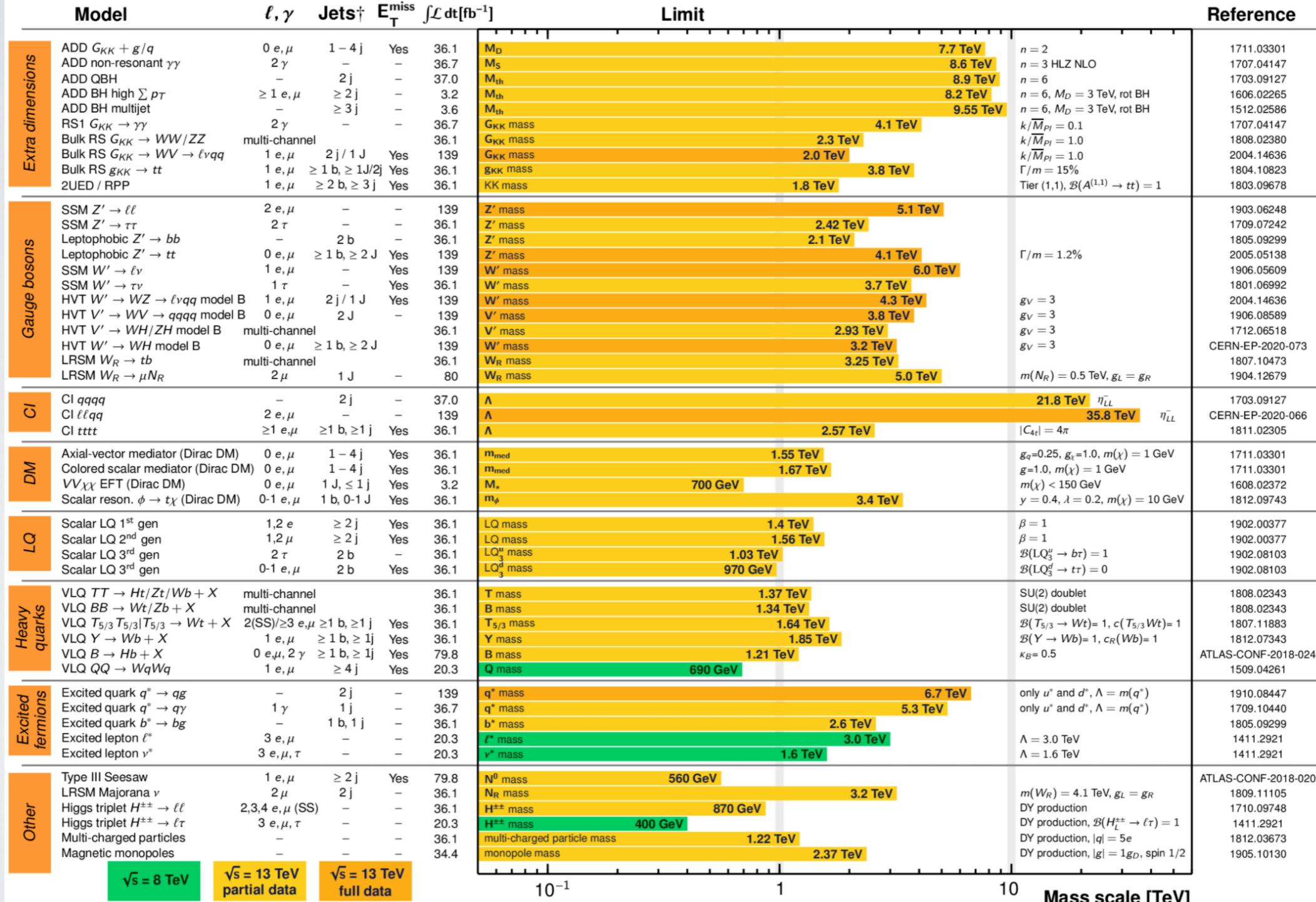
## ATLAS Exotics Searches\* - 95% CL Upper Exclusion Limits

Status: May 2020

ATLAS Preliminary

$\int \mathcal{L} dt = (3.2 - 139) \text{ fb}^{-1}$

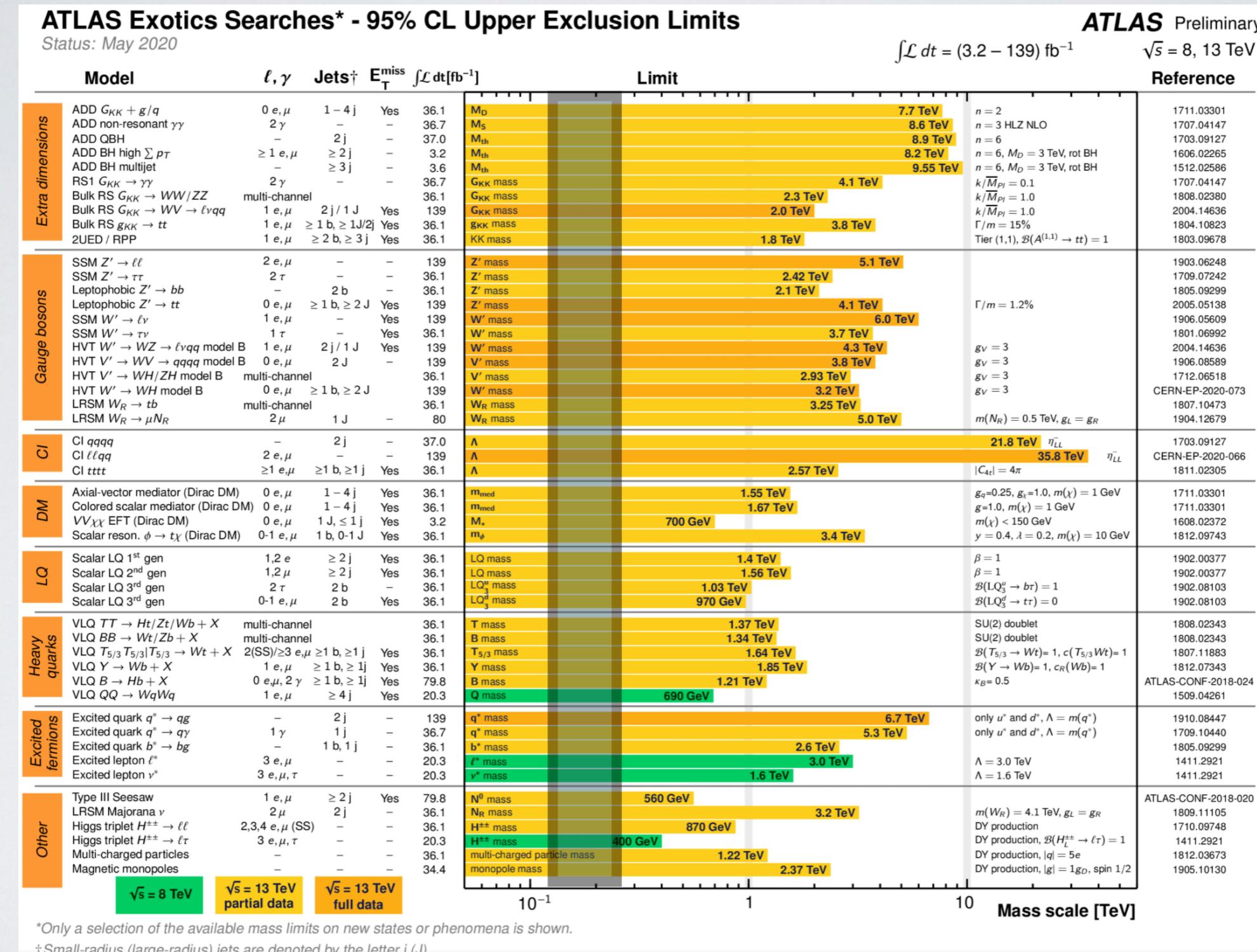
$\sqrt{s} = 8, 13 \text{ TeV}$



\*Only a selection of the available mass limits on new states or phenomena is shown.

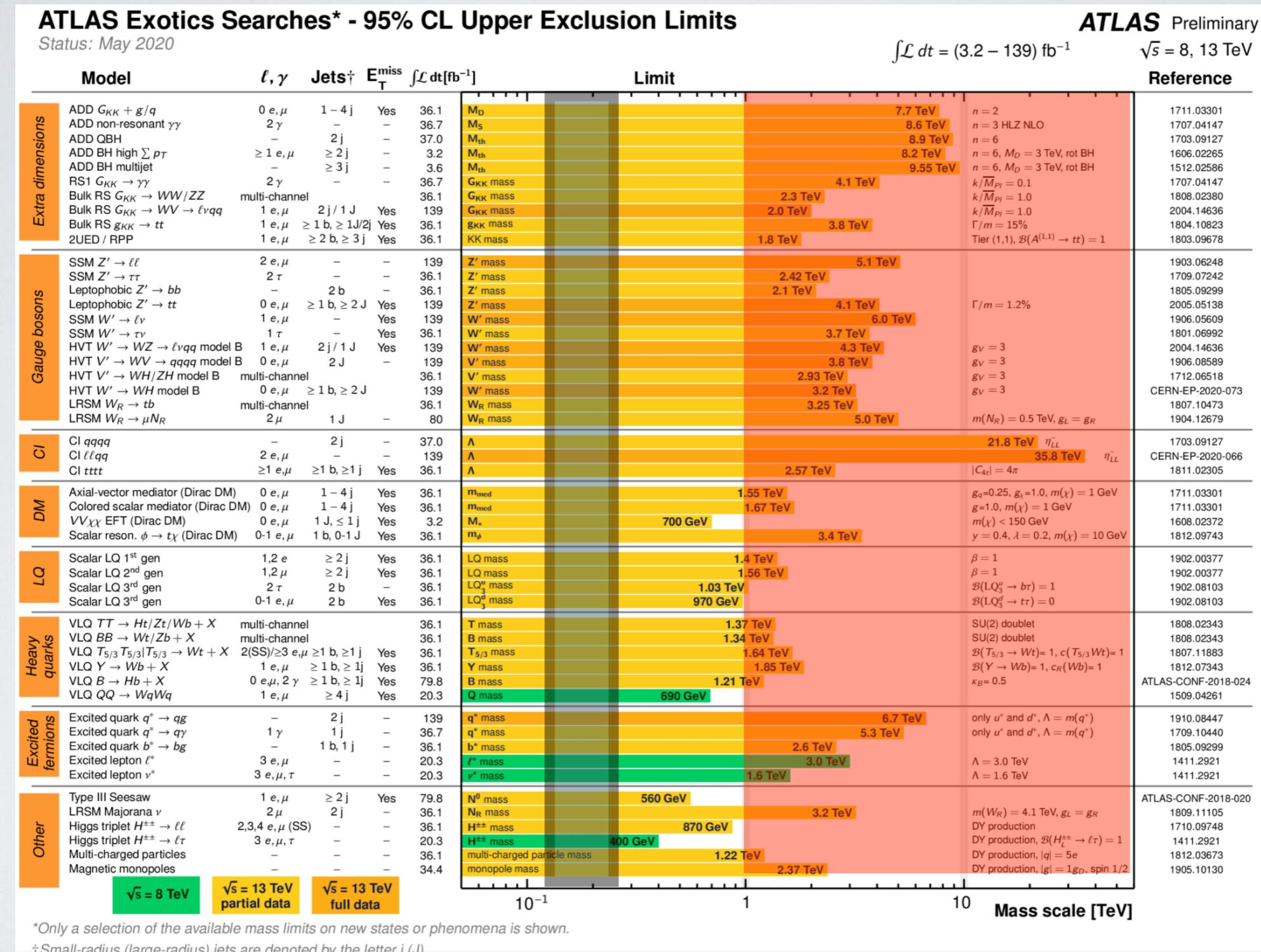
†Small-radius (large-radius) jets are denoted by the letter j (J).

# What wasn't discovered at LHC



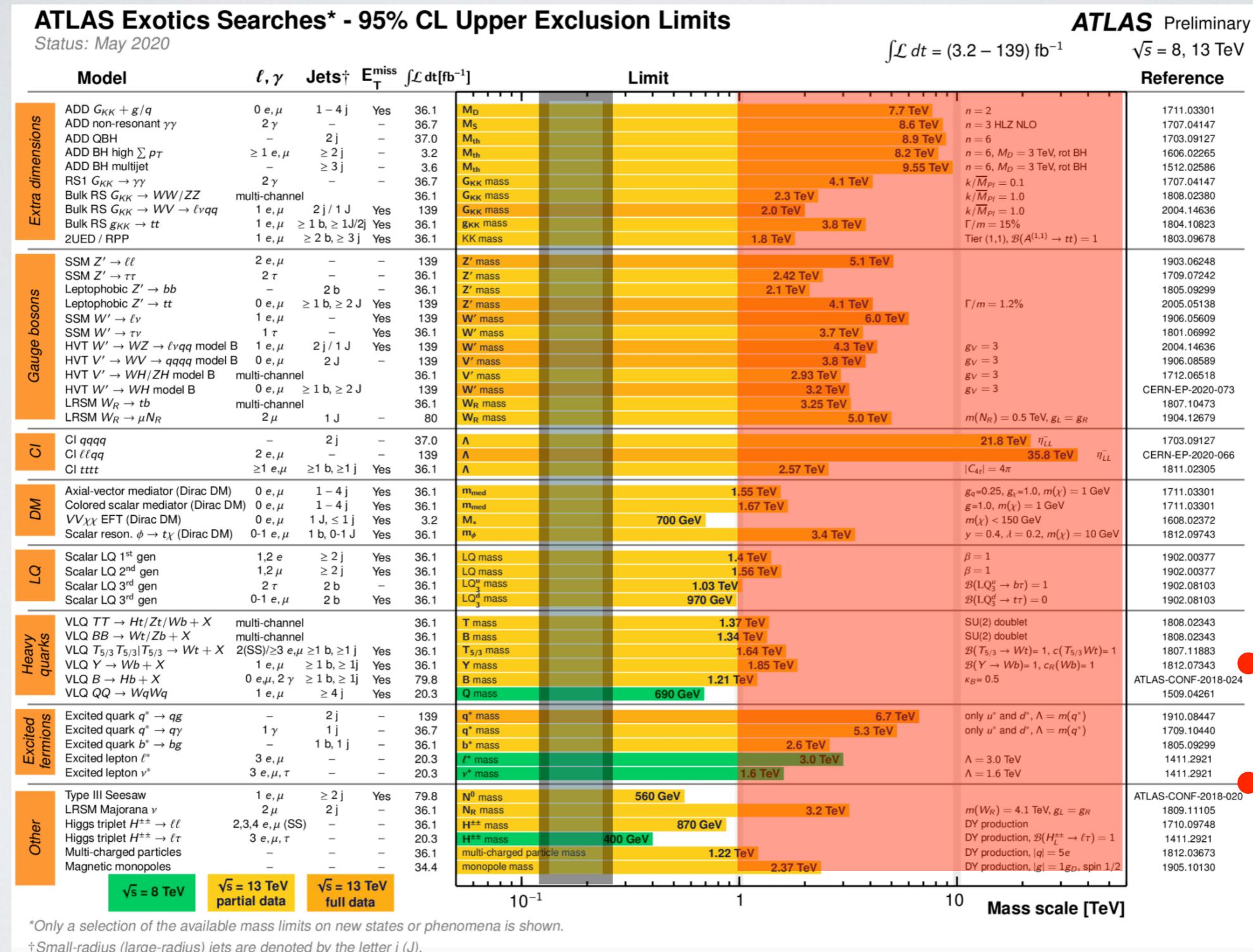
Masses of EW scale ( $\sim g v$ ) states  $m_W, m_Z, m_t, m_h$

# What wasn't discovered at LHC



Bounds have been pushed away from  
 $v \sim m_h$

# What wasn't discovered at LHC

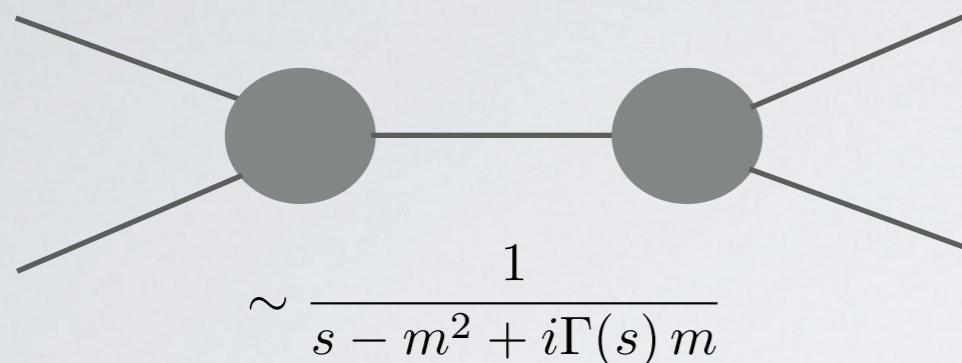


Bounds have been pushed away from  $v \sim m_h$

- USE that  $v/M < 1$  to simplify/for stronger conclusions:
- bound many models at once
- bound multiple resonances at same time

Deviations then look like local contact operator effects in EFT

# When you do measurements below a particle threshold



IF the collision probe does not reach  $\sim m_{heavy}^2$   
THEN observable's dependence on that scale simplified

- You can Taylor expand in LOCAL functions (operators)

$$\langle \rangle \sim O_{SM}^0 + \frac{f_1(s, t, u)}{M_{heavy}^2} + \frac{f_2(s, t, u)}{M_{heavy}^4} + \dots$$

This is the core idea of EFT interpretations of the data.

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots, \quad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

UV dependent Wilson coefficient  
and suppression scale

IR operator form

# SMEFT challenges.

# Parameters in the SMEFT

In Warsaw basis arXiv:1008.4884 (SMEFT standard basis)

Class	$N_{\text{op}}$	CP-even			CP-odd		
		$n_g$	1	3	$n_g$	1	3
1 $g^3 X^3$	4	2	2	2	2	2	2
2 $H^6$	1	1	1	1	0	0	0
3 $H^4 D^2$	2	2	2	2	0	0	0
4 $g^2 X^2 H^2$	8	4	4	4	4	4	4
5 $y\psi^2 H^3$	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6 $gy\psi^2 XH$	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7 $\psi^2 H^2 D$	8	$\frac{1}{2}n_g(9n_g + 7)$	8	51	$\frac{1}{2}n_g(9n_g - 7)$	1	30
8 : $(\bar{L}L)(LL)$	5	$\frac{1}{4}n_g^2(7n_g^2 + 13)$	5	171	$\frac{7}{4}n_g^2(n_g - 1)(n_g + 1)$	0	126
8 : $(\bar{R}R)(\bar{R}R)$	7	$\frac{1}{8}n_g(21n_g^3 + 2n_g^2 + 31n_g + 2)$	7	255	$\frac{1}{8}n_g(21n_g + 2)(n_g - 1)(n_g + 1)$	0	195
8 : $(\bar{L}L)(\bar{R}R)$	8	$4n_g^2(n_g^2 + 1)$	8	360	$4n_g^2(n_g - 1)(n_g + 1)$	0	288
8 : $(\bar{L}R)(\bar{R}L)$	1	$n_g^4$	1	81	$n_g^4$	1	81
8 : $(\bar{L}R)(\bar{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
8 : All	25	$\frac{1}{8}n_g(107n_g^3 + 2n_g^2 + 89n_g + 2)$	25	1191	$\frac{1}{8}n_g(107n_g^3 + 2n_g^2 - 67n_g - 2)$	5	1014
Total	59	$\frac{1}{8}(107n_g^4 + 2n_g^3 + 213n_g^2 + 30n_g + 72)$	53	1350	$\frac{1}{8}(107n_g^4 + 2n_g^3 + 57n_g^2 - 30n_g + 48)$	23	1149

**Table 2.** Number of CP-even and CP-odd coefficients in  $\mathcal{L}^{(6)}$  for  $n_g$  flavors. The total number of coefficients is  $(107n_g^4 + 2n_g^3 + 135n_g^2 + 60)/4$ , which is 76 for  $n_g = 1$  and 2499 for  $n_g = 3$ .

2499

arXiv:1312.2014 Alonso, Jenkins, Manohar, Trott

- Linearly realised symmetries (exact or softly broken) of the SMEFT relate parameters
- Thats a lot of parameters in general.

# Dim 6 SMEFT EW Lagrangian terms

- SMEFT at dimension 6 with all operators is already complicated.
- EW sector parameters redefined in the SMEFT

$$\begin{bmatrix} \mathcal{W}_\mu^3 \\ \mathcal{B}_\mu \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} v_T^2 C_{HWB} \\ -\frac{1}{2} v_T^2 C_{HWB} & 1 \end{bmatrix} \begin{bmatrix} \cos \bar{\theta} & \sin \bar{\theta} \\ -\sin \bar{\theta} & \cos \bar{\theta} \end{bmatrix} \begin{bmatrix} \mathcal{Z}_\mu \\ \mathcal{A}_\mu \end{bmatrix},$$

Mass redefinitions

$$M_W^2 = \frac{\bar{g}_2^2 v_T^2}{4},$$

$$M_Z^2 = \frac{v_T^2}{4} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{1}{8} v_T^4 C_{HD} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{1}{2} v_T^4 \bar{g}_1 \bar{g}_2 C_{HWB}.$$

Mixing angle redefinitions

$$\sin \bar{\theta} = \frac{\bar{g}_1}{\sqrt{\bar{g}_1^2 + \bar{g}_2^2}} \left[ 1 + \frac{v_T^2}{2} \frac{\bar{g}_2}{\bar{g}_1} \frac{\bar{g}_2^2 - \bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} C_{HWB} \right]$$

$$\cos \bar{\theta} = \frac{\bar{g}_2}{\sqrt{\bar{g}_1^2 + \bar{g}_2^2}} \left[ 1 - \frac{v_T^2}{2} \frac{\bar{g}_1}{\bar{g}_2} \frac{\bar{g}_2^2 - \bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} C_{HWB} \right]$$

Interactions to remaining SM fields via:

$$D_\mu = \partial_\mu + i \frac{\bar{g}_2}{\sqrt{2}} [\mathcal{W}_\mu^+ T^+ + \mathcal{W}_\mu^- T^-] + i \bar{g}_Z [T_3 - \bar{s}^2 Q] \mathcal{Z}_\mu + i \bar{e} Q \mathcal{A}_\mu,$$

$$\bar{e} = \frac{\bar{g}_1 \bar{g}_2}{\sqrt{\bar{g}_2^2 + \bar{g}_1^2}} \left[ 1 - \frac{\bar{g}_1 \bar{g}_2}{\bar{g}_2^2 + \bar{g}_1^2} v_T^2 C_{HWB} \right]$$

$$\bar{g}_Z = \sqrt{\bar{g}_2^2 + \bar{g}_1^2} + \frac{\bar{g}_1 \bar{g}_2}{\sqrt{\bar{g}_2^2 + \bar{g}_1^2}} v_T^2 C_{HWB}$$

$$\bar{s}^2 = \sin^2 \bar{\theta} = \frac{\bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} + \frac{\bar{g}_1 \bar{g}_2 (\bar{g}_2^2 - \bar{g}_1^2)}{(\bar{g}_1^2 + \bar{g}_2^2)^2} v_T^2 C_{HWB}.$$

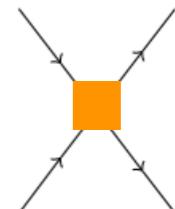
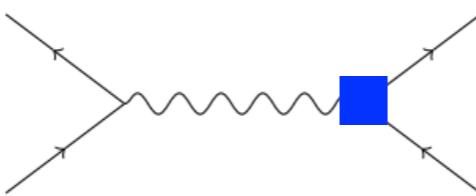
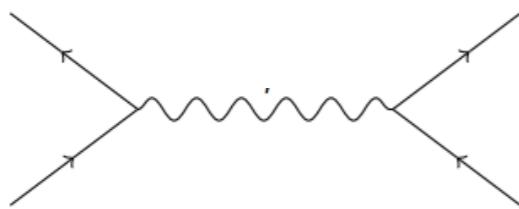
|3|2.2014 Alonso, Jenkins, Manohar, Trott

- Note the complications are proportional to the vev.

# Inputs also needed -SMEFT Muon decay

- Decay of  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$  still measured far below the W pole.
- Still probes the effective lagrangian

$$\mathcal{L}_{G_F} = -\frac{4\mathcal{G}_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\mu P_L \mu) (\bar{e} \gamma_\mu P_L \nu_e)$$



So now

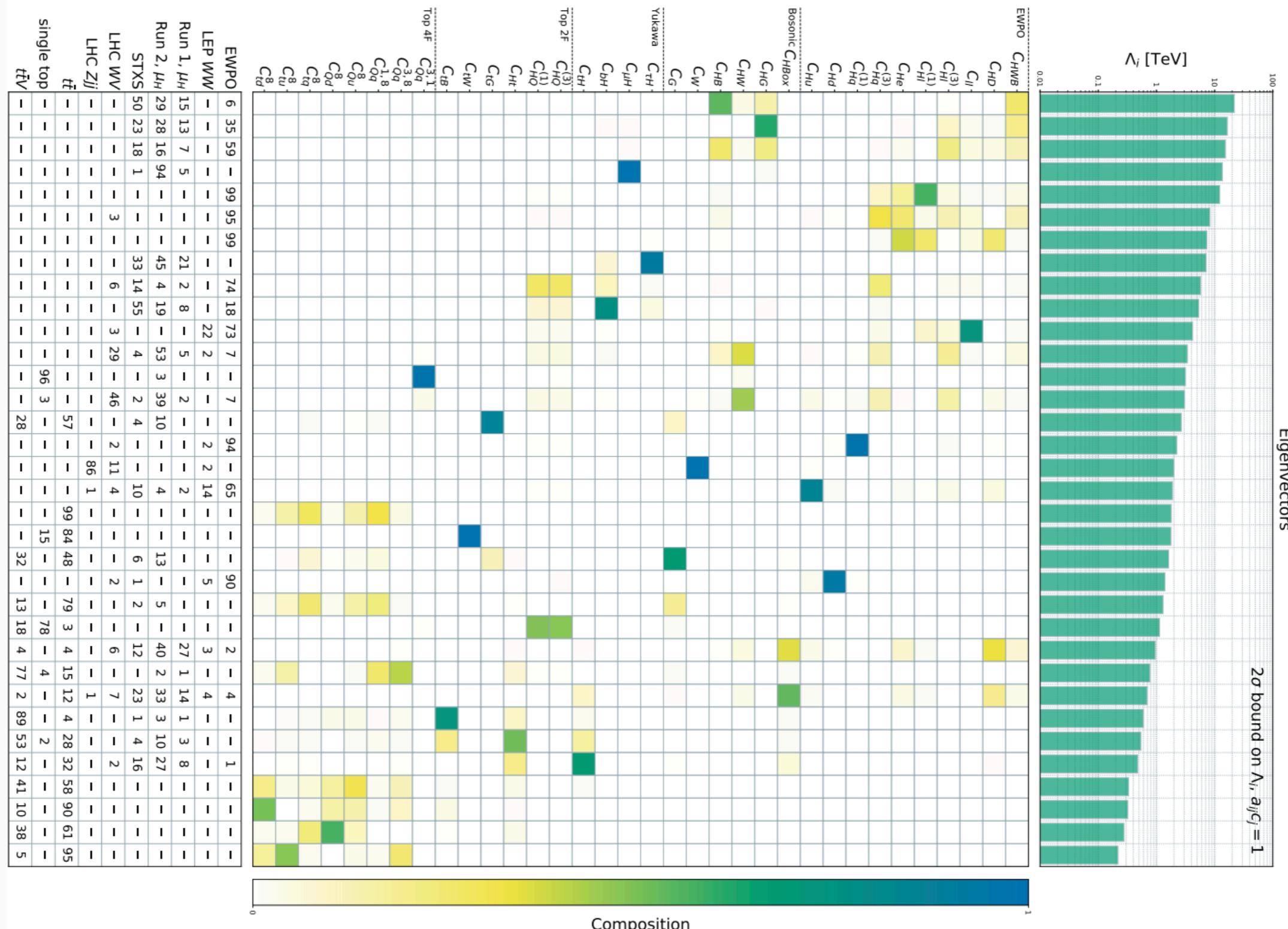
$$-\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{v_T^2} + \left( C_{\mu ee \mu} + C_{e \mu \mu e} \right) - 2 \left( C_{Hl ee}^{(3)} + C_{Hl \mu \mu}^{(3)} \right)$$

$\delta G_F$

- Tons of work to redefine things at dim 6, can we go to dim 8?

# Keep all operators gives eigenvectors of constraint

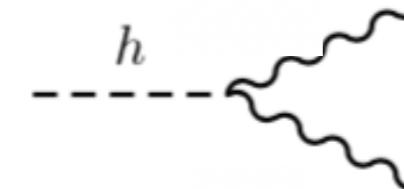
3) Properly eigenvectors of constraint, not individual op limits - what are the spaces?



# Can we do better re basis independence? Yes.

1b) More basis independent results are possible.

All orders expression for Higgs to gamma gamma  
can be defined in closed form as:



Yes! Its a Vielbein of a Higgs space defining asy.  
particle states in SMEFT

$$\langle h | \mathcal{A}(p_1) \mathcal{A}(p_2) \rangle = -\langle h A^{\mu\nu} A_{\mu\nu} \rangle \frac{\sqrt{h}^{44}}{4} \left[ \langle \frac{\delta g_{33}(\phi)}{\delta \phi_4} \rangle \frac{\bar{e}^2}{g_2^2} + 2 \langle \frac{\delta g_{34}(\phi)}{\delta \phi_4} \rangle \frac{\bar{e}^2}{g_1 g_2} + \langle \frac{\delta g_{44}(\phi)}{\delta \phi_4} \rangle \frac{\bar{e}^2}{g_1^2} \right],$$

Kinematic structure



Geometric Dressings



No explicit SMEFT expansion op forms. But all orders in vev expansion!  
How the heck do we get to such results?

Why all the geoSMEFT stuff.

# Consequences of the Higgs field becoming a number

The Higgs field takes on a vev, recall what happens:

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi$$
$$-\lambda \left( H^\dagger H - \frac{1}{2}v^2 \right)^2 - \left[ H^{\dagger j} \bar{d} Y_d q_j + \tilde{H}^{\dagger j} \bar{u} Y_u q_j + H^{\dagger j} \bar{e} Y_e l_j + \text{h.c.} \right],$$

$$D \leq 4$$

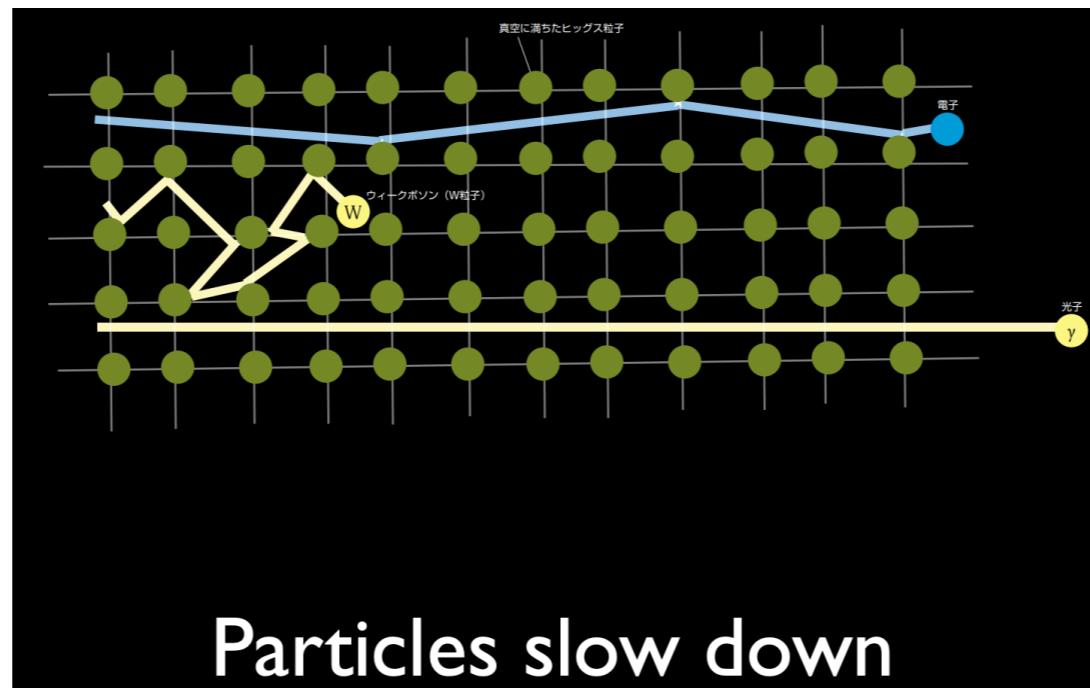
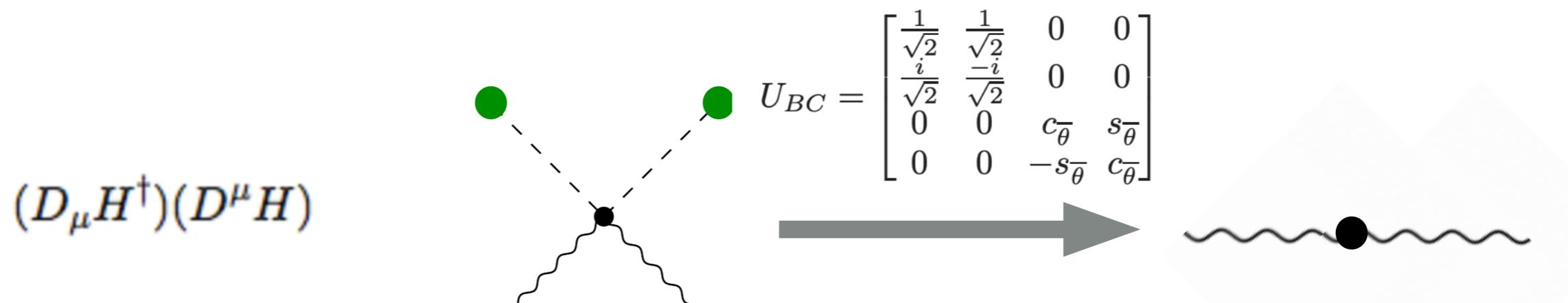


Image credit: Hitoshi's Higgs2020 talk

Gives masses, mass eigenstate fields, useful combinations of fields and couplings

# Consequences of the Higgs field becoming a number

The Higgs field takes on a vev, recall what happens:



4-point

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & c_{\bar{\theta}} & s_{\bar{\theta}} \\ 0 & 0 & -s_{\bar{\theta}} & c_{\bar{\theta}} \end{bmatrix}$$

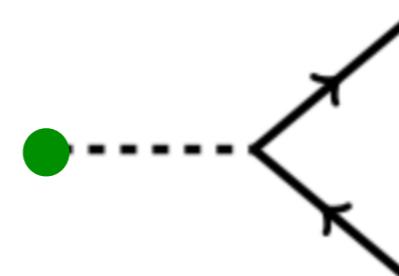
$$\mathcal{W}_B^\nu = U_{BC} \mathcal{A}^{C,\nu}$$

$$\mathcal{W}_B = \{W_1, W_2, W_3, B\}$$

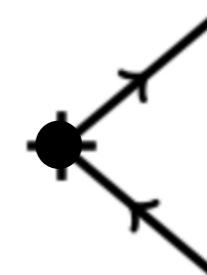
$$\mathcal{A}_C = \{W^+, W^-, Z, A\}$$

2-point (mass)

$$\left[ H^{\dagger j} \bar{d} Y_d q_j + \tilde{H}^{\dagger j} \bar{u} Y_u q_j + H^{\dagger j} \bar{e} Y_e l_j + \text{h.c.} \right]$$

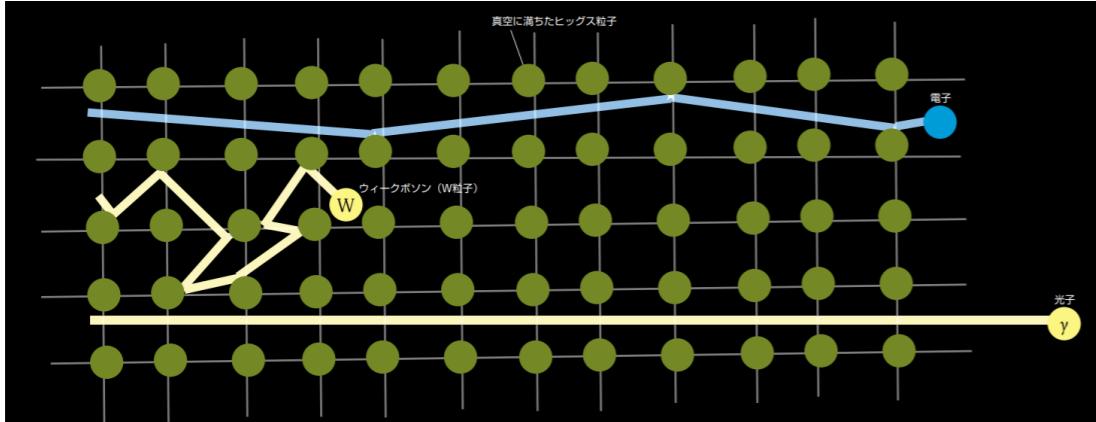


3-point

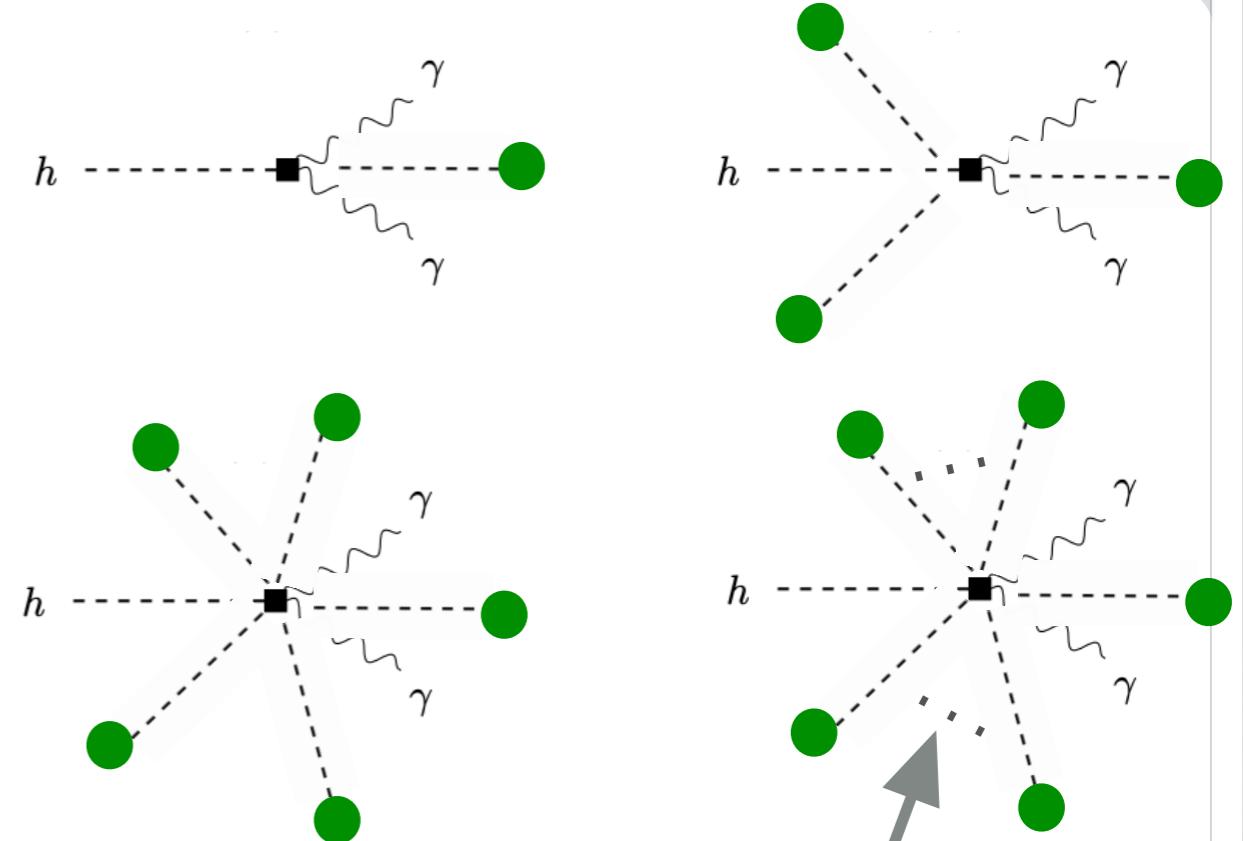


2-point (mass)

# What is the Geometric SMEFT?



Particles slow down



Powers of  $\frac{H^\dagger H}{\Lambda^2}$   
and symmetry generators

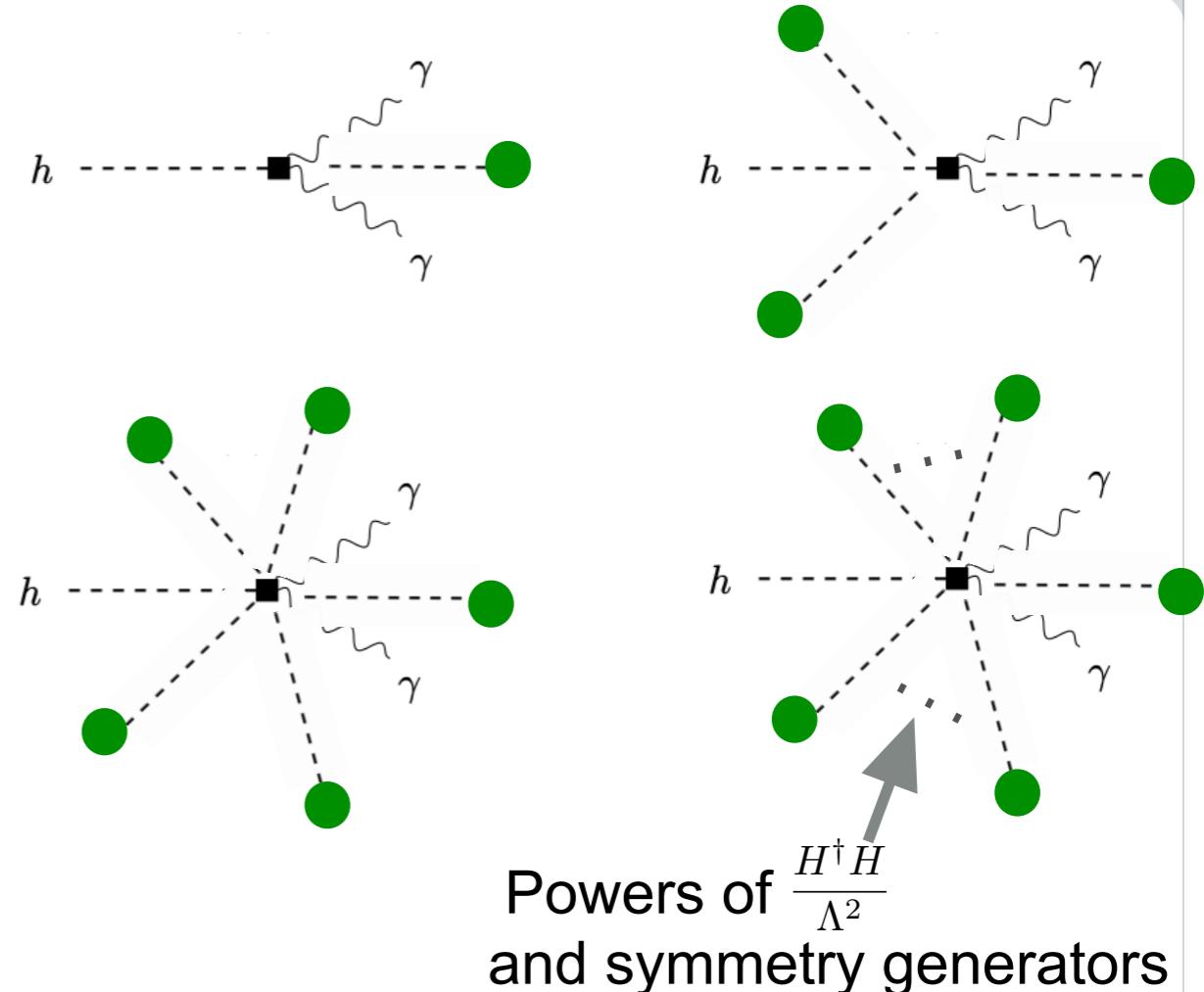
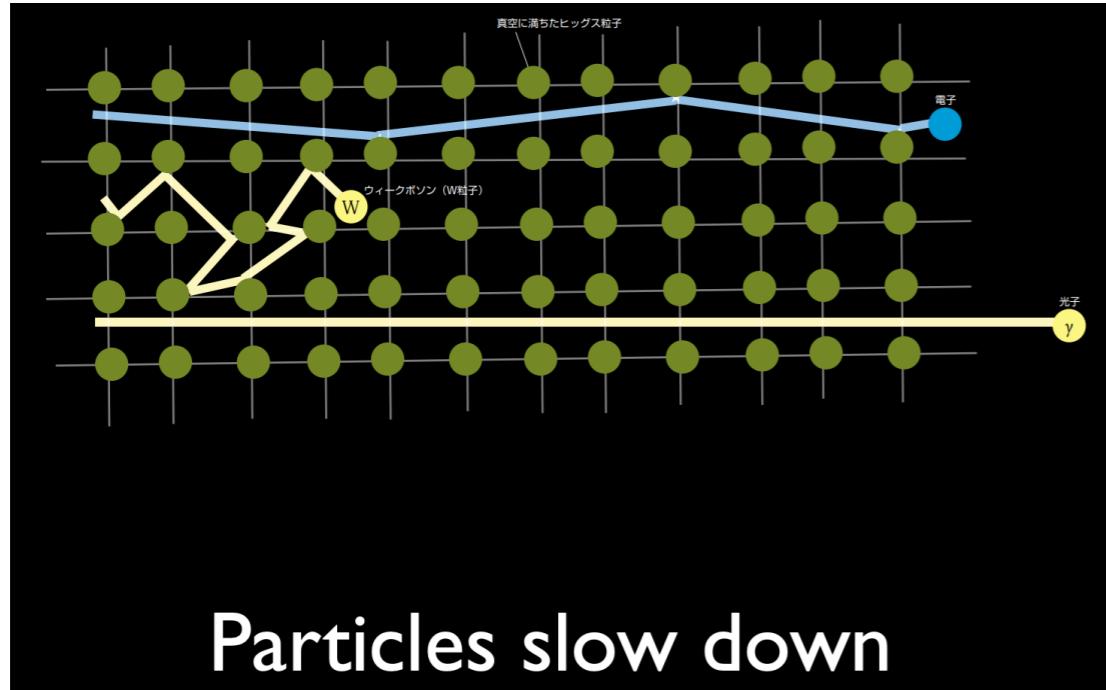
$$D \leq 4$$

$$D > 4$$

Gives masses, mass eigenstate fields.

Gives geometries that define the mass eigenstate fields and interactions in the EFT

# What is the Geometric SMEFT?



$$\langle h | \mathcal{A}(p_1) \mathcal{A}(p_2) \rangle = - \langle h A^{\mu\nu} A_{\mu\nu} \rangle \frac{\sqrt{h}^{44}}{4} \left[ \langle \frac{\delta g_{33}(\phi)}{\delta \phi_4} \rangle \frac{\bar{e}^2}{g_2^2} + 2 \langle \frac{\delta g_{34}(\phi)}{\delta \phi_4} \rangle \frac{\bar{e}^2}{g_1 g_2} + \langle \frac{\delta g_{44}(\phi)}{\delta \phi_4} \rangle \frac{\bar{e}^2}{g_1^2} \right],$$

Kinematic structure



Geometric Dressings

# Curved SMEFT spaces: scalar fields

- Curved SMEFT field space manifest in background field formulation

In general terms: G. A. Vilkovisky, Nucl. Phys. B234 (1984) 125.

Metric on Higgs field space, SM a **FLAT** field space

$$\mathcal{L}_{\text{scalar,kin}} = \frac{1}{2} h_{IJ}(\phi) (D_\mu \phi)^I (D^\mu \phi)^J, \quad \text{Where } H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

$$\sqrt{h}^{IJ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - \frac{1}{4}\tilde{C}_{HD} & 0 \\ 0 & 0 & 0 & 1 + \tilde{C}_{H\square} - \frac{1}{4}\tilde{C}_{HD} \end{bmatrix}$$

here  $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

Small perturbations so positive semi-definite  
Matrix and unique square root

| 002.2730 Burgess, Lee, Trott

(sqrt) Metric in SMEFT, a *curved* field space

$$R^I_{JKL} \neq 0$$

| 511.00724 Alonso, Jenkins, Manohar  
| 605.03602 Alonso, Jenkins, Manohar

# Curved SMEFT space: gauge fields

- Similarly in the gauge coupling space a curved field space

Metric on gauge field space, SM a **FLAT** field space

$$\mathcal{L}_{\text{gauge,kin}} = -\frac{1}{4}g_{AB}(\phi)\mathcal{W}_{\mu\nu}^A\mathcal{W}^{B,\mu\nu}, \quad \text{Where} \quad \mathcal{W}^A = (W^1, W^2, W^3, B)$$

$$\sqrt{g}^{AB} = \begin{bmatrix} 1 + \tilde{C}_{HW} & 0 & 0 & 0 \\ 0 & 1 + \tilde{C}_{HW} & 0 & 0 \\ 0 & 0 & 1 + \tilde{C}_{HW} & -\frac{\tilde{C}_{HWB}}{2} \\ 0 & 0 & -\frac{\tilde{C}_{HWB}}{2} & 1 + \tilde{C}_{HB} \end{bmatrix}$$

here  $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

|803.08001 Helset, Paraskevas,Trott  
|909.08470 Corbett, Helset,Trott

(sqrt) Metric in SMEFT, a *curved* field space

# All orders SM Lagrangian parameters

- Low n-point interactions of fields are parameterised in terms of couplings,

2001.01453 Helset, Martin, Trott

$$\begin{aligned}\bar{g}_2 &= g_2 \sqrt{g^{11}} = g_2 \sqrt{g^{22}}, \\ \bar{g}_Z &= \frac{g_2}{c_{\theta_Z}^2} \left( c_{\bar{\theta}} \sqrt{g^{33}} - s_{\bar{\theta}} \sqrt{g^{34}} \right) = \frac{g_1}{s_{\theta_Z}^2} \left( s_{\bar{\theta}} \sqrt{g^{44}} - c_{\bar{\theta}} \sqrt{g^{34}} \right), \\ \bar{e} &= g_2 \left( s_{\bar{\theta}} \sqrt{g^{33}} + c_{\bar{\theta}} \sqrt{g^{34}} \right) = g_1 \left( c_{\bar{\theta}} \sqrt{g^{44}} + s_{\bar{\theta}} \sqrt{g^{34}} \right),\end{aligned}$$

- Masses

$$\bar{m}_W^2 = \frac{\bar{g}_2^2}{4} \sqrt{h_{11}^{-2}} \bar{v}_T^2, \quad \bar{m}_Z^2 = \frac{\bar{g}_Z^2}{4} \sqrt{h_{33}^{-2}} \bar{v}_T^2 \quad \bar{m}_A^2 = 0.$$

- Mixing angles:

$$\begin{aligned}s_{\theta_Z}^2 &= \frac{g_1(\sqrt{g^{44}} s_{\bar{\theta}} - \sqrt{g^{34}} c_{\bar{\theta}})}{g_2(\sqrt{g^{33}} c_{\bar{\theta}} - \sqrt{g^{34}} s_{\bar{\theta}}) + g_1(\sqrt{g^{44}} s_{\bar{\theta}} - \sqrt{g^{34}} c_{\bar{\theta}})}, \\ s_{\bar{\theta}}^2 &= \frac{(g_1 \sqrt{g^{44}} - g_2 \sqrt{g^{34}})^2}{g_1^2[(\sqrt{g^{34}})^2 + (\sqrt{g^{44}})^2] + g_2^2[(\sqrt{g^{33}})^2 + (\sqrt{g^{34}})^2] - 2g_1g_2\sqrt{g^{34}}(\sqrt{g^{33}} + \sqrt{g^{44}})}.\end{aligned}$$

(Interesting way to think of the Weinberg angle)

# All orders expressions are known now

- All orders scalar metric -leading to gauge boson masses in SMEFT

$$h_{IJ} = \left[ 1 + \phi^2 C_{H\square}^{(6)} + \sum_{n=0}^{\infty} \left( \frac{\phi^2}{2} \right)^{n+2} (C_{HD}^{(8+2n)} - C_{H,D2}^{(8+2n)}) \right] \delta_{IJ}$$

$$+ \frac{\Gamma_{A,J}^I \phi_K \Gamma_{A,L}^K \phi^L}{2} \left( \frac{C_{HD}^{(6)}}{2} + \sum_{n=0}^{\infty} \left( \frac{\phi^2}{2} \right)^{n+1} C_{H,D2}^{(8+2n)} \right).$$

- All orders gauge metric - gives mass eigenstate couplings in SMEFT

$$g_{AB}(\phi_I) = \left[ 1 - 4 \sum_{n=0}^{\infty} (C_{HW}^{(6+2n)}(1 - \delta_{A4}) + C_{HB}^{(6+2n)}\delta_{A4}) \left( \frac{\phi^2}{2} \right)^{n+1} \right] \delta_{AB}$$

$$- \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)} \left( \frac{\phi^2}{2} \right)^n (\phi_I \Gamma_{A,J}^I \phi^J) (\phi_L \Gamma_{B,K}^L \phi^K) (1 - \delta_{A4})(1 - \delta_{B4})$$

$$+ \left[ \sum_{n=0}^{\infty} C_{HWB}^{(6+2n)} \left( \frac{\phi^2}{2} \right)^n \right] [(\phi_I \Gamma_{A,J}^I \phi^J) (1 - \delta_{A4})\delta_{B4} + (A \leftrightarrow B)],$$

- Number of operator forms saturate in geosmefit.

This is due to reducing possible generator insertions on the Higgs manifold

$$T_{ij}^a T_{kl}^a = \frac{1}{2} \left( \delta_{il} \delta_{jk} - \frac{1}{N} \delta_{ij} \delta_{kl} \right)$$

# SM weak-mass eigenstate relations

- Weak eigenstates

$$\hat{\mathcal{W}}^{A,\nu} = \delta^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$

$$\hat{\alpha}^A = \delta^{AB} U_{BC} \hat{\beta}^C,$$

$$\hat{\phi}^J = \delta^{JK} V_{KL} \hat{\Phi}^L,$$

## Mass eigenstate

## Rotations

Flat field space's.  
Due to  $D \leq 4$

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\bar{\theta}} & s_{\bar{\theta}} \\ 0 & 0 & -s_{\bar{\theta}} & c_{\bar{\theta}} \end{bmatrix}$$

$$V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi^J = \{\phi_1, \phi_2, \phi_3, \phi_4\}, \Phi^K = \{\Phi^-, \Phi^+, \chi, h\}$$

$$\alpha^A = \{g_2 g_2, g_2, g_1\},$$

$$\beta^C = \left\{ \frac{g_2(1-i)}{\sqrt{2}}, \frac{g_2(1+i)}{\sqrt{2}}, \sqrt{g_1^2 + g_2^2}(c_{\bar{\theta}}^2 - s_{\bar{\theta}}^2), \frac{2g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \right\},$$

$$\mathcal{W}^A = \{W_1, W_2, W_3, B\},$$

$$\mathcal{A}^C = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A}).$$

What else could you write?

# SMEFT weak-mass eigenstate relations

- Weak eigenstates

1909.08470 Corbett, Helset, Trott

## Mass eigenstate

### Generator transform

$$\gamma_{C,J}^I = \frac{1}{2} \tilde{\gamma}_{A,J}^I \sqrt{g^{AB}} U_{BC}.$$

SMEFT Field space metrics  
(Now known to all orders)

### Rotations

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & c_{\bar{\theta}} & s_{\bar{\theta}} \\ 0 & 0 & -s_{\bar{\theta}} & c_{\bar{\theta}} \end{bmatrix}$$

$$V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi^J = \{\phi_1, \phi_2, \phi_3, \phi_4\}, \Phi^K = \{\Phi^-, \Phi^+, \chi, h\}$$

$$\alpha^A = \{g_2 g_2, g_2, g_1\},$$

$$\beta^C = \left\{ \frac{g_2(1-i)}{\sqrt{2}}, \frac{g_2(1+i)}{\sqrt{2}}, \sqrt{g_1^2 + g_2^2}(c_{\bar{\theta}}^2 - s_{\bar{\theta}}^2), \frac{2g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \right\}, \quad \mathcal{W}^A = \{W_1, W_2, W_3, B\},$$

$$\mathcal{A}^C = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A}).$$

What else could you write? Nothing that generalises to all orders.

# Dim 6 SMEFT EW Lagrangian terms

- EW sector parameters redefined in the SMEFT (already in SMEFTsim)

$$\begin{bmatrix} \mathcal{W}_\mu^3 \\ \mathcal{B}_\mu \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} v_T^2 C_{HWB} \\ -\frac{1}{2} v_T^2 C_{HWB} & 1 \end{bmatrix} \begin{bmatrix} \cos \bar{\theta} & \sin \bar{\theta} \\ -\sin \bar{\theta} & \cos \bar{\theta} \end{bmatrix} \begin{bmatrix} \mathcal{Z}_\mu \\ \mathcal{A}_\mu \end{bmatrix},$$

Mass redefinitions

$$M_W^2 = \frac{\bar{g}_2^2 v_T^2}{4},$$

$$M_Z^2 = \frac{v_T^2}{4} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{1}{8} v_T^4 C_{HD} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{1}{2} v_T^4 \bar{g}_1 \bar{g}_2 C_{HWB}.$$

Mixing angle redefinitions

$$\sin \bar{\theta} = \frac{\bar{g}_1}{\sqrt{\bar{g}_1^2 + \bar{g}_2^2}} \left[ 1 + \frac{v_T^2}{2} \frac{\bar{g}_2}{\bar{g}_1} \frac{\bar{g}_2^2 - \bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} C_{HWB} \right]$$

$$\cos \bar{\theta} = \frac{\bar{g}_2}{\sqrt{\bar{g}_1^2 + \bar{g}_2^2}} \left[ 1 - \frac{v_T^2}{2} \frac{\bar{g}_1}{\bar{g}_2} \frac{\bar{g}_2^2 - \bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} C_{HWB} \right]$$

Interactions to remaining SM fields via:

$$D_\mu = \partial_\mu + i \frac{\bar{g}_2}{\sqrt{2}} [\mathcal{W}_\mu^+ T^+ + \mathcal{W}_\mu^- T^-] + i \bar{g}_Z [T_3 - \bar{s}^2 Q] \mathcal{Z}_\mu + i \bar{e} Q \mathcal{A}_\mu,$$

$$\bar{e} = \frac{\bar{g}_1 \bar{g}_2}{\sqrt{\bar{g}_2^2 + \bar{g}_1^2}} \left[ 1 - \frac{\bar{g}_1 \bar{g}_2}{\bar{g}_2^2 + \bar{g}_1^2} v_T^2 C_{HWB} \right]$$

$$\bar{g}_Z = \sqrt{\bar{g}_2^2 + \bar{g}_1^2} + \frac{\bar{g}_1 \bar{g}_2}{\sqrt{\bar{g}_2^2 + \bar{g}_1^2}} v_T^2 C_{HWB}$$

- Dim 8,10 etc solved in closed form. Just expand.

$$\bar{s}^2 = \sin^2 \bar{\theta} = \frac{\bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} + \frac{\bar{g}_1 \bar{g}_2 (\bar{g}_2^2 - \bar{g}_1^2)}{(\bar{g}_1^2 + \bar{g}_2^2)^2} v_T^2 C_{HWB}.$$

# Generalisation for composite ops

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots, \quad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$
$$v/M < 1$$

$$\mathcal{L}_{SMEFT} = \sum_i f_i(\alpha \cdots) G_i(I, A \cdots),$$

Derivative expansion

Composite operator form  
With minimal scalar field coordinate dependence

Vev expansion

Scalar field coordinate dependence  
And insertions of symmetry generators

$$D^\mu \phi$$

Mixes expansions, but grouped with derivative forms.

# Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta(D_\mu \phi)^I \delta(D_\nu \phi)^J} \Big|_{\mathcal{L}(\alpha, \beta \dots) \rightarrow 0}.$$

↑  
non-trivial Lorentz-index-carrying Lagrangian terms and spin connections  $\{\mathcal{W}_{\mu\nu}^A, (D^\mu \Phi)^K, \bar{\psi} \sigma^\mu \psi, \bar{\psi} \psi \dots\}$

- Limited number of such connections for up to three point functions

$$V(\phi) \quad h_{IJ}(\phi)(D_\mu \phi)^I (D_\mu \phi)^J, \quad g_{AB}(\phi) \mathcal{W}_{\mu\nu}^A \mathcal{W}^{B,\mu\nu}, \quad k_{IJ}^A(\phi) (D_\mu \phi)^I (D_\nu \phi)^J \mathcal{W}_A^{\mu\nu}, \\ f_{ABC}(\phi) \mathcal{W}_{\mu\nu}^A \mathcal{W}^{B,\nu\rho} \mathcal{W}_\rho^{C,\mu},$$

With fermions  $Y(\phi) \bar{\psi}_1 \psi_2, \quad L_{I,A}(\phi) \bar{\psi}_1 \gamma^\mu \tau_A \psi_2 (D_\mu \phi)^I, \quad d_A(\phi) \bar{\psi}_1 \sigma^{\mu\nu} \psi_2 \mathcal{W}_{\mu\nu}^A,$

Gluon fields  $k_{AB}(\phi) G_{\mu\nu}^A G^{B,\mu\nu}, \quad k_{ABC}(\phi) G_{\nu\mu}^A G^{B,\rho\nu} G^{C,\mu\rho}, \quad c(\phi) \bar{\psi}_1 \sigma^{\mu\nu} T_A \psi_2 G_{\mu\nu}^A.$

# Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta(D_\mu \phi)^I \delta(D_\nu \phi)^J} \Big|_{\mathcal{L}(\alpha, \beta \dots) \rightarrow 0}.$$

non-trivial Lorentz-index-carrying Lagrangian terms and spin connections  $\{\mathcal{W}_{\mu\nu}^A, (D^\mu \Phi)^K, \bar{\psi} \sigma^\mu \psi, \bar{\psi} \psi \dots\}$

- Limited number of such connections for up to three point functions

This is a non trivial fact proven in

2001.01453 Helset, Martin, Trott

There is a theory choice here - its REMOVE DERIVATIVE OPS, USE EOM.

Same reasoning built into, and led to the “Warsaw basis”.

Also why we were able to renormalise the Warsaw basis completely in 2013.

EFT Industry standard in flavour physics, chiral pert theory etc.

# An instant pay off of this approach

- Growth in operator forms in connections  
Always saturate to fixed number, this is just the simplest organization exploiting this

Field space connection	Mass Dimension	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}_{\mu\nu}^A\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}_{\mu\nu}^A$	0	3	4	4	4	4
$f_{ABC}(\phi)\mathcal{W}_{\mu\nu}^A\mathcal{W}^{B,\nu\rho}\mathcal{W}_\rho^{C,\mu}$	1	2	2	2	2	2
$Y_{pr}^u(\phi)\bar{Q}u + \text{h.c.}$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$
$Y_{pr}^d(\phi)\bar{Q}d + \text{h.c.}$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$
$Y_{pr}^e(\phi)\bar{L}e + \text{h.c.}$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$	$2N_f^2$
$d_A^{e,pr}(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}_A^{\mu\nu} + \text{h.c.}$	$4N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$
$d_A^{u,pr}(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}_A^{\mu\nu} + \text{h.c.}$	$4N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$
$d_A^{d,pr}(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}_A^{\mu\nu} + \text{h.c.}$	$4N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$	$6N_f^2$
$L_{pr,A}^{\psi_R}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	$N_f^2$	$N_f^2$	$N_f^2$	$N_f^2$	$N_f^2$	$N_f^2$
$L_{pr,A}^{\psi_L}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2N_f^2$	$4N_f^2$	$4N_f^2$	$4N_f^2$	$4N_f^2$	$4N_f^2$

- Once we have things to dim eight it is sufficient in many observables

Mases

Couplings and mixing angles

TGC, Higgs to ZZ,WW

QGC,TGC + Higgs

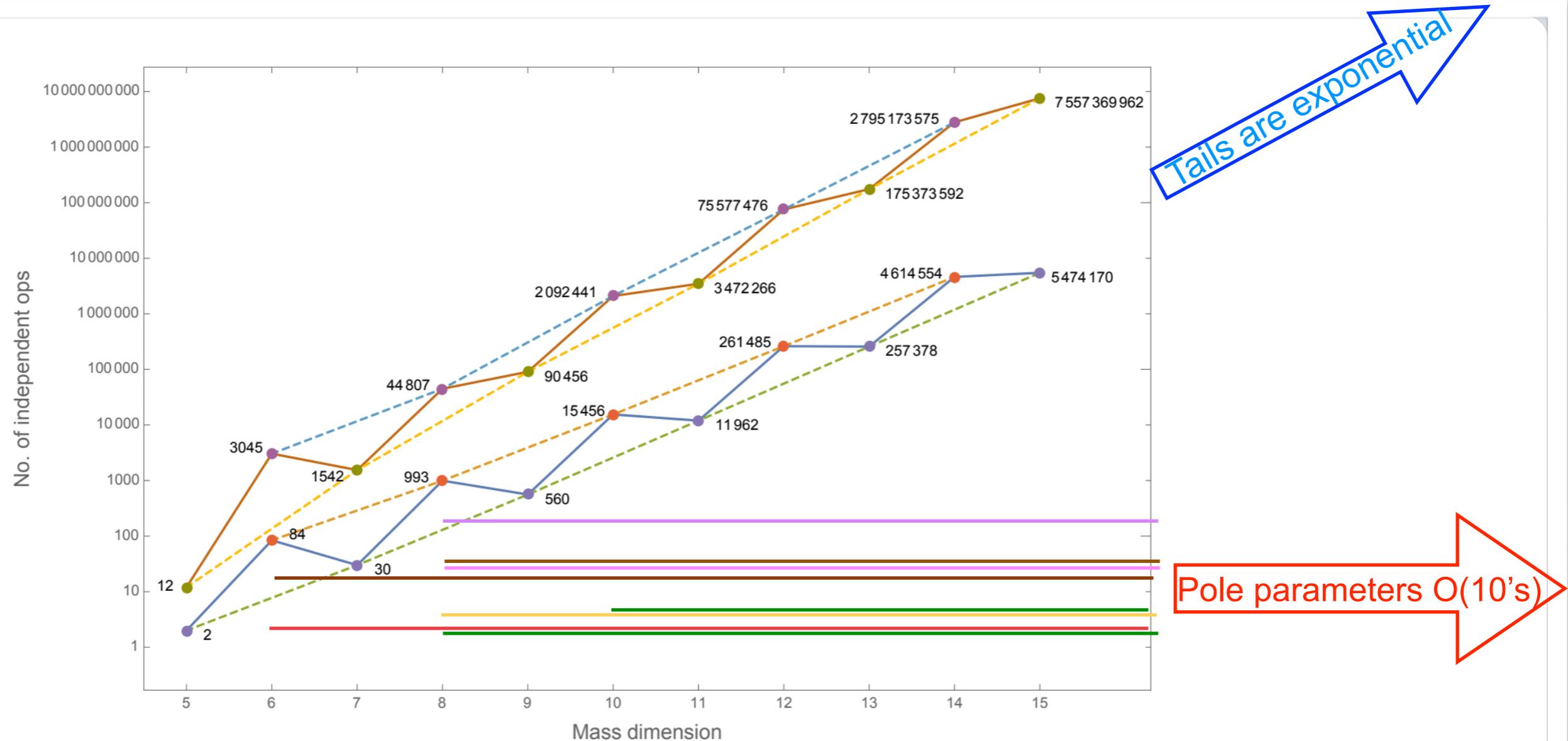
Yukawas

Dipoles

W,Z couplings to fermions + higgs

2001.01453 Helset, Martin, Trott

# Application to theory error

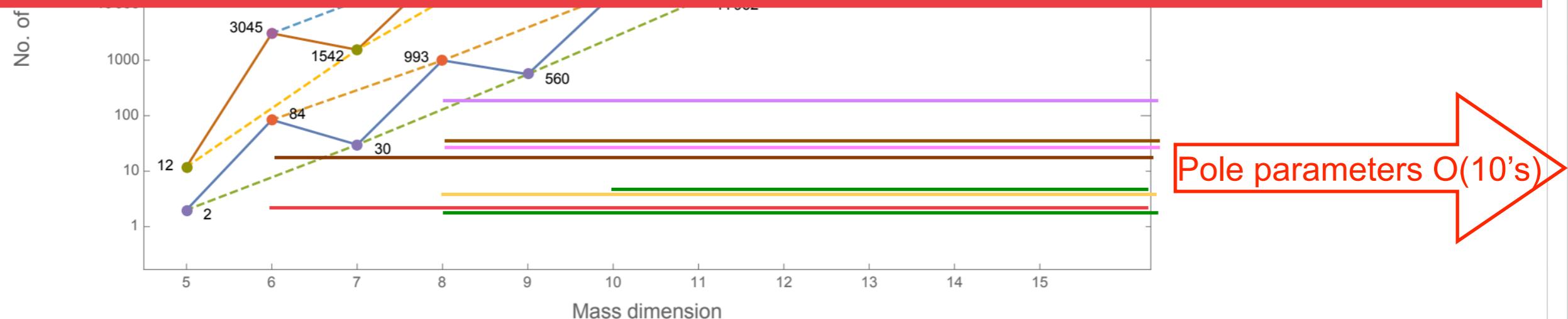


- General growth in operator forms from Hilbert series
  - <https://arxiv.org/abs/1503.07537>
  - <https://arxiv.org/pdf/1512.03433.pdf>
  - <https://arxiv.org/abs/1510.00372>
  - <https://arxiv.org/abs/1706.08520>

# Application to theory error



There is no free lunch using derivative expansions in SMEFT.  
The bill is pretty damn steep when you calc dim 8.  
That is where the  $n!$  sits.

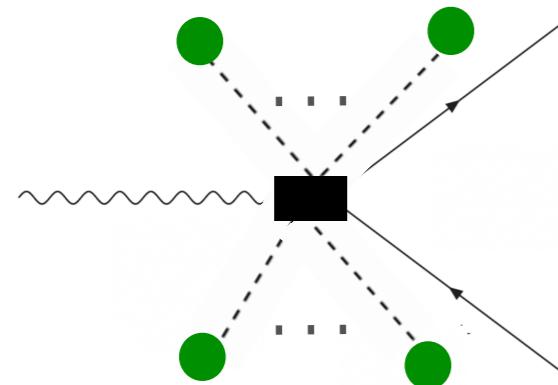


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# GeoSMEFT example

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



Consider a  $W^\pm, Z$  coupling to a fermion bilinear.

The all orders coupling in the SMEFT is a sum of two field space connections.

$\bar{\psi} i \not{D} \psi$  :with a consistent change weak to mass eigenstates in SMEFT

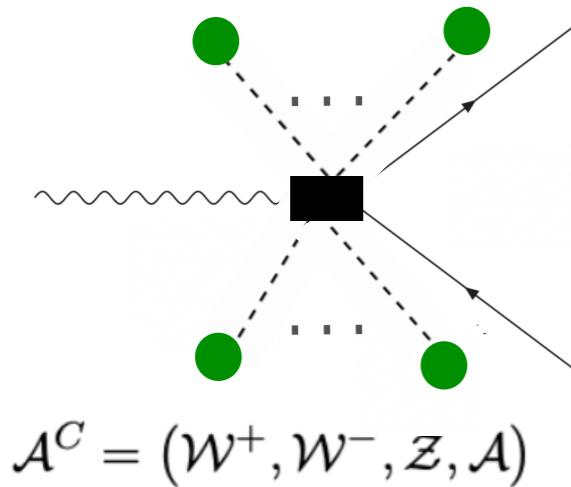
Added to this is the scalar, fermion connection  
(with a background field expectation)

$$L_{pr,A}^{\psi_R}(\phi) (D^\mu \phi)^J (\bar{\psi}_{p,R} \gamma_\mu \sigma_A \psi_{r,R})$$
$$L_{pr,A}^{\psi_L}(\phi) (D^\mu \phi)^J (\bar{\psi}_{p,L} \gamma_\mu \sigma_A \psi_{r,L})$$

# GeoSMEFT example

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



Consider a  $W^\pm, Z$  coupling to a fermion bilinear.

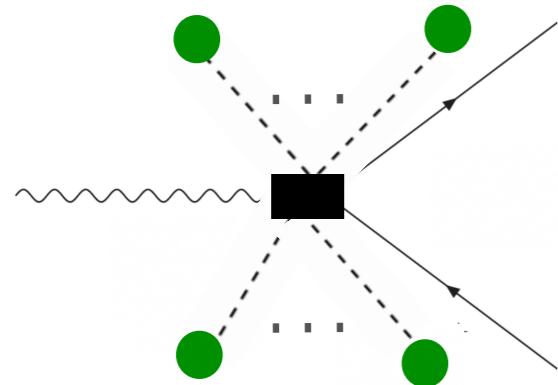
$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \tau_A \psi_r) \delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r) \langle L_{I,A}^{\psi,pr} \rangle (-\gamma_{C,4}^I) \bar{v}_T,$$

Compact all  $\bar{v}_T/\Lambda$  orders answer!

# GeoSMEFT example

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



Consider a  $W^\pm, Z$  coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \tau_A \psi_r) \delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r) \langle L_{I,A}^{\psi,pr} \rangle (-\gamma_{C,4}^I) \bar{v}_T,$$

The coupling of the canonically normalised mass eigenstate fields is then

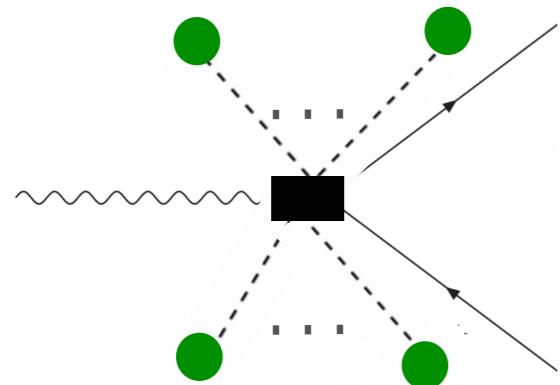
$$\langle \mathcal{Z} | \bar{\psi}_p \psi_r \rangle = \frac{\bar{g}_Z}{2} \bar{\psi}_p \not{\epsilon}_{\mathcal{Z}} \left[ (2s_{\theta_Z}^2 Q_\psi - \sigma_3) \delta_{pr} + \sigma_3 \bar{v}_T \langle L_{3,3}^{\psi,pr} \rangle + \bar{v}_T \langle L_{3,4}^{\psi,pr} \rangle \right] \psi_r,$$

$$\langle \mathcal{A} | \bar{\psi}_p \psi_r \rangle = -\bar{e} \bar{\psi}_p \not{\epsilon}_{\mathcal{A}} Q_\psi \delta_{pr} \psi_r,$$

$$\langle \mathcal{W}_\pm | \bar{\psi}_p \psi_r \rangle = -\frac{\bar{g}_2}{\sqrt{2}} \bar{\psi}_p (\not{\epsilon}_{\mathcal{W}^\pm}) T^\pm \left[ \delta_{pr} - \bar{v}_T \langle L_{1,1}^{\psi,pr} \rangle \pm i \bar{v}_T \langle L_{1,2}^{\psi,pr} \rangle \right] \psi_r.$$

# GeoSMEFT example

- Can build up observable quantities, such as a decay width.



Consider a  $W^\pm, Z$  coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \tau_A \psi_r) \delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r) \langle L_{I,A}^{\psi,pr} \rangle (-\gamma_{C,4}^I) \bar{v}_T,$$

- Two body decay widths:

$$\bar{\Gamma}_{Z \rightarrow \bar{\psi}\psi} = \sum_{\psi} \frac{N_c^{\psi}}{24\pi} \sqrt{\bar{m}_Z^2} |g_{\text{eff}}^{Z,\psi}|^2 \left(1 - \frac{4\bar{M}_{\psi}^2}{\bar{m}_Z^2}\right)^{3/2}$$

$$g_{\text{eff}}^{Z,\psi} = \frac{\bar{g}_Z}{2} \left[ (2s_{\theta_Z}^2 Q_{\psi} - \sigma_3) \delta_{pr} + \bar{v}_T \langle L_{3,4}^{\psi,pr} \rangle + \sigma_3 \bar{v}_T \langle L_{3,3}^{\psi,pr} \rangle \right]$$

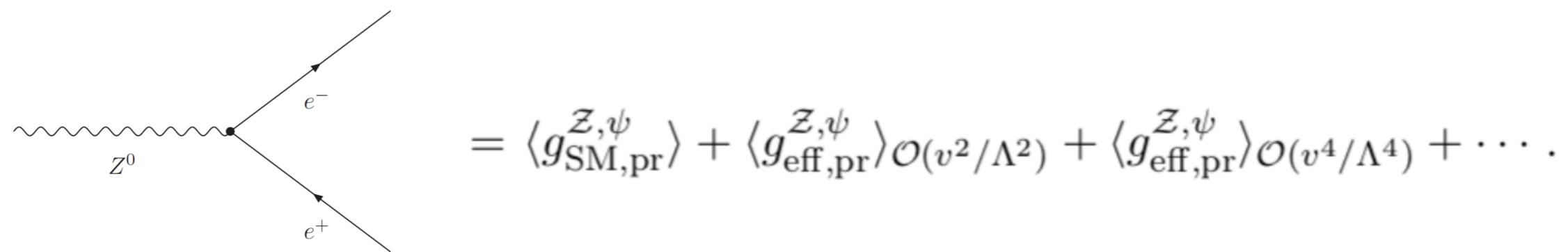
$$\bar{\Gamma}_{W \rightarrow \bar{\psi}\psi} = \sum_{\psi} \frac{N_c^{\psi}}{24\pi} \sqrt{\bar{m}_W^2} |g_{\text{eff}}^{W,\psi}|^2 \left(1 - \frac{4\bar{M}_{\psi}^2}{\bar{m}_W^2}\right)^{3/2}$$

$$g_{\text{eff}}^{W,q_L} = -\frac{\bar{g}_2}{\sqrt{2}} \left[ V_{\text{CKM}}^{pr} - \bar{v}_T \langle L_{1,1}^{q_L,pr} \rangle \pm i\bar{v}_T \langle L_{1,2}^{q_L,pr} \rangle \right],$$

$$g_{\text{eff}}^{W,\ell_L} = -\frac{\bar{g}_2}{\sqrt{2}} \left[ U_{\text{PMNS}}^{pr,\dagger} - \bar{v}_T \langle L_{1,1}^{\ell_L,pr} \rangle \pm i\bar{v}_T \langle L_{1,2}^{\ell_L,pr} \rangle \right],$$

# SMEFT expansion correction error estimate by process.

- Process by process many of the dimension 8 corrections are also fully known now. We can examine process by process to be informed in defining the error. Ex. <https://arxiv.org/pdf/2102.02819.pdf>



- LO SMEFT in SMEFTsim

SMEFT corrections in the $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}/\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme							
$\mathcal{O}(v^2/\Lambda^2)$	$\langle g_{\text{eff},\text{pp}}^{\mathcal{Z},u_R} \rangle$	$\langle g_{\text{eff},\text{pp}}^{\mathcal{Z},d_R} \rangle$	$\langle g_{\text{eff},\text{pp}}^{\mathcal{Z},\ell_R} \rangle$	$\langle g_{\text{eff},\text{pp}}^{\mathcal{Z},u_L} \rangle$	$\langle g_{\text{eff},\text{pp}}^{\mathcal{Z},d_L} \rangle$	$\langle g_{\text{eff},\text{pp}}^{\mathcal{Z},\ell_L} \rangle$	$\langle g_{\text{eff},\text{pp}}^{\mathcal{Z},\nu_L} \rangle$
$\delta G_F^{(6)}$	-0.08/0.15	0.04/-0.07	0.12/-0.22	0.18/0.41	-0.22/-0.34	-0.15/-0.49	0.26/0.26
$\tilde{C}_{HD}^{(6)}$	-0.22/0.05	0.11/-0.03	0.33/-0.08	-0.13/0.15	0.02/-0.12	0.24/-0.17	0.09/0.09
$\tilde{C}_{HWB}^{(6)}$	-0.21/0.39	0.10/-0.19	0.31/-0.58	-0.21/0.39	0.10/-0.19	0.31/-0.58	
$\tilde{C}_{H\psi}^{(6)}$	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37
$\tilde{C}_{H\psi}^{3,(6)}$				-0.37/-0.37	0.37/0.37	0.37/0.37	-0.37/-0.37

# SMEFT expansion correction error estimate by process.

- EWPD to dim 8 now known in both input parameter schemes.

2102.02819

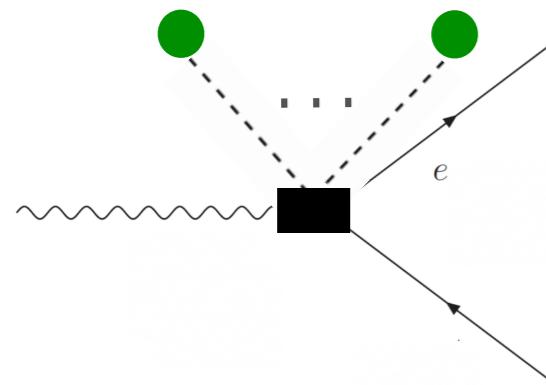
Many cross terms  
*NOT in quadratics*  
 but known for error  
 Estimates

Few new dimension  
 Eight parameters

SMEFT corrections in $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}/\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme			
$\mathcal{O}(\frac{v^4}{\Lambda^4})$	$\langle g_{\text{eff,pp}}^{\mathcal{Z},u_R} \rangle$	$\langle g_{\text{eff,pp}}^{\mathcal{Z},d_R} \rangle$	$\langle g_{\text{eff,pp}}^{\mathcal{Z},\ell_R} \rangle$
$\langle g_{\text{eff}}^{\mathcal{Z},\psi} \rangle^2$	14/5.5	-27/-11	-9.1/-3.6
$\tilde{C}_{HB} \tilde{C}_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}_{HD}^2$	0.28/-0.026	-0.14/0.013	-0.42/0.040
$\tilde{C}_{HD} \tilde{C}_{H\psi}^{(6)}$	-0.83/-0.19	-0.83/-0.19	-0.83/-0.19
$\tilde{C}_{HD} \tilde{C}_{HWB}$	0.59/-0.19	-0.29/0.097	-0.88/0.29
$\tilde{C}_{HD} \langle g_{\text{eff}}^{\mathcal{Z},\psi} \rangle$	4.0/0.50	4.0/0.50	4.0/0.50
$(\tilde{C}_{H\psi}^{(6)})^2$	0.62/1.4	-1.2/-2.8	-0.42/-0.93
$\tilde{C}_{HWB} \tilde{C}_{H\psi}^{(6)}$	-0.69/0.58	-0.69/0.58	-0.69/0.58
$\tilde{C}_{H\psi}^{(6)} \langle g_{\text{eff}}^{\mathcal{Z},\psi} \rangle$	-6.7/-5.8	13/12	4.5/3.9
$\tilde{C}_{HWB} \langle g_{\text{eff}}^{\mathcal{Z},\psi} \rangle$	3.7/0.26	3.7/0.26	3.7/0.26
$\tilde{C}_{HW} \tilde{C}_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}_{HD}^{(8)}$	-0.014/0.026	0.0069/-0.013	0.021/-0.040
$\tilde{C}_{HD,2}^{(8)}$	-0.21/0.026	0.10/-0.013	0.31/-0.040
$\tilde{C}_{H\psi}^{(8)}$	0.19/0.19	0.19/0.19	0.19/0.19
$\tilde{C}_{HW,2}^{(8)}$	-0.38/0	0.19/0	0.58/0
$\tilde{C}_{HWB}^{(8)}$	-0.10/0.19	0.051/-0.097	0.15/-0.29
$\delta G_F^{(8)}$	-0.078/0.15	0.039/-0.075	0.12/-0.22
$(\tilde{C}_{HWB}^{(6)})^2$	0.19/-0.35	-0.096/0.18	-0.29/0.53

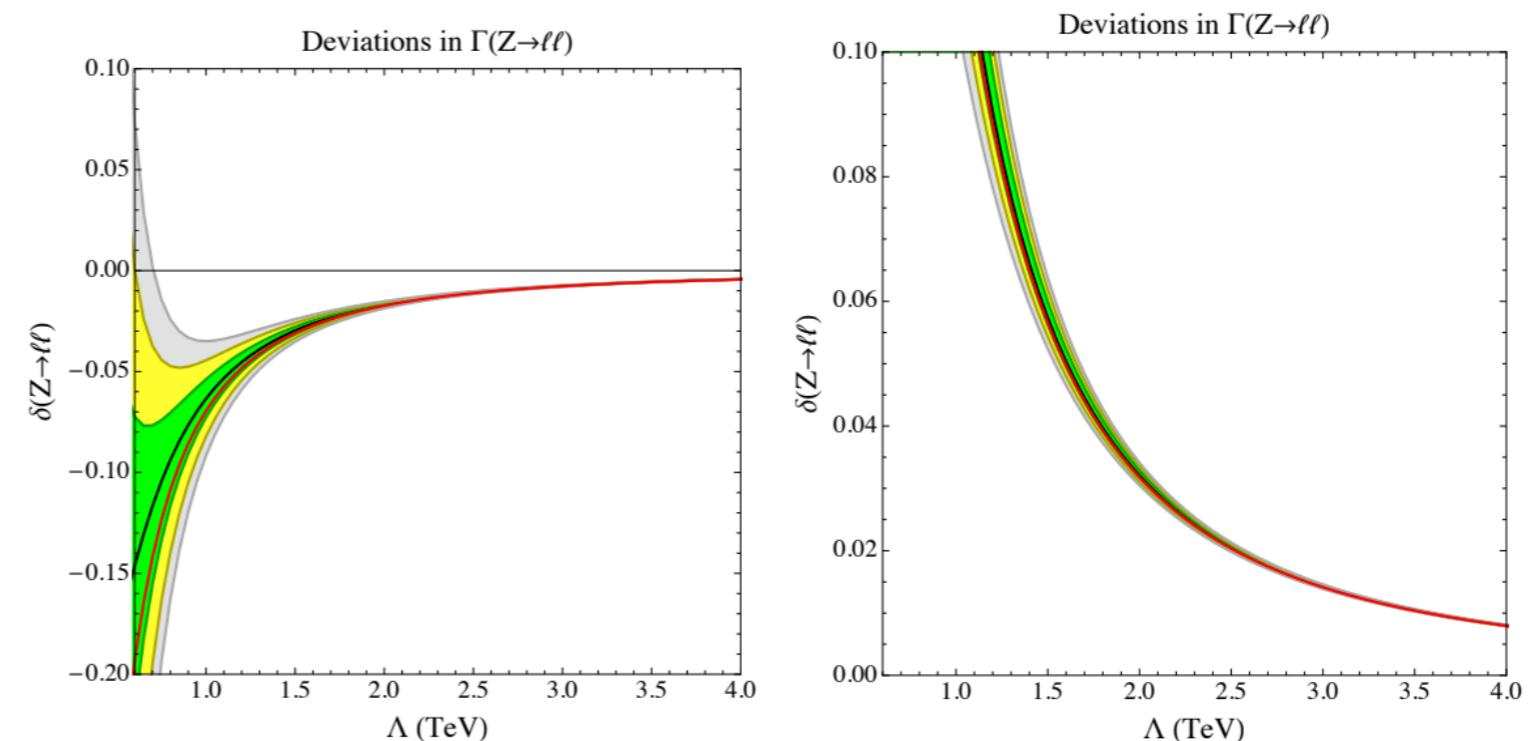
# GeoSMEFT directly informs theory error

- Can build up observable quantities, such as a decay width.



Consider a  $W^\pm, Z$  coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \tau_A \psi_r) \delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r) \langle L_{I,A}^{\psi,pr} \rangle (-\gamma_{C,4}^I) \bar{v}_T,$$



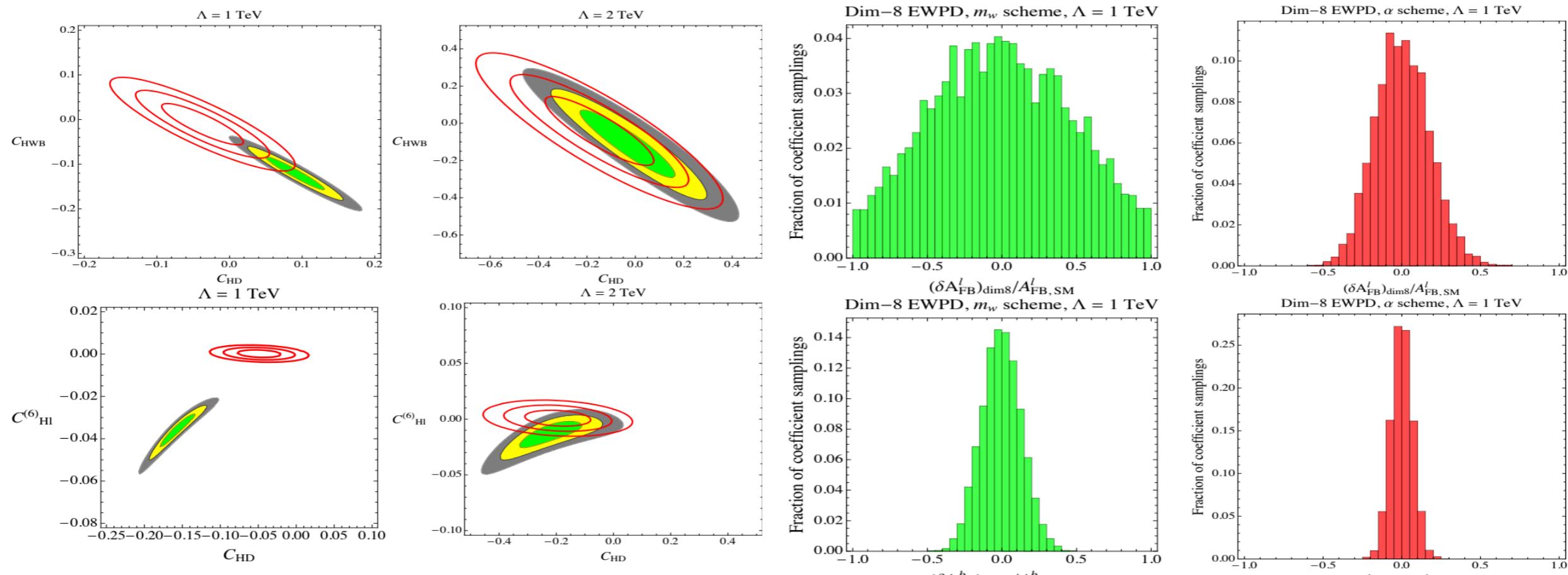
- Dim 8 effects on Z decay now known

**Figure 3.** The deviations in  $Z \rightarrow \ell\ell$  from the  $\mathcal{O}(v^2/\Lambda^2)$  (red line) and partial-square (black line) results, and the full  $\mathcal{O}(v^4/\Lambda^4)$  results (green  $\pm 1\sigma_\delta$ , yellow  $\pm 2\sigma_\delta$ , and grey  $\pm 3\sigma_\delta$  regions). In the left panel the coefficients determining the  $\mathcal{O}(v^2/\Lambda^2)$  and partial-square results are  $C_{H\ell}^{1,(6)} = -0.46$ ,  $C_{H\ell}^{3,(6)} = 1.24$ ,  $C_{He}^{(6)} = 1.53$ ,  $C_{HD}^{(6)} = -0.79$ ,  $C_{HWB}^{(6)} = 0.007$ , and  $\delta G_F^{(6)} = 0.16$ . In the right panel they are  $C_{H\ell}^{1,(6)} = 1.55$ ,  $C_{H\ell}^{3,(6)} = -0.71$ ,  $C_{He}^{(6)} = 0.23$ ,  $C_{HD}^{(6)} = -0.51$ ,  $C_{HWB}^{(6)} = -0.008$ , and  $\delta G_F^{(6)} = -0.44$ .

# Yes people do over interpret EWPD.

- (Heretical) Doubts were raised years ago that dim 8 neglect and loop neglect in EWPD can significantly impact naive bounds with significant implications

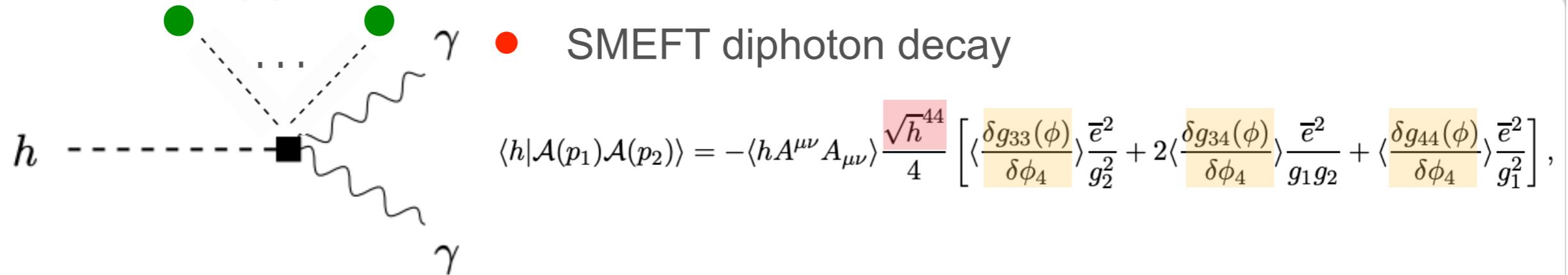
1502.02570, 1508.05060, Berthier, MT , 1606.06693 Berthier, Bjorn, MT , arXiv:1701.06424 Brivio, MT



**Figure 3.** The green/yellow/gray contours correspond to the 68%/95%/99.9% CL two parameter fit determined by  $\Delta\chi^2_{\mathcal{O}(v^4/\Lambda^4)}$ , while the red rings correspond to the same CL determined using  $\Delta\chi^2_{\mathcal{O}(v^2/\Lambda^2)}$ . In the top panels the free parameters are  $C_{HD}$  and  $C_{HWB}$ , while in the bottom panels the free parameters are  $C_{HD}$  and  $C_{H\ell}^{(6)}$ . Note that the axes ranges vary from panel to panel. In the left panels, we have taken the scale  $\Lambda = 1 \text{ TeV}$ , while in the right panels  $\Lambda = 2 \text{ TeV}$ . All calculations use the  $\hat{m}_W$  scheme.

- Significant implications for future collider studies.**

# GeoSMEFT for the Higgs



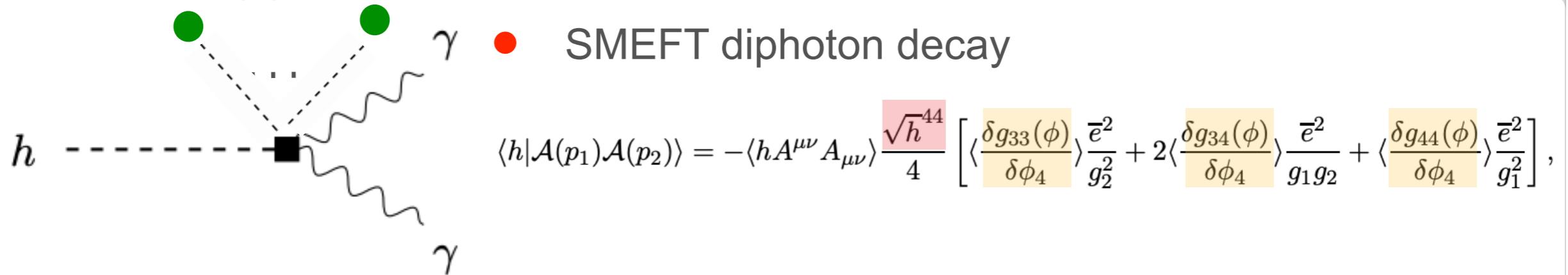
- What is the detailed difference between squaring the dimension 6 correction, and the full dimension 8 result?
- Now we know. Using the expression above, just expand.

“Naive square” proportional to

$$\left| \mathcal{A}_{SM}^{h\gamma\gamma} \right|^2 + 2 \operatorname{Re}(\mathcal{A}_{SM}^{h\gamma\gamma}) \langle h | \gamma\gamma \rangle_{\mathcal{L}^{(6)}} + \langle h | \gamma\gamma \rangle_{\mathcal{L}^{(6)}}^2$$

Where:  $\langle h | \gamma\gamma \rangle_{\mathcal{L}^{(6)}} = \left[ \frac{g_2^2 \tilde{C}_{HB}^{(6)} + g_1^2 \tilde{C}_{HW}^{(6)} - g_1 g_2 \tilde{C}_{HWB}^{(6)}}{(g_1^2 + g_2^2) \bar{v}_T} \right]$

# GeoSMEFT for the Higgs



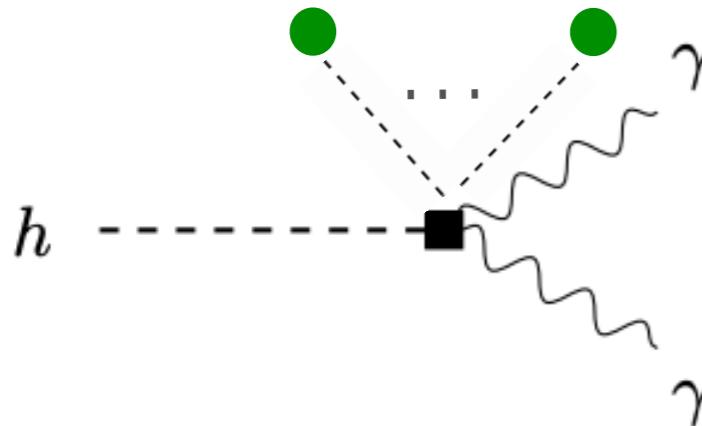
- What is the detailed difference between squaring the dimension 6 correction, and the full dimension 8 result?
- Now we know. Using the expression above, just expand.

Correct result:

Hays, Helset, Martin Trott: 2007.00565

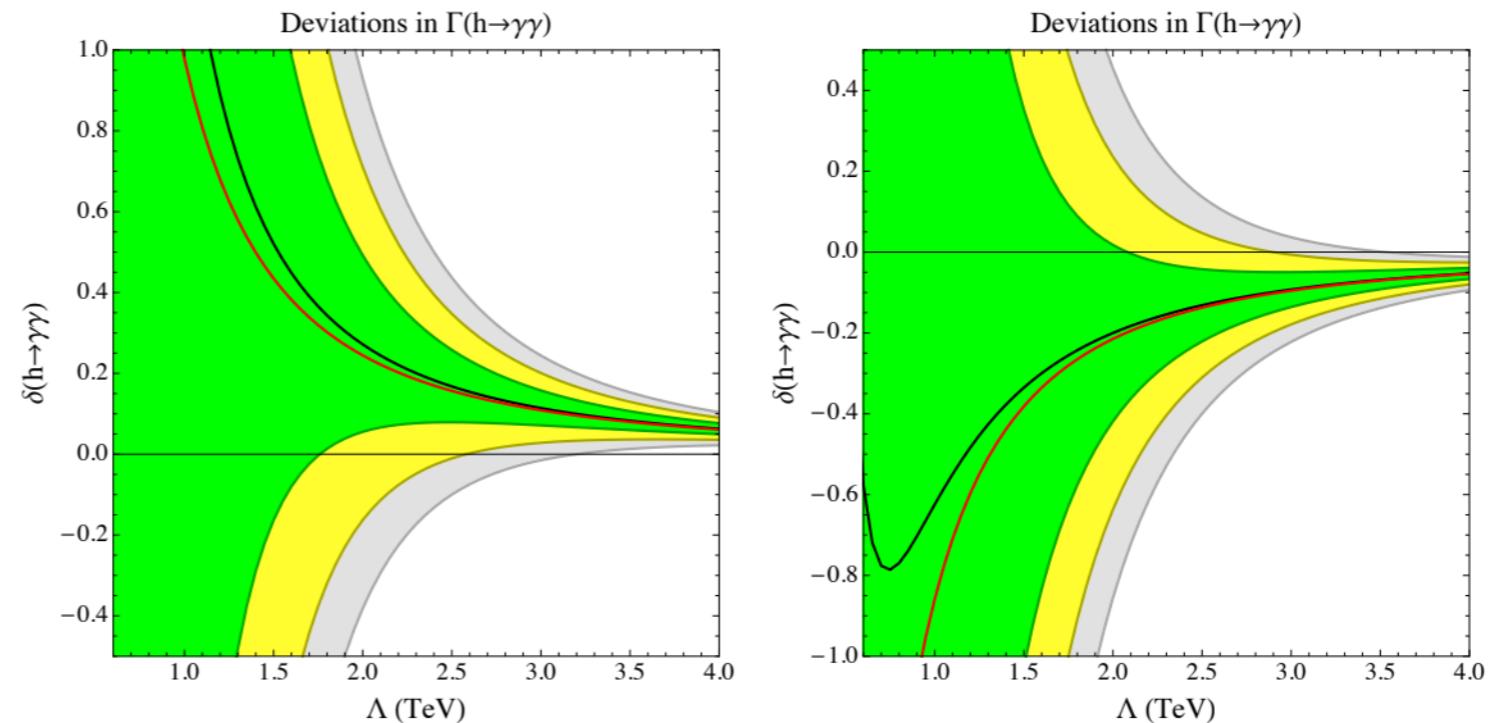
$$\begin{aligned} & \left| \mathcal{A}_{SM}^{h\gamma\gamma} \right|^2 + 2 \operatorname{Re}(\mathcal{A}_{SM}^{h\gamma\gamma}) (1 + \langle \sqrt{h^{44}} \rangle_{\mathcal{L}^{(6)}}) \langle h | \gamma\gamma \rangle_{\mathcal{L}^{(6)}} + (1 + 4 \bar{v}_T \operatorname{Re}(\mathcal{A}_{SM}^{h\gamma\gamma})) (\langle h | \gamma\gamma \rangle_{\mathcal{L}^{(6)}})^2, \\ & + 2 \operatorname{Re}(\mathcal{A}_{SM}^{h\gamma\gamma}) \left[ \frac{g_2^2 \tilde{C}_{HB}^{(8)} + g_1^2 (\tilde{C}_{HW}^{(8)} - \tilde{C}_{HW,2}^{(8)}) - g_1 g_2 \tilde{C}_{HWB}^{(8)}}{(g_1^2 + g_2^2) \bar{v}_T} \right]. \end{aligned}$$

# GeoSMEFT for the Higgs



- How much does dim 8 effect things? A lot.

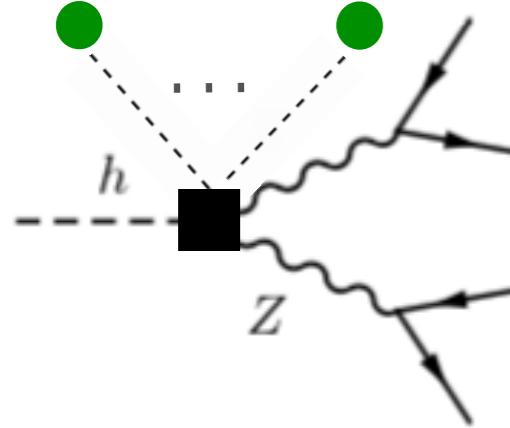
Hays, Helset, Martin Trott: 2007.00565



**Figure 1.** The deviations in  $h \rightarrow \gamma\gamma$  from the  $\mathcal{O}(v^2/\Lambda^2)$  (red line) and partial-square (black line) results, and the full  $\mathcal{O}(v^4/\Lambda^4)$  results (green  $\pm 1\sigma_\delta$ , yellow  $\pm 2\sigma_\delta$ , and grey  $\pm 3\sigma_\delta$  regions). In the left panel the coefficients determining the  $\mathcal{O}(v^2/\Lambda^2)$  and partial-square results are  $C_{HB}^{(6)} = -0.01$ ,  $C_{HW}^{(6)} = 0.004$ ,  $C_{HWB}^{(6)} = 0.007$ ,  $C_{HD}^{(6)} = -0.74$ , and  $\delta G_F^{(6)} = -1.6$ . In the right panel they are  $C_{HB}^{(6)} = 0.007$ ,  $C_{HW}^{(6)} = 0.007$ ,  $C_{HWB}^{(6)} = -0.015$ ,  $C_{HD}^{(6)} = 0.50$ , and  $\delta G_F^{(6)} = 1.26$ .

- EFT studies that ignore this geoSMEFT enabled information can be misleading.
- Loop processes are more subject to SMEFT theoretical errors in the cases we looked at.

# GeoSMEFT for the Higgs

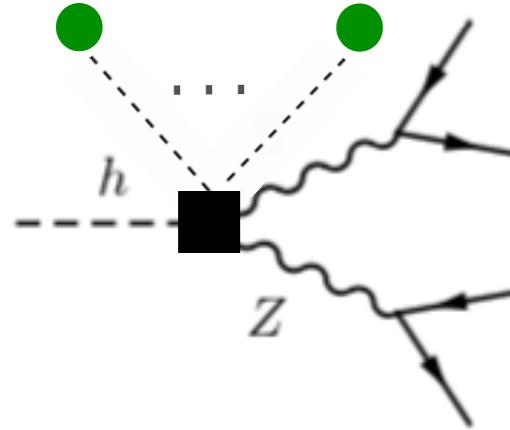


- All orders SMEFT higgs coupling to  $W^\pm, Z$

$$\begin{aligned} \langle h | \mathcal{Z}(p_1) \mathcal{Z}(p_2) \rangle &= -\frac{\sqrt{h}^{44}}{4} \bar{g}_Z^2 \left[ \langle \frac{\delta g_{33}(\phi)}{\delta \phi_4} \rangle \frac{c_{\theta_Z}^4}{g_2^2} - 2 \langle \frac{\delta g_{34}(\phi)}{\delta \phi_4} \rangle \frac{c_{\theta_Z}^2 s_{\theta_Z}^2}{g_1 g_2} + \langle \frac{\delta g_{44}(\phi)}{\delta \phi_4} \rangle \frac{s_{\theta_Z}^4}{g_1^2} \right] \langle h \mathcal{Z}_{\mu\nu} \mathcal{Z}^{\mu\nu} \rangle \\ &\quad + \sqrt{h}^{44} \frac{\bar{g}_Z^2}{2} \left[ \langle \frac{\delta h_{33}(\phi)}{\delta \phi_4} \rangle \left( \frac{\bar{v}_T}{2} \right)^2 + \langle h_{33}(\phi) \rangle \frac{\bar{v}_T}{2} \right] \langle h \mathcal{Z}_\mu \mathcal{Z}^\mu \rangle \\ &\quad + \sqrt{h}^{44} \bar{g}_Z^2 \bar{v}_T \left[ \langle k_{34}^3 \rangle \frac{c_{\theta_Z}^2}{g_2} - \langle k_{34}^4 \rangle \frac{s_{\theta_Z}^2}{g_1} \right] \langle \partial^\nu h \mathcal{Z}_\mu \mathcal{Z}^{\mu\nu} \rangle, \end{aligned}$$

$$\begin{aligned} \langle h | \mathcal{W}(p_1) \mathcal{W}(p_2) \rangle &= -\frac{\sqrt{h}^{44}}{2} \bar{g}_2^2 \left[ \langle \frac{\delta g_{11}(\phi)}{\delta \phi_4} \rangle \frac{1}{g_2^2} \right] \langle h \mathcal{W}_{\mu\nu} \mathcal{W}^{\mu\nu} \rangle \\ &\quad + \sqrt{h}^{44} \bar{g}_2^2 \left[ \langle \frac{\delta h_{11}(\phi)}{\delta \phi_4} \rangle \left( \frac{\bar{v}_T}{2} \right)^2 + \langle h_{11}(\phi) \rangle \frac{\bar{v}_T}{2} \right] \langle h \mathcal{W}_\mu \mathcal{W}^\mu \rangle \\ &\quad + 2 \sqrt{h}^{44} \frac{\bar{g}_2^2}{g_2} \frac{\bar{v}_T}{4} [i \langle k_{42}^1 \rangle - \langle k_{42}^2 \rangle] \langle (\partial^\mu h) (\mathcal{W}_{\mu\nu}^+ W_-^\nu + \mathcal{W}_{\mu\nu}^- W_+^\nu) \rangle. \end{aligned}$$

# GeoSMEFT for the Higgs



- All orders SMEFT higgs coupling to  $W^\pm, Z$

$$\langle h | \mathcal{Z}(p_1) \mathcal{Z}(p_2) \rangle = -\frac{\sqrt{h}^{44}}{4} \bar{g}_Z^2 \left[ \langle \frac{\delta g_{33}(\phi)}{\delta \phi_4} \rangle \frac{c_{\theta_Z}^4}{g_2^2} - 2 \langle \frac{\delta g_{34}(\phi)}{\delta \phi_4} \rangle \frac{c_{\theta_Z}^2 s_{\theta_Z}^2}{g_1 g_2} + \langle \frac{\delta g_{44}(\phi)}{\delta \phi_4} \rangle \frac{s_{\theta_Z}^4}{g_1^2} \right] \langle h \mathcal{Z}_{\mu\nu} \mathcal{Z}^{\mu\nu} \rangle$$

← SM like kinematics

$$+ \sqrt{h}^{44} \frac{\bar{g}_Z^2}{2} \left[ \langle \frac{\delta h_{33}(\phi)}{\delta \phi_4} \rangle \left( \frac{\bar{v}_T}{2} \right)^2 + \langle h_{33}(\phi) \rangle \frac{\bar{v}_T}{2} \right] \langle h \mathcal{Z}_\mu \mathcal{Z}^\mu \rangle$$

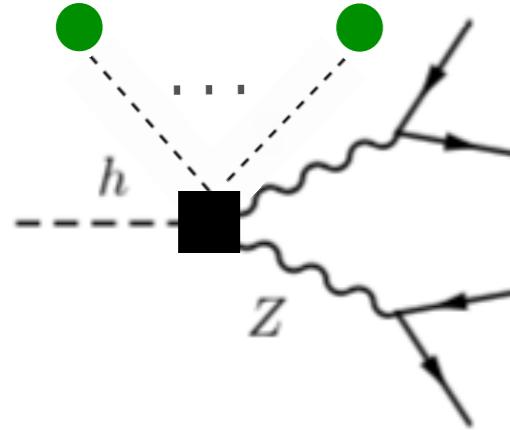
$$+ \sqrt{h}^{44} \bar{g}_Z^2 \bar{v}_T \left[ \langle k_{34}^3 \rangle \frac{c_{\theta_Z}^2}{g_2} - \langle k_{34}^4 \rangle \frac{s_{\theta_Z}^2}{g_1} \right] \langle \partial^\nu h \mathcal{Z}_\mu \mathcal{Z}^{\mu\nu} \rangle,$$

$$\langle h | \mathcal{W}(p_1) \mathcal{W}(p_2) \rangle = -\frac{\sqrt{h}^{44}}{2} \bar{g}_2^2 \left[ \langle \frac{\delta g_{11}(\phi)}{\delta \phi_4} \rangle \frac{1}{g_2^2} \right] \langle h \mathcal{W}_{\mu\nu} \mathcal{W}^{\mu\nu} \rangle$$

$$+ \sqrt{h}^{44} \bar{g}_2^2 \left[ \langle \frac{\delta h_{11}(\phi)}{\delta \phi_4} \rangle \left( \frac{\bar{v}_T}{2} \right)^2 + \langle h_{11}(\phi) \rangle \frac{\bar{v}_T}{2} \right] \langle h \mathcal{W}_\mu \mathcal{W}^\mu \rangle$$

$$+ 2 \sqrt{h}^{44} \frac{\bar{g}_2^2}{g_2} \frac{\bar{v}_T}{4} [i \langle k_{42}^1 \rangle - \langle k_{42}^2 \rangle] \langle (\partial^\mu h) (\mathcal{W}_{\mu\nu}^+ W_-^\nu + \mathcal{W}_{\mu\nu}^- W_+^\nu) \rangle.$$

# GeoSMEFT for the Higgs



- All orders SMEFT higgs coupling to  $W^\pm, Z$

$$\begin{aligned} \langle h | \mathcal{Z}(p_1) \mathcal{Z}(p_2) \rangle &= -\frac{\sqrt{h}^{44}}{4} \bar{g}_Z^2 \left[ \langle \frac{\delta g_{33}(\phi)}{\delta \phi_4} \rangle \frac{c_{\theta_Z}^4}{g_2^2} - 2 \langle \frac{\delta g_{34}(\phi)}{\delta \phi_4} \rangle \frac{c_{\theta_Z}^2 s_{\theta_Z}^2}{g_1 g_2} + \langle \frac{\delta g_{44}(\phi)}{\delta \phi_4} \rangle \frac{s_{\theta_Z}^4}{g_1^2} \right] \langle h \mathcal{Z}_{\mu\nu} \mathcal{Z}^{\mu\nu} \rangle \\ &+ \sqrt{h}^{44} \frac{\bar{g}_Z^2}{2} \left[ \langle \frac{\delta h_{33}(\phi)}{\delta \phi_4} \rangle \left( \frac{\bar{v}_T}{2} \right)^2 + \langle h_{33}(\phi) \rangle \frac{\bar{v}_T}{2} \right] \langle h \mathcal{Z}_\mu \mathcal{Z}^\mu \rangle \quad \leftarrow \text{SM like kinematics} \\ &+ \sqrt{h}^{44} \bar{g}_Z^2 \bar{v}_T \left[ \langle k_{34}^3 \rangle \frac{c_{\theta_Z}^2}{g_2} - \langle k_{34}^4 \rangle \frac{s_{\theta_Z}^2}{g_1} \right] \langle \partial^\nu h \mathcal{Z}_\mu \mathcal{Z}^{\mu\nu} \rangle, \quad \text{Anomalous kinematic population correction factor - not many!} \end{aligned}$$

$$\begin{aligned} \langle h | \mathcal{W}(p_1) \mathcal{W}(p_2) \rangle &= -\frac{\sqrt{h}^{44}}{2} \bar{g}_2^2 \left[ \langle \frac{\delta g_{11}(\phi)}{\delta \phi_4} \rangle \frac{1}{g_2^2} \right] \langle h \mathcal{W}_{\mu\nu} \mathcal{W}^{\mu\nu} \rangle \\ &+ \sqrt{h}^{44} \bar{g}_2^2 \left[ \langle \frac{\delta h_{11}(\phi)}{\delta \phi_4} \rangle \left( \frac{\bar{v}_T}{2} \right)^2 + \langle h_{11}(\phi) \rangle \frac{\bar{v}_T}{2} \right] \langle h \mathcal{W}_\mu \mathcal{W}^\mu \rangle \\ &+ 2\sqrt{h}^{44} \frac{\bar{g}_2^2}{g_2} \frac{\bar{v}_T}{4} [i \langle k_{42}^1 \rangle - \langle k_{42}^2 \rangle] \langle (\partial^\mu h) (\mathcal{W}_{\mu\nu}^+ W_-^\nu + \mathcal{W}_{\mu\nu}^- W_+^\nu) \rangle. \end{aligned}$$

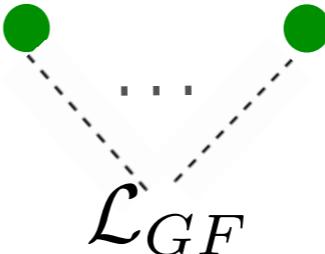
In the hands of ATLAS for kinematic studies  
Thx to Tyler Corbett for coding SMEFTsim mod.

# GeoSMEFT based loop corrections?

- The simplicity of the results for two and three point functions points to radiative corrections being more elegant than expected.

The renormalisation follows the dependence on the Wilson coefficients.

- Do we have hints of this yet? Yes.



- Background field gauge fixing with preserved background Gauge invariance 1803.08001 Helset, Paraskevas, Trott

$$\mathcal{L}_{GF} = -\frac{\hat{g}_{AB}}{2\xi} \mathcal{G}^A \mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu \mathcal{W}^{X,\mu} - \tilde{\epsilon}_{CD}^X \hat{W}_\mu^C \mathcal{W}^{D,\mu} + \frac{\xi}{2} \hat{g}^{XC} \phi^I \hat{h}_{IK} \tilde{\gamma}_{C,J}^K \hat{\phi}^J.$$

- Gauge fixing confusion directly solved generalising to GeoSMEFT

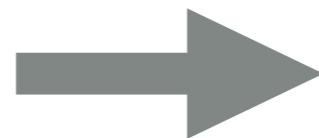
Further exploration of gauge fixing based on this idea: 1812.11513 Misiak et al

# GeoSMEFT based loop corrections?

- Will this simplify the NLO SMEFT radiative correction program?(Yes)

Immediate BFM Ward Identities have already been derived:

$$\frac{\delta\Gamma[\hat{F}, 0]}{\delta\hat{\alpha}^B} = 0.$$



Background field gauge transformation

1909.08470 Corbett, Helset, Trott

$$0 = \left( \partial^\mu \delta_B^A - \tilde{\epsilon}_{BC}^A \hat{W}^{C,\mu} \right) \frac{\delta\Gamma}{\delta\hat{W}_A^\mu} - \frac{\tilde{\gamma}_{B,J}^I}{2} \hat{\phi}^J \frac{\delta\Gamma}{\delta\hat{\phi}^I} \\ + \sum_j \left( \bar{f}_j \bar{\Lambda}_{B,i}^j \frac{\delta\Gamma}{\delta\bar{f}_i} - \frac{\delta\Gamma}{\delta f_i} \Lambda_{B,j}^i f_j \right).$$

Photon identities:

$$0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{4\mu}\delta\hat{\mathcal{A}}^{Y\nu}}, \quad 0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{4\mu}\delta\hat{\Phi}^I}.$$



$$\Sigma_{L,\text{SMEFT}}^{\hat{\mathcal{A}}, \hat{\mathcal{A}}}(k^2) = 0, \quad \Sigma_{T,\text{SMEFT}}^{\hat{\mathcal{A}}, \hat{\mathcal{A}}}(0) = 0.$$

One loop behaviour works!

Z identities:

$$0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{3\mu}\delta\hat{\mathcal{A}}^{Y\nu}} - \bar{M}_Z \frac{\delta^2\Gamma}{\delta\hat{\Phi}^3\delta\hat{\mathcal{A}}^{Y\nu}}, \\ 0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{3\mu}\delta\hat{\Phi}^I} - \bar{M}_Z \frac{\delta^2\Gamma}{\delta\hat{\Phi}^3\delta\hat{\Phi}^I} \\ + \frac{\bar{g}_Z}{2} \frac{\delta\Gamma}{\delta\hat{\Phi}^4} \left( \sqrt{h}_{[4,4]} \sqrt{h}^{[3,3]} - \sqrt{h}_{[4,3]} \sqrt{h}^{[4,3]} \right) \delta_I^3 \\ - \frac{\bar{g}_Z}{2} \frac{\delta\Gamma}{\delta\hat{\Phi}^4} \left( \sqrt{h}_{[4,4]} \sqrt{h}^{[3,4]} - \sqrt{h}_{[4,3]} \sqrt{h}^{[4,4]} \right) \delta_I^4,$$

Geometric mass

# GeoSMEFT based loop corrections?

- Expanding out the Ward ID you get expressions like this:

$$0 = \Sigma_L^{\hat{Z}\hat{Z}}(k^2) - i\bar{M}_{\mathcal{Z}}\Sigma^{\hat{Z}\hat{\chi}}(k^2),$$

Consider the operator  $C_{HWB}$

$$\begin{aligned} -i\bar{M}_{\mathcal{Z}}\Sigma^{\hat{Z}\hat{\chi}}(k^2) &= -i \frac{\sqrt{g_1^2 + g_2^2} \bar{v}_T}{2} \left[ \Sigma^{\hat{Z}\hat{\chi}}(k^2) \right]_{\tilde{C}_{HWB}}^{div} - i \frac{g_1 g_2 \tilde{C}_{HWB} \bar{v}_T}{2 \sqrt{g_1^2 + g_2^2}} \left[ \Sigma^{\hat{Z}\hat{\chi}}(k^2) \right]_{SM}^{div}, \\ &= -\tilde{C}_{HWB} \bar{v}_T^2 (\xi + 3) \left[ \frac{g_1 g_2 (3g_1^2 + 5g_2^2)}{256 \pi^2 \epsilon} + \frac{g_1 g_2 (g_1^2 + 3g_2^2)}{256 \pi^2 \epsilon} \right], \\ &= -\tilde{C}_{HWB} \bar{v}_T^2 (\xi + 3) \frac{g_1^2(g_1^2 + 2g_2^2)}{64 \pi^2 \epsilon}. \end{aligned}$$

Geometric mass

Corbett Trott, 2010.08451

Corbett 2010.15852

Which exactly cancels:  $\left[ \Sigma_L^{\hat{Z}\hat{Z}}(k^2) \right]_{\tilde{C}_{HWB}}^{div} = \tilde{C}_{HWB} \bar{v}_T^2 (\xi + 3) \frac{g_1^2(g_1^2 + 2g_2^2)}{64 \pi^2 \epsilon}$ .

All one and two point ward ID working out at one loop. Powerful new NLO code tool is being developed using this as a theory cross check by Tyler Corbett.

# GeoSMEFT based Theory error.

- GeoSMEFT efficiently organised the physics for dimension 8 calc that follow from rescaling dimension 6 results with the same kinematics.
- An algorithm to develop calc to dimension 8 efficiently to use, and also inform theory errors 2106.13794 Methodology paper

*SMEFT RGE is transparent to simulation chain.* Thus infer logs of missing pert corrections for theory error.

Use geoSMEFT to rescale dim 6 results with common kinematics to post-facto (avoid redundant Monte Carlo) get dim 8 from SMEFTsim

Extra dim 8 bits with novel kinematics define in geoSMEFT consistent basis. Put in code. (SMEFTsim mod ongoing of this form with T Corbett)

- Being applied to associated production, already done for  $\Gamma(h \rightarrow \gamma\gamma)$   
 $\sigma(\mathcal{G}\mathcal{G} \rightarrow h)$ ,  $\Gamma(h \rightarrow \mathcal{G}\mathcal{G})$

# Need for consistent dim 8 and loop

- Operator expansion and loop expansion are NOT independent.  
Can use geoSMEFT to get dim 8 in many cases, a background field formulation so Background Field Method pert corrections preferred.

$$\begin{aligned} \frac{\sigma_{\text{SMEFT}}^{\hat{\alpha}}(\mathcal{G}\mathcal{G} \rightarrow h)}{\sigma_{\text{SM}}^{\hat{\alpha}, 1/m_t^2}(\mathcal{G}\mathcal{G} \rightarrow h)} &\simeq 1 + 519 \tilde{C}_{HG}^{(6)} + 504 \tilde{C}_{HG}^{(6)} \left( \tilde{C}_{H\square}^{(6)} - \frac{1}{4} \tilde{C}_{HD}^{(6)} \right) + 8.15 \times 10^4 (\tilde{C}_{HG}^{(6)})^2 + 504 \tilde{C}_{HG}^{(8)} \\ &+ 1.58 \left( \tilde{C}_{H\square}^{(6)} - \frac{1}{4} \tilde{C}_{HD}^{(6)} \right) + 362 \tilde{C}_{HG}^{(6)} - 1.59 \tilde{C}_{uH}^{(6)} - 12.6 \text{Re } \tilde{C}_{uG}^{(6)} - 1.12 \delta G_F^{(6)} - 7.70 \text{Re } \tilde{C}_{uG}^{(6)} \log \left( \frac{\hat{m}_h^2}{\Lambda^2} \right) \\ &- 0.19 \text{Re } \tilde{C}_{dG}^{(6)} \log \left( \frac{\hat{m}_h^2}{\Lambda^2} \right) - 0.09 \text{Re } \tilde{C}_{dG}^{(6)} + 3.54 \tilde{C}_{dH}^{(6)}. \end{aligned}$$

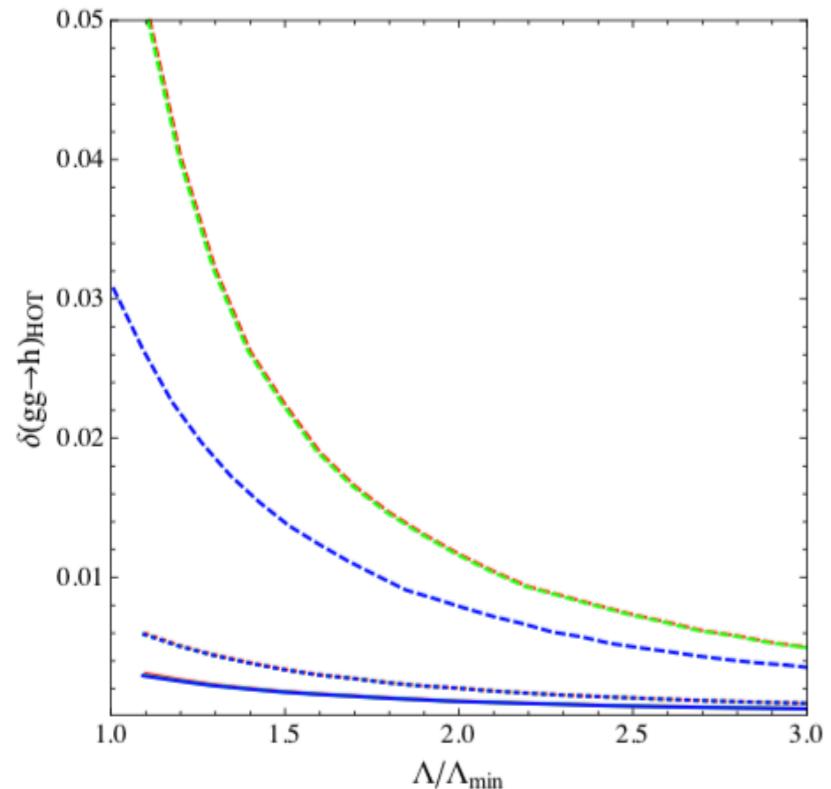


FIG. 1: Uncertainty on  $\sigma(gg \rightarrow h)$  from higher order terms as a function of the new physics scale  $\Lambda$  relative to the min-

- Justified triage for the SMEFT global fit program with theory errors



- Otherwise

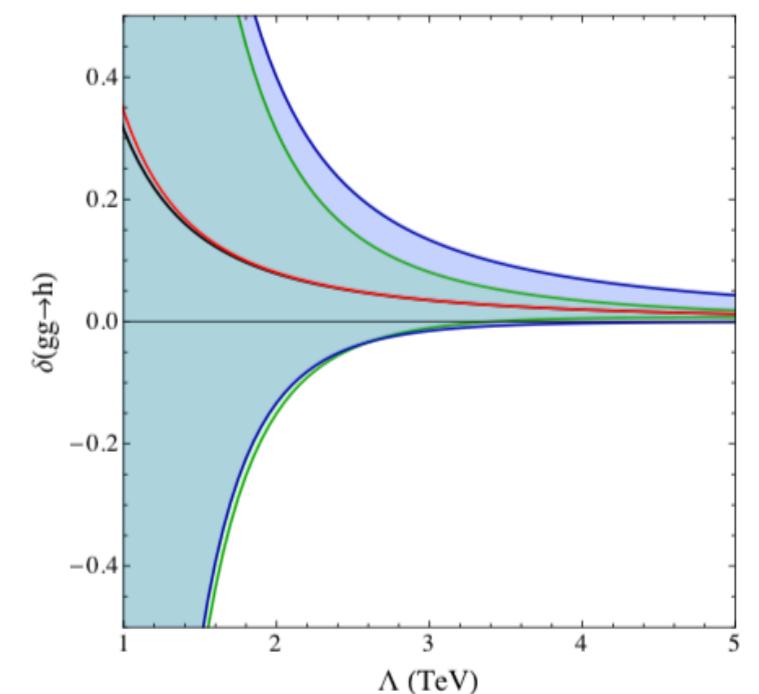


FIG. 4: Deviation in  $\sigma(\mathcal{G}\mathcal{G} \rightarrow h)$  relative to the SM with  $C_{HG}^{(6)} = 0.01$ , and all other coefficients sampled according to gaussian distributions with zero mean and width of either 1.0 or .01 depending on whether the corresponding operator is generated at tree or loop level following the classification in Ref. [7–9]. The deviation is plotted as a function of  $\Lambda$ , and the color scheme for the lines and bands is the same as in Figs. 2, 3.

# Conclusions.

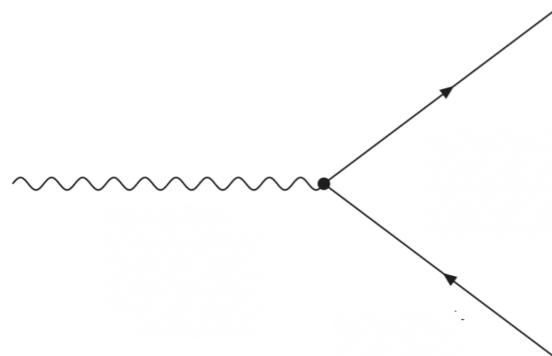
Higgs physics is the physics  
of curved field space.

Backup slides for discussion.

# Current geosmeft limitations

# GeoSMEFT Pushing to higher n points

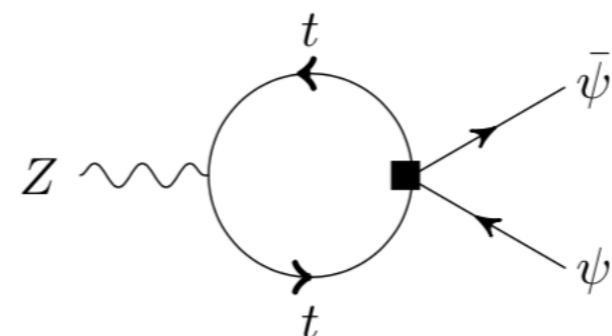
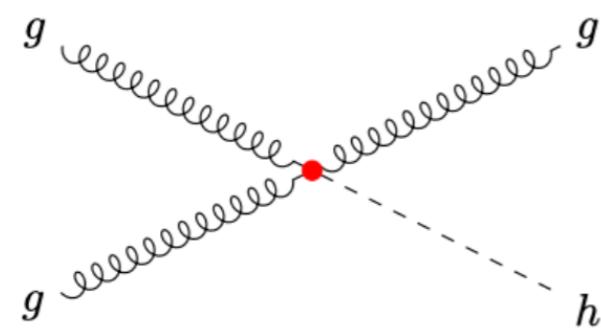
- Can build up observable quantities, such as a decay width.



Consider a  $W^\pm, Z$  coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \tau_A \psi_r) \delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r) \langle L_{I,A}^{\psi,pr} \rangle (-\gamma_{C,4}^I) \bar{v}_T,$$

- Not all physics is derivable from two and three point functions



# GeoSMEFT Pushing to higher n points

- Limited number of such connections for up to three point functions  
This is a non trivial fact proven for:  $F = \{H, \psi, \mathcal{W}^{\mu\nu}\}$  via the following:

$D^2 F \Rightarrow \boxed{\text{EOM}}$  and higher-points,

2001.01453 Helset, Martin, Trott

$f(H)(D_\mu F_1)(D_\nu F_2)D_{\{\mu\nu\}}F_3 \Rightarrow \boxed{\text{EOM}}$  and higher-points.

$$f(\phi) F_1 (D_\mu F_2) (D_\mu F_3) \Rightarrow (D_\mu f(\phi)) (D_\mu F_1) F_2 F_3 + \frac{1}{2}(D^2 f(\phi)) F_1 F_2 F_3 + \boxed{\text{EOM}},$$



- How to incorporate such higher n-point effects is the key challenge.
- Pert corrections advancing fast- higher n points also moving.

# GeoSMEFT Pushing to higher n points

- Note these integration by parts steps were used

$$\begin{aligned} & f(H)(D_\mu F_1)(D_\nu F_2)D_{\{\mu\nu\}}F_3 \\ &= -f(H) \left[ (D^2 F_1)(D_\nu F_2) + (D_\mu F_1)(D_\mu D_\nu F_2) + (D_\mu D_\nu F_1)(D_\mu F_2) + (D_\nu F_1)(D^2 F_2) \right] (D_\nu F_3) \\ &\quad - (D_\mu f(H)) [(D_\mu F_1)(D_\nu F_2) + (D_\nu F_1)(D_\mu F_2)] (D_\nu F_3) \end{aligned}$$

$$f(\phi) F_1 (D_\mu F_2) (D_\mu F_3) \Rightarrow (D_\mu f(\phi)) (D_\mu F_1) F_2 F_3 + \frac{1}{2} (D^2 f(\phi)) F_1 F_2 F_3 + \boxed{\text{EOM}},$$

These steps were critical to reducing the number of connections for two and three point functions. This just fails for four points and higher.

One knows that there are an infinite set of higher derivative terms lurking in higher n points, dependent on  $\{D_\mu \phi^I, D_{\{\mu,\nu\}} \phi^I, D_{\{\mu,\nu,\rho\}} \phi^I, \dots\}$ ,

*This is a problem for measurements away from SM resonances.*

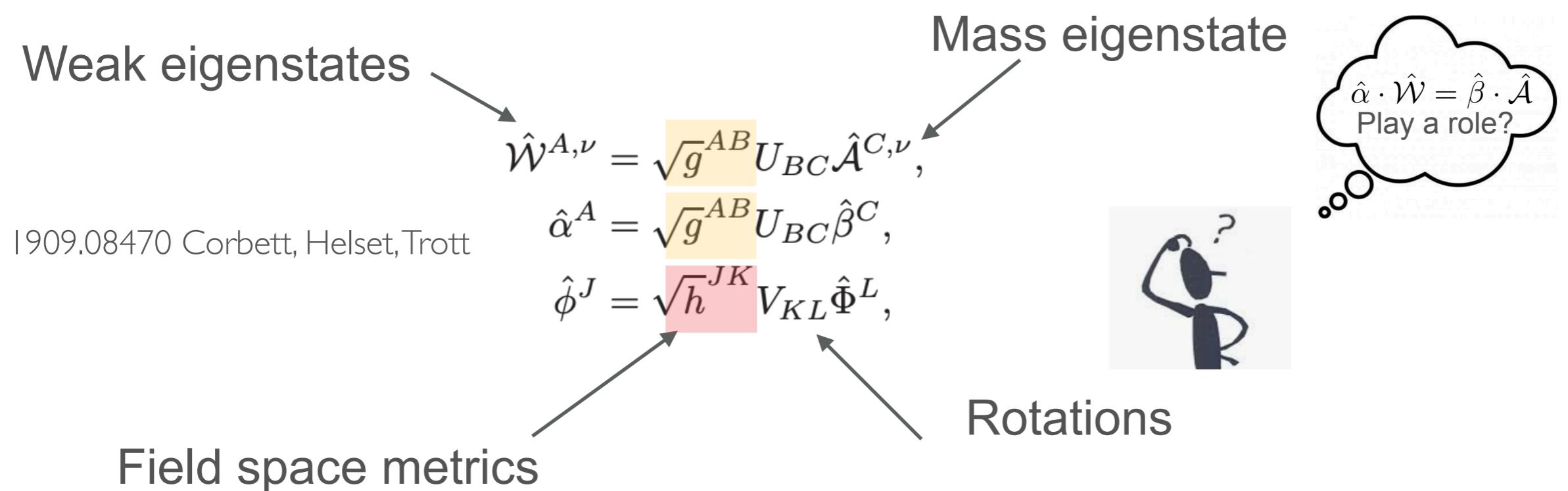
# Flat directions and vev scaling

# GeoSMEFT and flat directions

- Deep irony of the GeoSMEFT. As soon as people started to use the full SMEFT - immediately data analysis indicated this hidden structure.

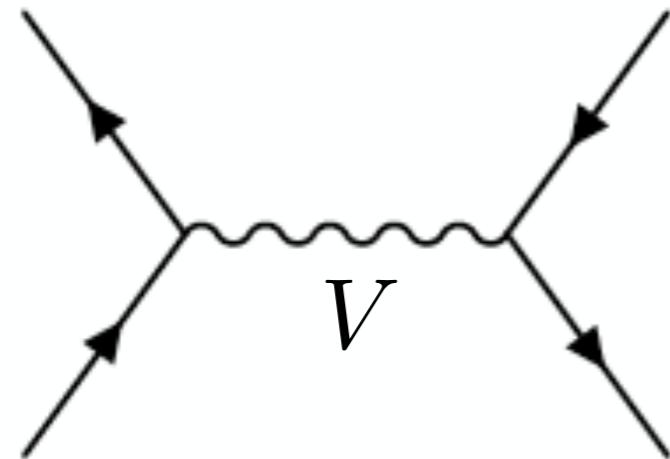
Key early one going in right direction: Han,Skiba 0412166

They found flat directions in LEP data. We now know due an invariance.



# EWPD flat directions

- Flat directions due the invariance, fundamentally its  $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$



arXiv:1701.06424 Reparameterization!

$$(V, g) \leftrightarrow (V' (1 + \epsilon), g' (1 - \epsilon)),$$

$\bar{\psi}\psi \rightarrow \bar{\psi}\psi$  scattering has a reparamatrization invariance

- LEP data can't see EOM equivalent to parameters cancelling in  $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$

$$\langle y_h g_1^2 Q_{HB} \rangle_{S_R} = \langle \sum_{\substack{\psi_\kappa=u,d, \\ q,e,l}} y_k g_1^2 \bar{\psi}_\kappa \gamma_\beta \psi_\kappa (H^\dagger i \overleftrightarrow{D}_\beta H) + \frac{g_1^2}{2} (Q_{H\square} + 4Q_{HD}) - \frac{1}{2} g_1 g_2 Q_{HWB} \rangle_{S_R},$$

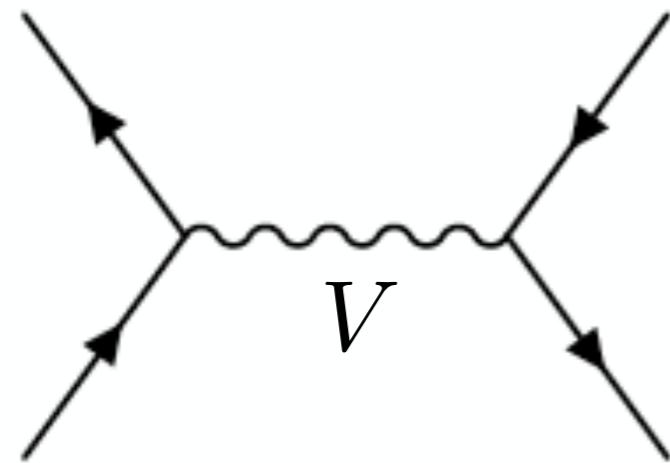
$$\langle g_2^2 Q_{HW} \rangle_{S_R} = \langle g_2^2 (\bar{q} \tau^I \gamma_\beta q + \bar{l} \tau^I \gamma_\beta l) (H^\dagger i \overleftrightarrow{D}_\beta^I H) + 2 g_2^2 Q_{H\square} - 2 g_1 g_2 y_h Q_{HWB} \rangle_{S_R}.$$

# EWPD flat directions

- Flat directions due the invariance, fundamentally its  $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$



arXiv:1701.06424 Reparameterization! Ilaria Brivio, MT



$$(V, g) \leftrightarrow (V' (1 + \epsilon), g' (1 - \epsilon)),$$

$\bar{\psi}\psi \rightarrow \bar{\psi}\psi$  scattering has a  
reparamatrization invariance

- Flat directions in many data sets project onto EOM equivalents to what cancels in the invariant  $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$   $(\tilde{C}_{HB}, \tilde{C}_{HW})$

$$w_1^\alpha = -w_B - 2.59 w_W$$
$$w_2^\alpha = -w_B + 4.31 w_W,$$

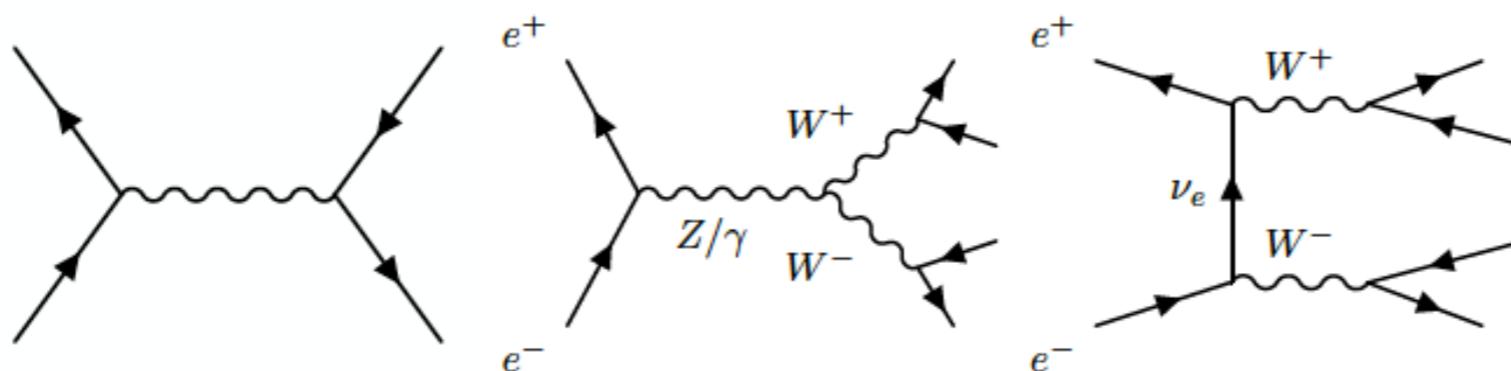
$$w_1^{m_W} = -w_B - 2.48 w_W$$
$$w_2^{m_W} = -w_B + 4.40 w_W.$$

be careful and keep all operators!

- Input scheme independent.

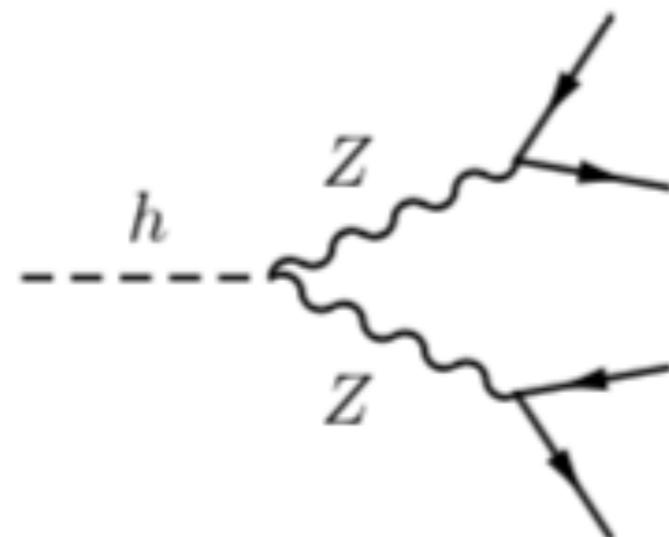
# SMEFT reparameterization invariance

- Must combine data sets in a well defined SMEFT, so no matter what operator basis you choose you get consistent results



Breaks the invariance. This channel dominant at LEP2

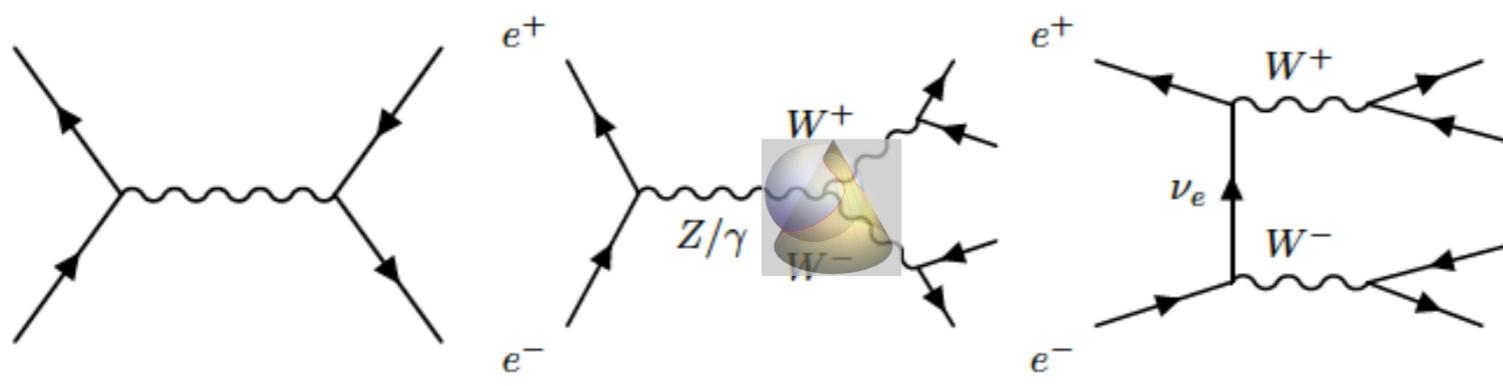
- Precision Higgs physics data will compete and we need to combine it consistently



Breaks the invariance.  
Probes the scalar and gauge  
metric connections directly.

# SMEFT reparameterization invariance

- Must combine data sets in a well defined SMEFT, so no matter what operator basis you choose you get consistent results

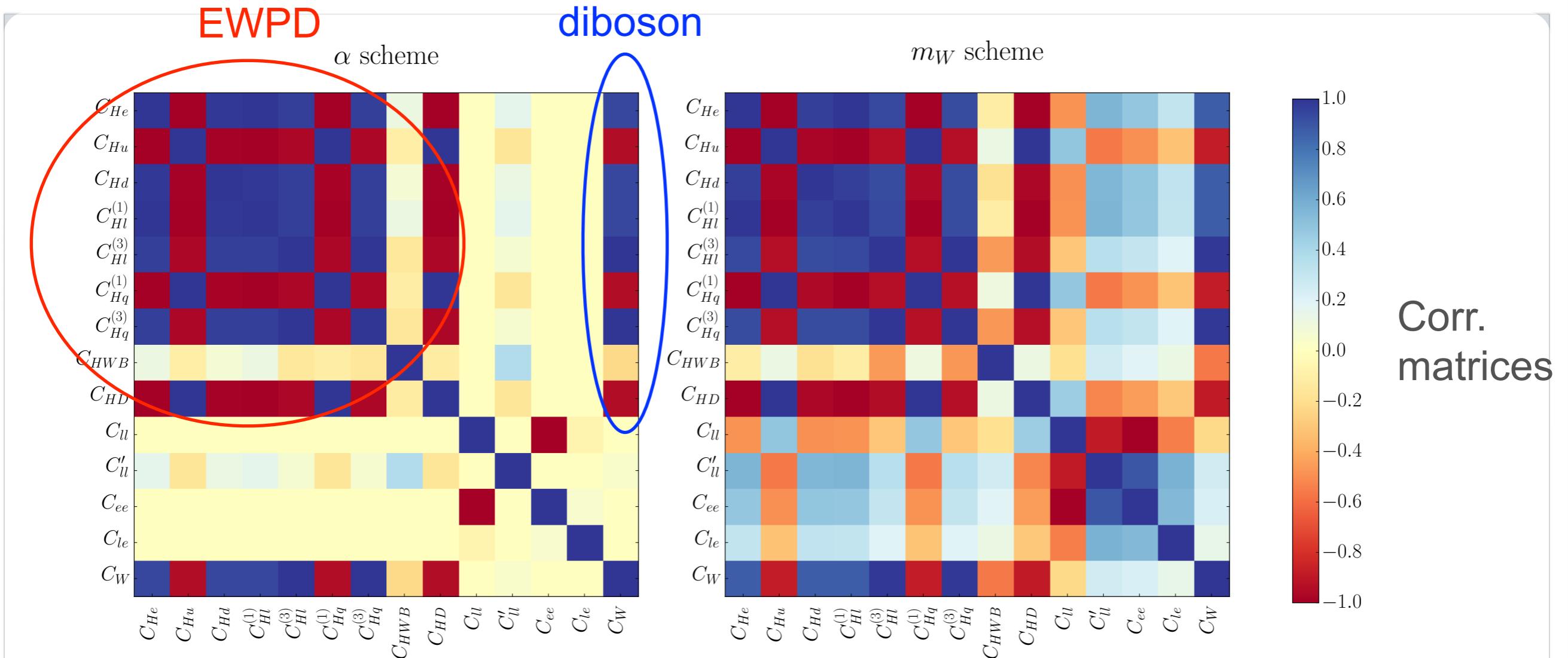


This channel dominant at LEP2

- Precision Higgs physics data will compete and we need to combine it consistently



# Flat directions reflect a consistent analysis



- Global analysis of data from PEP, PETRA, TRISTAN, SpS, Tevatron, SLAC, LEPI and LEP II
- Correlation matrices in a likelihood for the SMEFT (before higgs data)

$$L(C) = \frac{1}{\sqrt{(2\pi)^n |V|}} \exp \left( -\frac{1}{2} (\hat{\theta} - \bar{\theta})^T V^{-1} (\hat{\theta} - \bar{\theta}) \right),$$

1502.02570, 1508.05060, Berthier, MT , 1606.06693 Berthier, Bjorn, MT , arXiv:1701.06424 Brivio, MT

# Generators for SMEFT vs SM

# Generators on scalar SMEFT space

- To think in a unified gauge space manifold need generators (reformulate SM generators)

$$(D^\mu \phi)^I = (\partial^\mu \delta_J^I - \frac{1}{2} \mathcal{W}^{A,\mu} \tilde{\gamma}_{A,J}^I) \phi^J$$

$$\tilde{\epsilon}_B^A = g_2 \epsilon_B^A, \quad \text{with } \tilde{\epsilon}_{23}^1 = +g_2,$$

$$\tilde{\gamma}_{A,J}^I = \begin{cases} g_2 \gamma_{A,J}^I, & \text{for } A = 1, 2, 3 \\ g_1 \gamma_{A,J}^I, & \text{for } A = 4. \end{cases}$$

$$\gamma_{1,J}^I = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma_{2,J}^I = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\gamma_{3,J}^I = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \gamma_{4,J}^I = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

(this last one also “i”)

| 1803.08001 Helset, Paraskevas, Trott

- Some interesting math here, we also define

$$\Gamma_{A,K}^I = \gamma_{A,J}^I \gamma_{4,K}^J$$

$$\Gamma_{1,J}^I = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \Gamma_{2,J}^I = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_{3,J}^I = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Gamma_{4,J}^I = -\mathbb{I}_{4 \times 4}.$$

# Generators on scalar SMEFT space

- The mapping of operator forms works via: 2001.01453 Helset, Martin, Trott

$$\begin{aligned} H^\dagger \sigma_a H &= -\frac{1}{2} \phi_I \Gamma_{a,J}^I \phi^J, \\ H^\dagger i \overleftrightarrow{D}^\mu H &= -\phi_I \gamma_{4,J}^I (D^\mu \phi)^J = (D^\mu \phi)_I \gamma_{4,J}^I \phi^J, \\ H^\dagger i \overleftrightarrow{D}_a^\mu H &= -\phi_I \gamma_{a,J}^I (D^\mu \phi)^J = (D^\mu \phi)_I \gamma_{a,J}^I \phi^J, \\ 2 \tilde{H}^\dagger D^\mu H &= \tilde{\phi}_I (-\Gamma_{4,J}^I + i \gamma_{4,J}^I) (D^\mu \phi)^J. \end{aligned}$$

It is useful to “real” the SM symmetry representation on the scalar manifold for lots of reasons. Makes more manifest possible contractions

$$\phi_I \Gamma_{A,J}^I \phi^J \neq 0, \quad \phi_I \gamma_{a,J}^I \phi^J = \phi_I \gamma_{4,J}^I \phi^J = 0.$$

- All orders results follow, for example:

$$\begin{aligned} h_{IJ} &= \left[ 1 + \phi^2 C_{H\square}^{(6)} + \sum_{n=0}^{\infty} \left( \frac{\phi^2}{2} \right)^{n+2} (C_{HD}^{(8+2n)} - C_{H,D2}^{(8+2n)}) \right] \delta_{IJ} \\ &\quad + \frac{\Gamma_{A,J}^I \phi_K \Gamma_{A,L}^K \phi^L}{2} \left( \frac{C_{HD}^{(6)}}{2} + \sum_{n=0}^{\infty} \left( \frac{\phi^2}{2} \right)^{n+1} C_{H,D2}^{(8+2n)} \right). \end{aligned}$$

# Field space connections to all orders

2001.01453 Helset, Martin, Trott

- Field space connection for W,Z coupling to fermion pairs  $(D^\mu \phi)^I \bar{\psi} \Gamma_\mu \psi$

$$\begin{aligned}\mathcal{Q}_{H\psi_{pr}}^{1,(6+2n)} &= (H^\dagger H)^n H^\dagger i \overleftrightarrow{D}^\mu H \bar{\psi}_p \gamma_\mu \psi_r, \\ \mathcal{Q}_{H\psi_{pr}}^{3,(6+2n)} &= (H^\dagger H)^n H^\dagger i \overleftrightarrow{D}_a^\mu H \bar{\psi}_p \gamma_\mu \sigma_a \psi_r, \\ \mathcal{Q}_{H\psi_{pr}}^{2,(8+2n)} &= (H^\dagger H)^n (H^\dagger \sigma_a H) H^\dagger i \overleftrightarrow{D}^\mu H \bar{\psi}_p \gamma_\mu \sigma_a \psi_r, \\ \mathcal{Q}_{H\psi_{pr}}^{\epsilon,(8+2n)} &= \epsilon_{bc}^a (H^\dagger H)^n (H^\dagger \sigma_c H) H^\dagger i \overleftrightarrow{D}_b^\mu H \bar{\psi}_p \gamma_\mu \sigma_a \psi_r.\end{aligned}$$

Not that many op forms. Closed form field space connection.

$$\begin{aligned}L_{J,A}^{\psi,pr} &= -(\phi \gamma_4)_J \delta_{A4} \sum_{n=0}^{\infty} C_{H\psi_{pr}}^{1,(6+2n)} \left( \frac{\phi^2}{2} \right)^n - (\phi \gamma_A)_J (1 - \delta_{A4}) \sum_{n=0}^{\infty} C_{H\psi_L}^{3,(6+2n)} \left( \frac{\phi^2}{2} \right)^n \\ &\quad + \frac{1}{2} (\phi \gamma_4)_J (1 - \delta_{A4}) (\phi_K \Gamma_{A,L}^K \phi^L) \sum_{n=0}^{\infty} C_{H\psi_L}^{2,(8+2n)} \left( \frac{\phi^2}{2} \right)^n \\ &\quad + \frac{\epsilon_{BC}^A}{2} (\phi \gamma_B)_J (\phi_K \Gamma_{C,L}^K \phi^L) \sum_{n=0}^{\infty} C_{H\psi_L}^{\epsilon,(8+2n)} \left( \frac{\phi^2}{2} \right)^n.\end{aligned}$$

Notice the clean form due to generator structure and real fields.

# Field space connections to all orders

2001.01453 Helset, Martin, Trott

- Off shell operators contributing to three points  $(D_\mu \phi)^I \sigma_A (D_\nu \phi)^J \mathcal{W}_{\mu\nu}^A$

$$\begin{aligned} Q_{HDHB}^{(8+2n)} &= i(H^\dagger H)^{n+1} (D_\mu H)^\dagger (D_\nu H) B^{\mu\nu}, \\ Q_{HDHW}^{(8+2n)} &= i\delta_{ab} (H^\dagger H)^{n+1} (D_\mu H)^\dagger \sigma^a (D_\nu H) W_b^{\mu\nu}, \\ Q_{HDHW,2}^{(8+2n)} &= i\epsilon_{abc} (H^\dagger H)^n (H^\dagger \sigma^a H) (D_\mu H)^\dagger \sigma^b (D_\nu H) W_c^{\mu\nu}, \\ Q_{HDHW,3}^{(10+2n)} &= i\delta_{ab}\delta_{cd} (H^\dagger H)^n (H^\dagger \sigma^a H) (H^\dagger \sigma^c H) (D_\mu H)^\dagger \sigma^b (D_\nu H) W_d^{\mu\nu}. \end{aligned}$$

This connection saturates last in op dimension. This is due to EOM reduction. No entries at dim 6 in Warsaw basis.

$$\begin{aligned} k_{IJ}^A(\phi) &= -\frac{1}{2} \gamma_{4,J}^I \delta_{A4} \sum_{n=0}^{\infty} C_{HDHB}^{(8+2n)} \left(\frac{\phi^2}{2}\right)^{n+1} - \frac{1}{2} \gamma_{A,J}^I (1 - \delta_{A4}) \sum_{n=0}^{\infty} C_{HDHW}^{(8+2n)} \left(\frac{\phi^2}{2}\right)^{n+1} \\ &\quad - \frac{1}{8} (1 - \delta_{A4}) [\phi_K \Gamma_{A,L}^K \phi^L] [\phi_M \Gamma_{B,L}^K \phi^N] \gamma_{B,J}^I \sum_{n=0}^{\infty} C_{HDHW,3}^{(10+2n)} \left(\frac{\phi^2}{2}\right)^n \\ &\quad + \frac{1}{4} \epsilon_{ABC} [\phi_K \Gamma_{B,L}^K \phi^L] \gamma_{C,J}^I \sum_{n=0}^{\infty} C_{HDHW,2}^{(8+2n)} \left(\frac{\phi^2}{2}\right)^n. \end{aligned}$$

# Input parameter/scheme dependence

# Strong input scheme dependence in SMEFT

$\{\hat{\alpha}_{ew}, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$  Scheme

$$\delta m_Z^2 = \frac{\hat{M}_Z^2}{2} \tilde{C}_{HD} + \frac{2^{3/4} \sqrt{\pi \hat{\alpha}} \hat{M}_Z}{\hat{G}_F^{1/2}} \tilde{C}_{HWB},$$

$$\begin{aligned} \delta s_\theta^2 &= 0.17 \tilde{C}_{HD} + 0.79 \tilde{C}_{HWB} + 0.76 \tilde{C}_{Hl}^{(3)} - 0.34 \tilde{C}'_{ll}, \\ \frac{\delta \Gamma_Z}{\Gamma_Z^{SM}} &= -0.82 \tilde{C}_{HWB} - 0.67 \tilde{C}_{HD} - 0.19 \tilde{C}_{Hl}^{(1)} - 2.06 \tilde{C}_{Hl}^{(3)} - 0.19 \tilde{C}_{He} \\ &\quad + 0.47 \tilde{C}_{Hq}^{(1)} + 1.61 \tilde{C}_{Hq}^{(3)} + 0.26 \tilde{C}_{Hu} - 0.19 \tilde{C}_{Hd} + 1.35 \tilde{C}'_{ll} \\ \frac{\delta \alpha}{2 \hat{\alpha}} &= 0, \\ \frac{\delta M_W^2}{\hat{M}_W^2} &= -\frac{s_{2\hat{\theta}}}{4 c_{2\hat{\theta}}} \left( \frac{c_{\hat{\theta}}}{s_{\hat{\theta}}} \tilde{C}_{HD} + \frac{s_{\hat{\theta}}}{c_{\hat{\theta}}} 2\sqrt{2} \delta G_F + 4 \tilde{C}_{HWB} \right), \\ \frac{\delta \Gamma_W}{\Gamma_W^{SM}} &= -3.97 \tilde{C}_{HWB} - 1.80 \tilde{C}_{HD} - 3.52 \tilde{C}_{Hl}^{(3)} + 1.33 \tilde{C}_{Hq}^{(3)} + 2.10 \tilde{C}'_{ll}. \end{aligned}$$

$\{\hat{M}_W, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$  Scheme

$$\begin{aligned} \delta m_Z^2 &= \frac{\hat{M}_Z^2}{2} \tilde{C}_{HD} + 2 \hat{M}_Z \hat{M}_W \sqrt{1 - \frac{\hat{M}_W^2}{\hat{M}_Z^2}} \tilde{C}_{HWB}, \\ \delta s_\theta^2 &= -0.39 \tilde{C}_{HD} - 0.42 \tilde{C}_{HWB}, \\ \frac{\delta \Gamma_Z}{\Gamma_Z^{SM}} &= 0.46 \tilde{C}_{HWB} - 0.07 \tilde{C}_{HD} - 0.18 \tilde{C}_{Hl}^{(1)} - 1.37 \tilde{C}_{Hl}^{(3)} - 0.18 \tilde{C}_{He} \\ &\quad + 0.47 \tilde{C}_{Hq}^{(1)} + 1.61 \tilde{C}_{Hq}^{(3)} + 0.24 \tilde{C}_{Hu} - 0.18 \tilde{C}_{Hd} + \tilde{C}'_{ll}, \\ \frac{\delta \alpha}{2 \hat{\alpha}} &= -\frac{\delta G_F}{\sqrt{2}} + \frac{\delta m_Z^2}{\hat{M}_Z^2} \frac{\hat{M}_W^2}{2(\hat{M}_W^2 - \hat{M}_Z^2)} - \tilde{C}_{HWB} \frac{\hat{M}_W}{\hat{M}_Z} \sqrt{1 - \frac{\hat{M}_W^2}{\hat{M}_Z^2}}, \\ \frac{\delta M_W^2}{\hat{M}_W^2} &= 0, \\ \frac{\delta \Gamma_W}{\Gamma_W^{SM}} &= \frac{4}{3} \left( \tilde{C}_{Hq}^{(3)} - \tilde{C}_{Hl}^{(3)} \right) + \tilde{C}'_{ll}. \end{aligned}$$

- At leading order (tree level) already strong input parameter dependence, different than case in SM!

# Strong input scheme dependence in SMEFT

$\{\hat{\alpha}_{ew}, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$  Scheme

$$\delta m_Z^2 = \frac{\hat{M}_Z^2}{2} \tilde{C}_{HD} + \frac{2^{3/4} \sqrt{\pi \hat{\alpha}} \hat{M}_Z}{\hat{G}_F^{1/2}} \tilde{C}_{HWB},$$

$$\begin{aligned} \delta s_\theta^2 &= 0.17 \tilde{C}_{HD} + 0.79 \tilde{C}_{HWB} + 0.76 \tilde{C}_{Hl}^{(3)} - 0.34 \tilde{C}'_{ll}, \\ \frac{\delta \Gamma_Z}{\Gamma_Z^{SM}} &= -0.82 \tilde{C}_{HWB} - 0.67 \tilde{C}_{HD} - 0.19 \tilde{C}_{Hl}^{(1)} - 2.06 \tilde{C}_{Hl}^{(3)} - 0.19 \tilde{C}_{He} \\ &\quad + 0.47 \tilde{C}_{Hq}^{(1)} + 1.61 \tilde{C}_{Hq}^{(3)} + 0.26 \tilde{C}_{Hu} - 0.19 \tilde{C}_{Hd} + 1.35 \tilde{C}'_{ll} \end{aligned}$$

$$\frac{\delta \alpha}{2 \hat{\alpha}} = 0,$$

$$\frac{\delta M_W^2}{\hat{M}_W^2} = -\frac{s_{2\hat{\theta}}}{4 c_{2\hat{\theta}}} \left( \frac{c_{\hat{\theta}}}{s_{\hat{\theta}}} \tilde{C}_{HD} + \frac{s_{\hat{\theta}}}{c_{\hat{\theta}}} 2\sqrt{2} \delta G_F + 4 \tilde{C}_{HWB} \right),$$

$$\frac{\delta \Gamma_W}{\Gamma_W^{SM}} = -3.97 \tilde{C}_{HWB} - 1.80 \tilde{C}_{HD} - 3.52 \tilde{C}_{Hl}^{(3)} + 1.33 \tilde{C}_{Hq}^{(3)} + 2.10 \tilde{C}'_{ll}.$$

$\{\hat{M}_W, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$  Scheme

$$\delta m_Z^2 = \frac{\hat{M}_Z^2}{2} \tilde{C}_{HD} + 2 \hat{M}_Z \hat{M}_W \sqrt{1 - \frac{\hat{M}_W^2}{\hat{M}_Z^2}} \tilde{C}_{HWB},$$

$$\begin{aligned} \delta s_\theta^2 &= -0.39 \tilde{C}_{HD} - 0.42 \tilde{C}_{HWB}, \\ \frac{\delta \Gamma_Z}{\Gamma_Z^{SM}} &= 0.46 \tilde{C}_{HWB} - 0.07 \tilde{C}_{HD} - 0.18 \tilde{C}_{Hl}^{(1)} - 1.37 \tilde{C}_{Hl}^{(3)} - 0.18 \tilde{C}_{He} \\ &\quad + 0.47 \tilde{C}_{Hq}^{(1)} + 1.61 \tilde{C}_{Hq}^{(3)} + 0.24 \tilde{C}_{Hu} - 0.18 \tilde{C}_{Hd} + \tilde{C}'_{ll}, \end{aligned}$$

$$\frac{\delta \alpha}{2 \hat{\alpha}} = -\frac{\delta G_F}{\sqrt{2}} + \frac{\delta m_Z^2}{\hat{M}_Z^2} \frac{\hat{M}_W^2}{2(\hat{M}_W^2 - \hat{M}_Z^2)} - \tilde{C}_{HWB} \frac{\hat{M}_W}{\hat{M}_Z} \sqrt{1 - \frac{\hat{M}_W^2}{\hat{M}_Z^2}},$$

$$\frac{\delta M_W^2}{\hat{M}_W^2} = 0,$$

$$\frac{\delta \Gamma_W}{\Gamma_W^{SM}} = \frac{4}{3} \left( \tilde{C}_{Hq}^{(3)} - \tilde{C}_{Hl}^{(3)} \right) + \tilde{C}'_{ll}.$$

- Completely expected from decoupling theorem. **UV physics preserving SM symmetries being absorbed into measured low scale parameters now.**

# Need input parameters defined at all orders

## $\{\hat{M}_W, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$ Scheme

Sorted in

Hays, Helset, Martin Trott: 2007.00565

Input parameter dependence  
Increased order by order  
due to Lagrangian parameters  
being redefined geometrically

### D $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$ input-parameter scheme at all orders in $(\bar{v}_T^2/\Lambda^2)^n$

In this scheme we can again use Eqn. (E.2) to define a shift to  $\bar{g}_Z$ . We also use

$$\bar{g}_2 = g_2 \sqrt{g^{11}} = \frac{2\hat{m}_W}{\sqrt{h_{11}\bar{v}_T}}. \quad (\text{D.1})$$

and

$$g_1 = g_2 \frac{(s_{\bar{\theta}}\sqrt{g^{33}} + c_{\bar{\theta}}\sqrt{g^{34}})}{(c_{\bar{\theta}}\sqrt{g^{44}} + s_{\bar{\theta}}\sqrt{g^{34}})} \quad (\text{D.2})$$

to solve for  $s_{\bar{\theta}}^2$  via

$$s_{\bar{\theta}}^2 = \frac{1}{[(\sqrt{g^{44}})^2 + (\sqrt{g^{34}})^2]^2} \left\{ - \left( \frac{g_2 \sqrt{g_-}}{\bar{g}_Z} \right)^2 \left[ (\sqrt{g^{44}})^2 - (\sqrt{g^{34}})^2 \right] + (\sqrt{g^{44}})^2 \left[ (\sqrt{g^{44}})^2 + (\sqrt{g^{34}})^2 \right] - 2 \left( \frac{g_2 \sqrt{g_-}}{\bar{g}_Z} \right) \sqrt{(\sqrt{g^{44}})^2 (\sqrt{g^{34}})^2 \left[ (\sqrt{g^{44}})^2 + (\sqrt{g^{34}})^2 - \left( \frac{g_2 \sqrt{g_-}}{\bar{g}_Z} \right)^2 \right]} \right\}. \quad (\text{D.3})$$

The remaining Lagrangian parameters can then be defined via

$$\bar{e} = \frac{\bar{g}_2}{\sqrt{g^{11}}} (s_{\bar{\theta}}\sqrt{g^{33}} + c_{\bar{\theta}}\sqrt{g^{34}}), \quad (\text{D.4})$$

and

$$s_{\theta_Z}^2 = \frac{\bar{e}}{\bar{g}_Z} \frac{(s_{\bar{\theta}}\sqrt{g^{44}} - c_{\bar{\theta}}\sqrt{g^{34}})}{(c_{\bar{\theta}}\sqrt{g^{44}} + s_{\bar{\theta}}\sqrt{g^{34}})}. \quad (\text{D.5})$$

In both schemes,  $\bar{g}_Z$  and  $s_{\theta_Z}^2$  have the same definition in terms of other “barred” Lagrangian parameters.

# Model Example

# Consider a kinetic mixing model to dim 8

- Lets see how this works in the simplest benchmark model to dim 8

$$\Delta\mathcal{L} = -\frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{1}{2}m_K^2K_\mu K^\mu - \frac{k}{2}B^{\mu\nu}K_{\mu\nu},$$

Only has 2 parameters. Dimension 6 matching:

	$H^4 D^2$	$\psi^4 : (\bar{R}R)(\bar{R}R)$	$\psi^4 : (\bar{L}L)(\bar{R}R)$
$H^2\psi^2 D$			
$C_{H\ell}^{1,(6)}$	$-\frac{y_\ell g_1^2}{2m_K^2} b_1$	$C_{ee}^{(6)}$	$C_{\ell e}^{(6)}$
$C_{He}^{(6)}$	$-\frac{y_e g_1^2}{2m_K^2} b_1$	$C_{uu}^{(6)}$	$C_{\ell u}^{(6)}$
$C_{Hq}^{1,(6)}$	$-\frac{y_q g_1^2}{2m_K^2} b_1$	$C_{dd}^{(6)}$	$C_{\ell d}^{(6)}$
$C_{Hu}^{(6)}$	$-\frac{y_u g_1^2}{2m_K^2} b_1$	$C_{eu}^{(6)}$	$C_{qe}^{(6)}$
$C_{Hd}^{(6)}$	$-\frac{y_d g_1^2}{2m_K^2} b_1$	$C_{ed}^{(6)}$	$C_{qu}^{1,(6)}$
		$C_{ud}^{1,(6)}$	$C_{qd}^{1,(6)}$

# Consider a kinetic mixing model to dim 8

- Lets see how this works in the simplest benchmark model to dim 8

$$\Delta\mathcal{L} = -\frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{1}{2}m_K^2 K_\mu K^\mu - \frac{k}{2}B^{\mu\nu}K_{\mu\nu},$$

Only has 2 parameters. Dimension 8 matching:

$H^4\psi^2 D$	
$C_{H\ell}^{1,(8)}$	$\frac{y_\ell g_1^4}{4 m_K^4} k^4 - \frac{g_1^2 y_\ell}{m_K^4} (k^2 - k^4) (2\lambda + \frac{g_1^2 + g_2^2}{4})$
$C_{He}^{1,(8)}$	$\frac{y_e g_1^4}{4 m_K^4} k^4 - \frac{g_1^2 y_e}{m_K^4} (k^2 - k^4) (2\lambda + \frac{g_1^2 + g_2^2}{4})$
$C_{Hq}^{1,(8)}$	$\frac{y_q g_1^4}{4 m_K^4} k^4 - \frac{g_1^2 y_q}{m_K^4} (k^2 - k^4) (2\lambda + \frac{g_1^2 + g_2^2}{4})$
$C_{Hu}^{1,(8)}$	$\frac{y_u g_1^4}{4 m_K^4} k^4 - \frac{g_1^2 y_u}{m_K^4} (k^2 - k^4) (2\lambda + \frac{g_1^2 + g_2^2}{4})$
$C_{Hd}^{1,(8)}$	$\frac{y_d g_1^4}{4 m_K^4} k^4 - \frac{g_1^2 y_d}{m_K^4} (k^2 - k^4) (2\lambda + \frac{g_1^2 + g_2^2}{4})$
$C_{H\ell}^{2,(8)}$	$-\frac{g_1^2 g_2^2}{16 m_K^4} (k^2 - k^4)$
$C_{Hq}^{2,(8)}$	$-\frac{g_1^2 g_2^2}{16 m_K^4} (k^2 - k^4)$
$C_{H\ell}^{3,(8)}$	$-\frac{g_1^2 g_2^2}{16 m_K^4} (k^2 - k^4)$
$C_{Hq}^{3,(8)}$	$-\frac{g_1^2 g_2^2}{16 m_K^4} (k^2 - k^4)$

$H^6 D^2$	
$C_{H,D2}^{(8)}$	$\frac{g_1^4 k^4}{8 m_K^4} - \frac{g_1^2 g_2^2}{2 m_K^4} (k^2 - k^4)$
$C_{HD}^{(8)}$	$\frac{3 g_1^4 k^4}{16 m_K^4} - \frac{g_1^2 g_2^2}{2 m_K^4} (k^2 - k^4)$

$X^2 H^4$	
$C_{HB}^{(8)}$	$-\frac{g_1^4}{16 m_K^4} (k^2 - k^4)$
$C_{HW}^{(8)}$	$\frac{g_1^2 g_2^2}{16 m_K^4} (k^2 - k^4)$

No loop suppression.

# Consider a kinetic mixing model to dim 8

- Lets see how this works in the simplest benchmark model to dim 8

$$\Delta\mathcal{L} = -\frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{1}{2}m_K^2 K_\mu K^\mu - \frac{k}{2}B^{\mu\nu}K_{\mu\nu},$$

- Dimension 6 squared approximation:

$$\frac{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{p.s., \hat{\alpha}_{ew}}}{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{\text{SM}, \hat{\alpha}_{ew}}} = 1 + 4.5 \times 10^{-3} \frac{\bar{v}_T^2 k^2}{m_K^2} + 4.4 \times 10^{-3} k^4 \frac{\bar{v}_T^4}{m_K^4},$$

$$\frac{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{p.s., \hat{m}_W}}{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{\text{SM}, \hat{m}_W}} = 1 - 3.1 \times 10^{-2} \frac{\bar{v}_T^2 k^2}{m_K^2} + 2.3 \times 10^{-4} k^4 \frac{\bar{v}_T^4}{m_K^4}.$$

# Consider a kinetic mixing model to dim 8

- Lets see how this works in the simplest benchmark model to dim 8

$$\Delta\mathcal{L} = -\frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{1}{2}m_K^2 K_\mu K^\mu - \frac{k}{2}B^{\mu\nu}K_{\mu\nu},$$

- To Dimension 8 consistently:

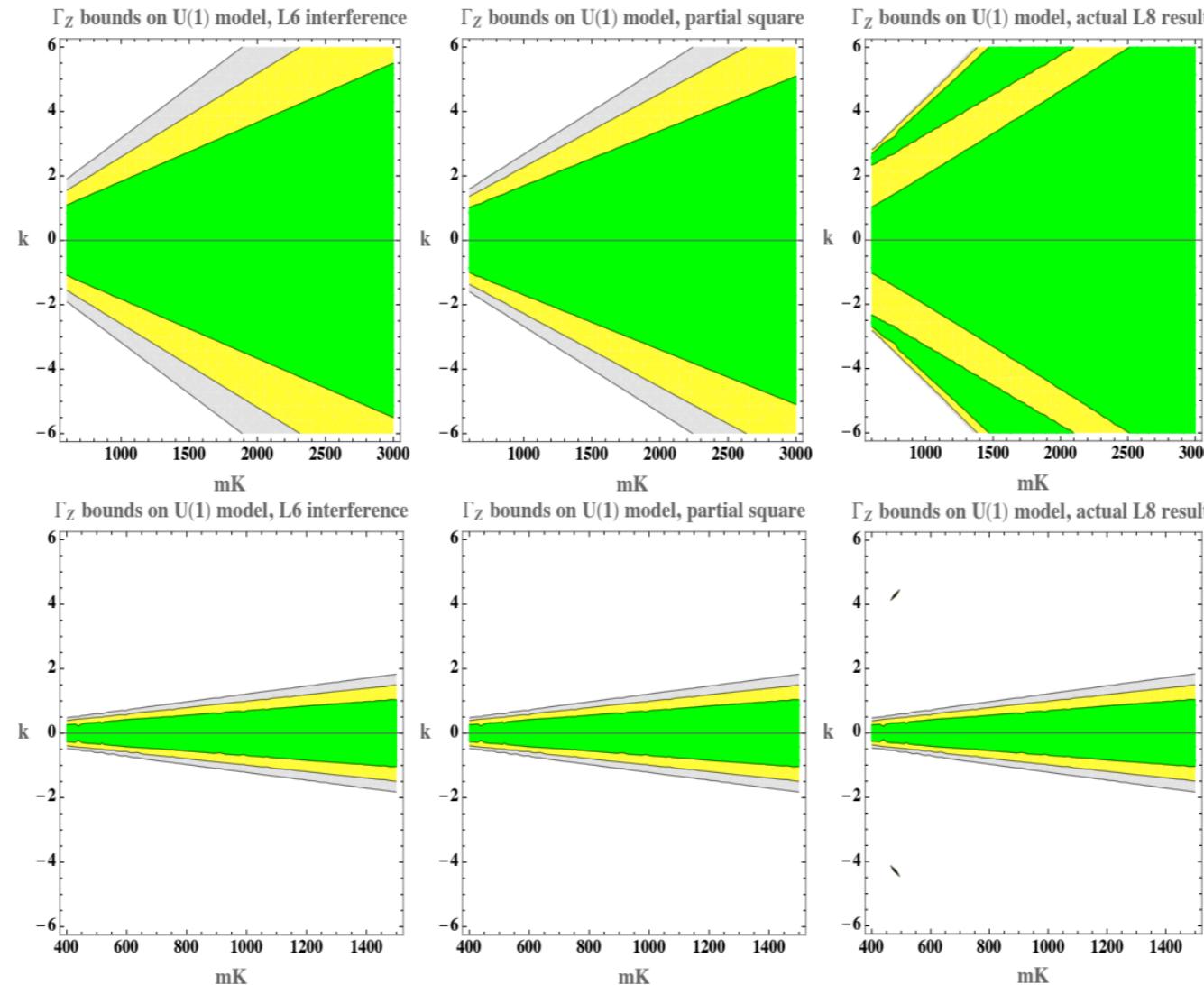
$$\frac{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{\text{SMEFT}, \hat{\alpha}_{ew}}}{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{\text{SM}, \hat{\alpha}_{ew}}} = 1 + 4.5 \times 10^{-3} \frac{\bar{v}_T^2 k^2}{m_K^2} - 5.7 \times 10^{-3} (k^4 - 1.74k^2) \frac{\bar{v}_T^4}{m_K^4},$$

$$\frac{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{\text{SMEFT}, \hat{m}_W}}{\sum_\psi \bar{\Gamma}_{Z \rightarrow \bar{\psi}_p \psi_p}^{\text{SM}, \hat{m}_W}} = 1 - 3.1 \times 10^{-2} \frac{\bar{v}_T^2 k^2}{m_K^2} + 7.9 \times 10^{-3} (k^4 - 0.88k^2) \frac{\bar{v}_T^4}{m_K^4}.$$

# Consider a kinetic mixing model to dim 8

- Strong input parameter dependence in any one observable:

- L6<sup>2</sup> can manifestly mislead.



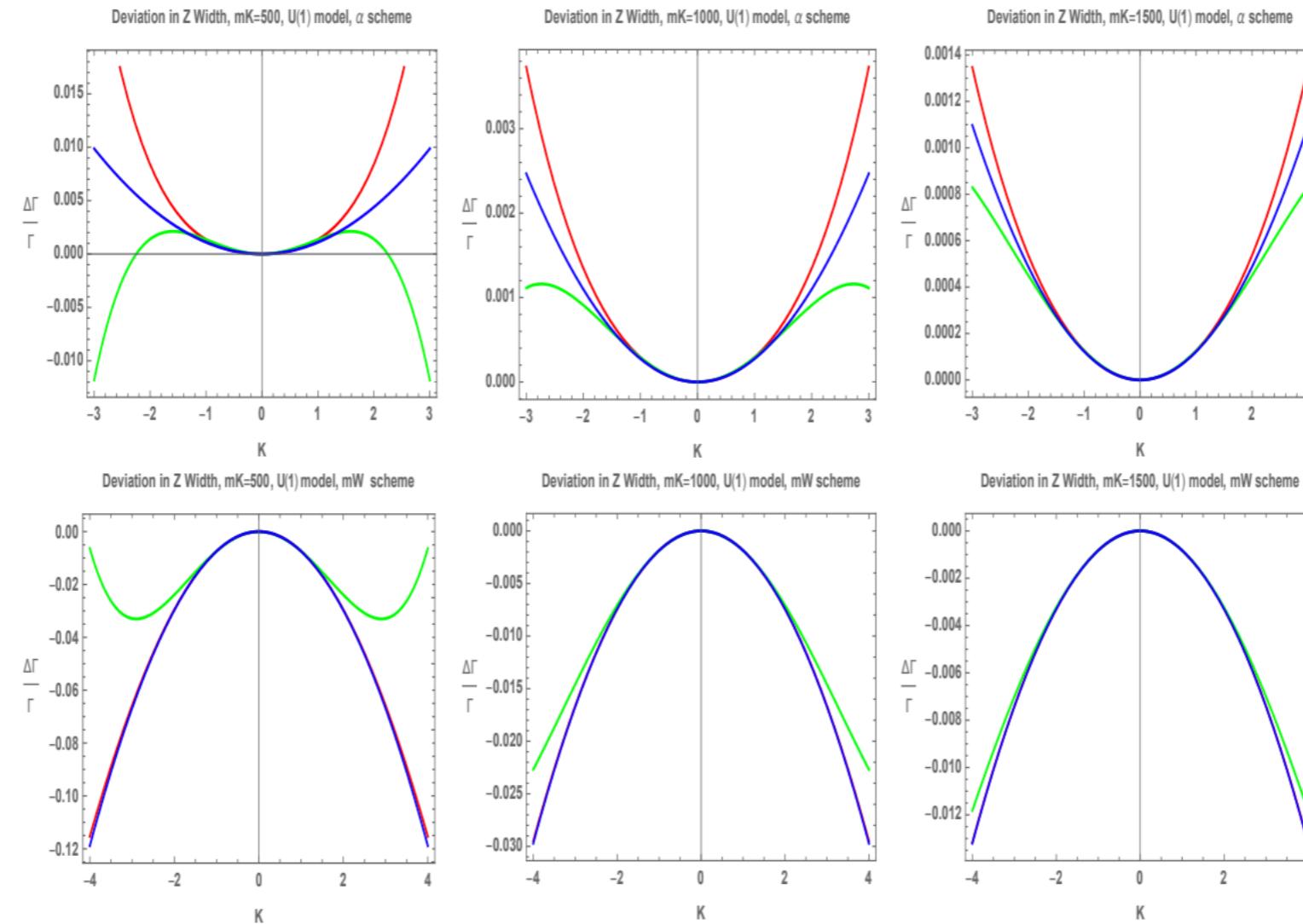
**Figure 8.** Illustrative bounds on a U(1) mixing model parameters due to bounds on  $\Gamma_Z$ . Shown is the  $\{1, 2, 3\}\sigma$  allowed region in green, yellow, gray. Here  $1\sigma$  for  $\delta\Gamma_Z = 0.0023/2.4952$ . Results shown are for the  $\hat{\alpha}_{ew}$  input-parameter scheme in the first row. The results in the second row are in the  $\hat{m}_W$  input-parameter scheme.

- Clearly need to go beyond quadratic. With geoSMEFT we can proceed efficiently to full dim 8.

# Consider a kinetic mixing model to dim 8

- Strong input parameter dependence in any one observable:

- L6^2 can Mislead



**Figure 9.** Deviation in the  $Z$  width in the  $U(1)$  mixing model for fixed  $m_K$  comparing the partial-square result in red and the full SMEFT result at  $\mathcal{L}^{(8)}$  in green, and the full SMEFT result at  $\mathcal{L}^{(6)}$  in blue.

- The power of the EFT is that you can combine observables and you have to do it

# One loop consistency checks

# Consistency checks at one loop/dim8

- The operator and loop expansion are not independent.

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)} a_i + \dots$$

If you choose to rescale (or not) the Wilson coefficient at L6 it changes the one loop result and the dimension 8 result in a correlated way.

Only game in town for full dimension 8 with all input redefinitions etc is geoSMEFT. Loops should be done in BFM for consistency with background field independent formulation defining geoSMEFT.

# Consistency checks at one loop/dim8

Benefits of the Background Field method one loop approach in SMEFT.

- Many cross checks afforded (Ward identities and more).
- Clean understanding of ward identities.
- One loop redefinition of input parameters INDIVIDUALLY gauge independent.
- Cross checks of  $\Delta Z_e = -\frac{1}{2}\Delta Z_{\hat{\mathcal{A}}}$ ,  $\Delta R_e = -\frac{1}{2}\Delta R_{\hat{\mathcal{A}}}$ . Our calc in 2107.07470, Stoffer/Denkens in [1908.05295](#)

$$\Delta R_{\hat{\mathcal{A}}} = \frac{\bar{g}_1^2 \bar{g}_2^2}{(\bar{g}_1^2 + \bar{g}_2^2)} \left[ -\frac{7}{16\pi^2} \log\left(\frac{\mu^2}{\bar{m}_W^2}\right) + \sum_{\psi} \frac{N_c^\psi Q_\psi^2}{12\pi^2} \log\left(\frac{\mu^2}{\bar{m}_\psi^2}\right) - \frac{1}{24\pi^2} \right].$$

# Consistency checks at one loop/dim8

Cancelation of large mt dependent logs in relations between observables:  
Expected and anticipated in Hartmann/Trott. [1505.02646](#)

- Expected cancelation confirmed in 2107.07470 and [1908.05295](#)

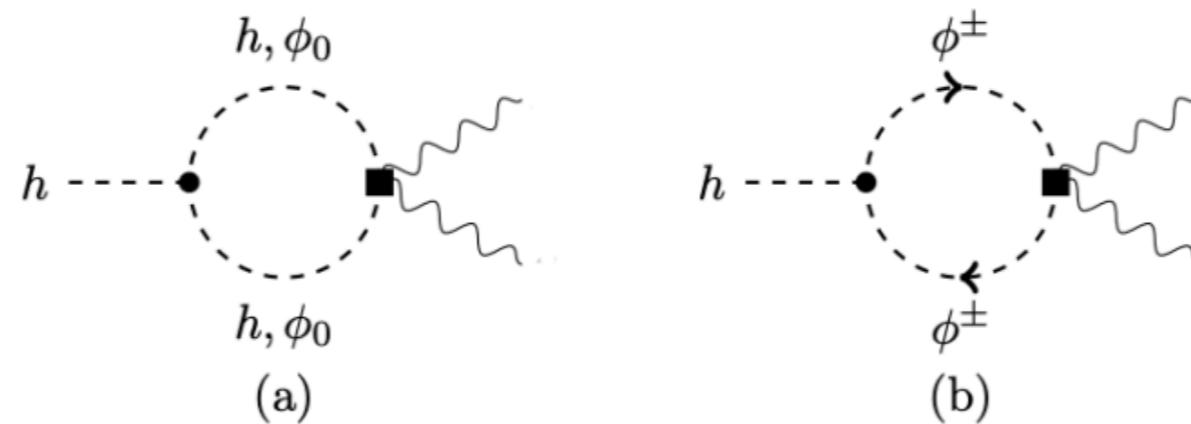
$$\bar{v}_T = \hat{v}_T \left[ 1 + \frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2} \left[ 1 + \log \left( \frac{\mu^2}{m_f^2} \right) \right] + \dots \right].$$

$$\frac{\Delta v}{\bar{v}_T} \propto -\frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2} \left[ 1 + \log \left( \frac{\mu^2}{m_f^2} \right) \right].$$

- Cancelation in single Higgs, single dev observables with tadpole term and GF extraction.

# Consistency checks at one loop/dim8

Gauge independence of a common partial matrix element in single Higgs processes in BFM:



**Figure 2.** One loop contributions to  $\langle \phi_4 | F F \rangle \langle \frac{\delta M_{AB}}{\delta \phi_4} \rangle$ .

$$\frac{\langle \phi_4 F(p_1) F(p_2) \rangle^1}{\langle \phi_4 F^{\mu\nu} F_{\mu\nu} \rangle^0 \langle \frac{\delta M_{AB}(\phi)}{\delta \phi_4} \rangle^0} \propto M_1$$

$$M_1 \equiv \left( \frac{\Delta R_h}{2} + \frac{\Delta v}{v} + \frac{(\sqrt{3}\pi - 6)\lambda}{16\pi^2} + \frac{1}{16\pi^2} \left( \frac{\bar{g}_1^2}{4} + \frac{3\bar{g}_2^2}{4} + 6\lambda \right) \log \left[ \frac{\bar{m}_h^2}{\mu^2} \right] \right), \\ + \frac{1}{16\pi^2} \left( \frac{\bar{g}_1^2}{4} \mathcal{I}[\bar{m}_Z] + \left( \frac{\bar{g}_2^2}{4} + \lambda \right) (\mathcal{I}[\bar{m}_Z] + 2\mathcal{I}[\bar{m}_W]) \right).$$