Soft gluon resummation for non-global observables in SCET

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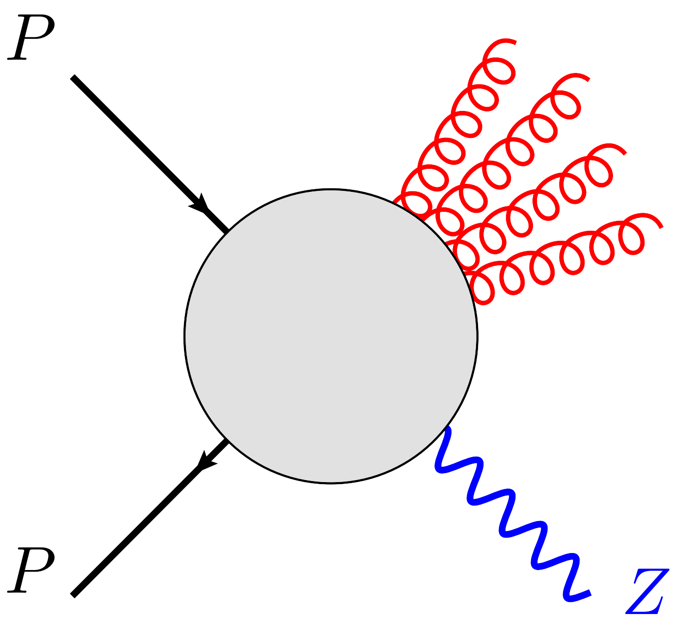
Resummation: all-order

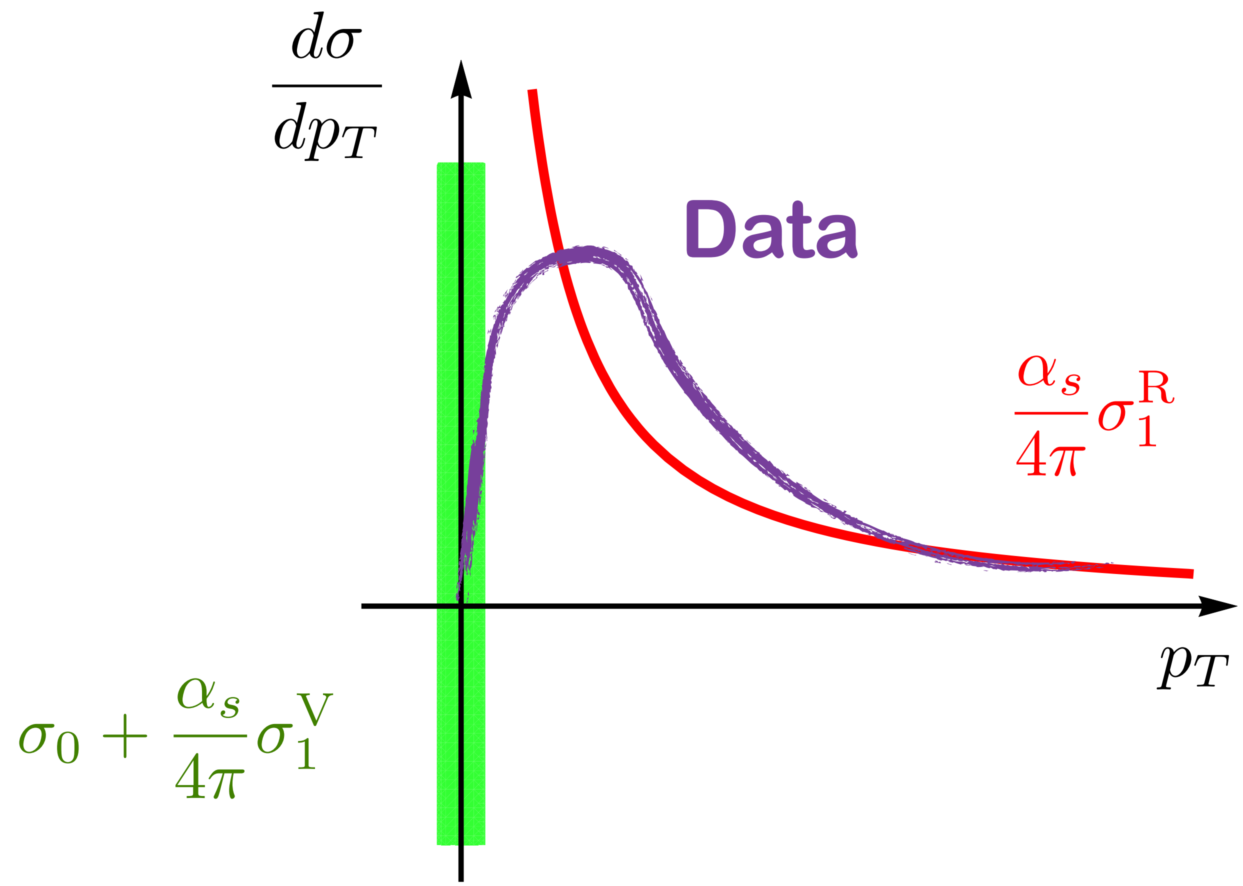
- In collider process many observables are (often by construction) sensitive to multi-parton production in the final state

- The production of many particles is typically suppressed. However, the rate could be enhanced by large logarithmic terms in the perturbation series.

- Fixed order expansions are not always sufficient! Need all-order results

A classical example: Drell-Yan P_T





$$L = \ln(m_Z^2/p_T^2)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \frac{\mathrm{d}}{\mathrm{d}p_T}\left(\sigma_0 \exp\left(-\frac{\alpha_s C_F}{2\pi}L^2\right)\right)$$

- Fixed order in α_s fails if $L \gg 1$
- All-order resummation necessary

$$\sigma \sim 1 + \alpha_s L^2 + \alpha_s^2 L^4 + \alpha_s^3 L^6 + \cdots \sim e^{-\alpha_s L^2}$$

All-order structure

- All order result generally exponentiate

$$L = \ln(\lambda)$$

λ: small parameter (depends on observable)

$$\sigma \sim \sigma_0 \exp(\alpha_s L^2 + \alpha_s L + \alpha_s^2 L + \dots)$$

- Based on this exponentiation we can define a resumed perturbative order

$$\begin{array}{rl} \sigma \sim \sigma_0 \big[1 + \alpha_s & (L^2 + L + c_1) \qquad \text{NLO} \\ + \alpha_s^2 & (L^4 + L^3 + L^2 + L + c_2) \qquad \text{NNLO} \\ + \vdots & (\vdots + \vdots + \vdots + \vdots + \dots) \big] \qquad \text{N}^n\text{LO} \\ & \text{LL} \quad \text{NLL} \quad \text{NNLL} \end{array}$$

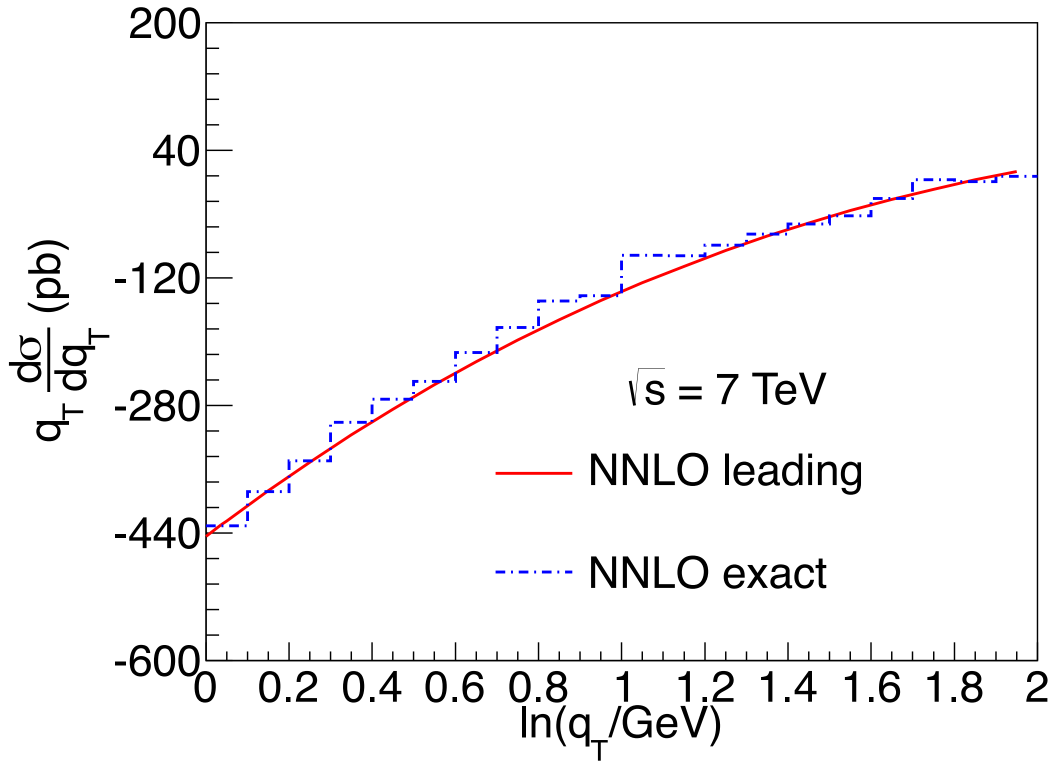
- Only correct for global observables (see later)

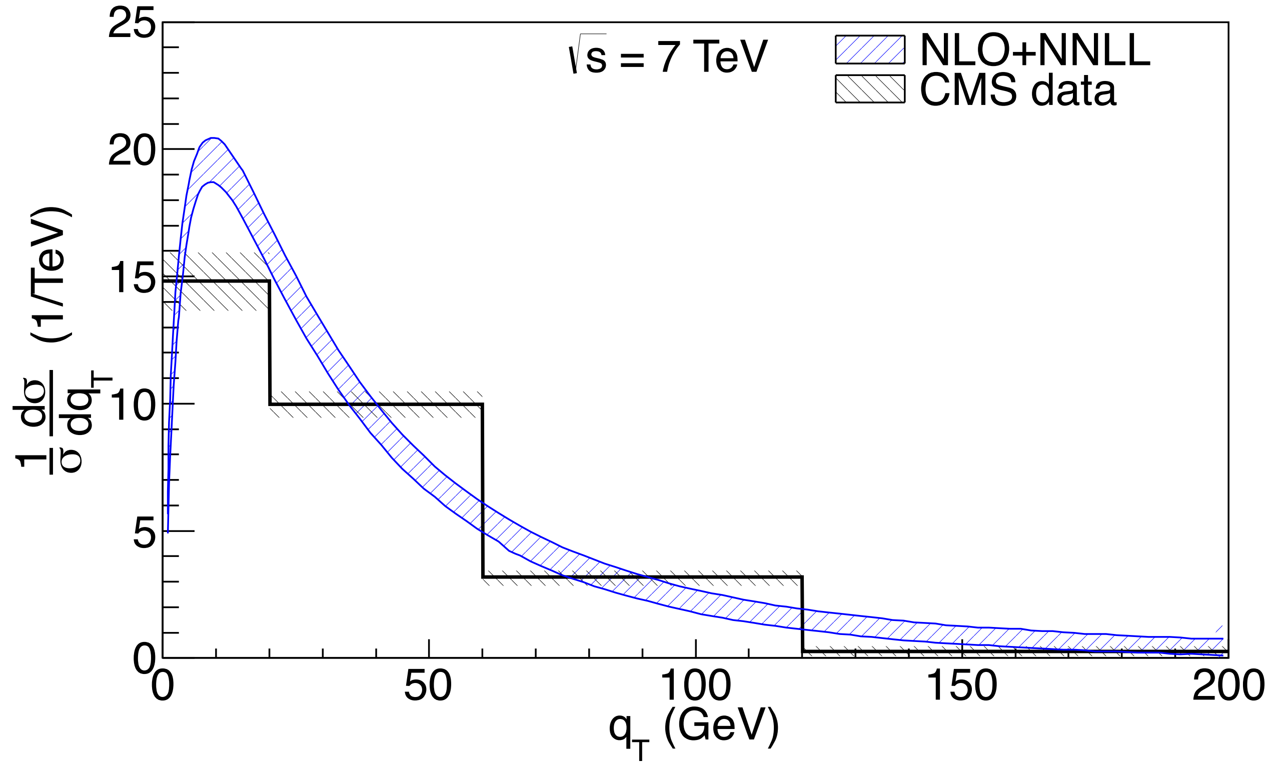
Top quark pair production at small q_T

Li, Li, DYS, Yang & Zhu '13 PRL;
Bonciani, Catani, Grazzini, Sargsyan & Torre '15

$$\frac{d^4\sigma}{dq_T^2\,dy\,dM\,d\cos\theta} = \sum_{i=q,\bar{q},g} \frac{8\pi\beta_t}{3sM}\,\frac{1}{2} \int x_T dx_T J_0(x_T q_T) \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{i\bar{i}}(x_T^2,\mu)} B_{i/N_1}(\xi_1, x_T^2, \mu) B_{\bar{i}/N_2}(\xi_2, x_T^2, \mu)$$
$$\times \mathrm{Tr}[\boldsymbol{H}_{i\bar{i}}(M, m_t, \cos\theta, \mu)\underline{\boldsymbol{S}_{i\bar{i}}(L_\perp, M, m_t, \cos\theta, \mu)}].$$

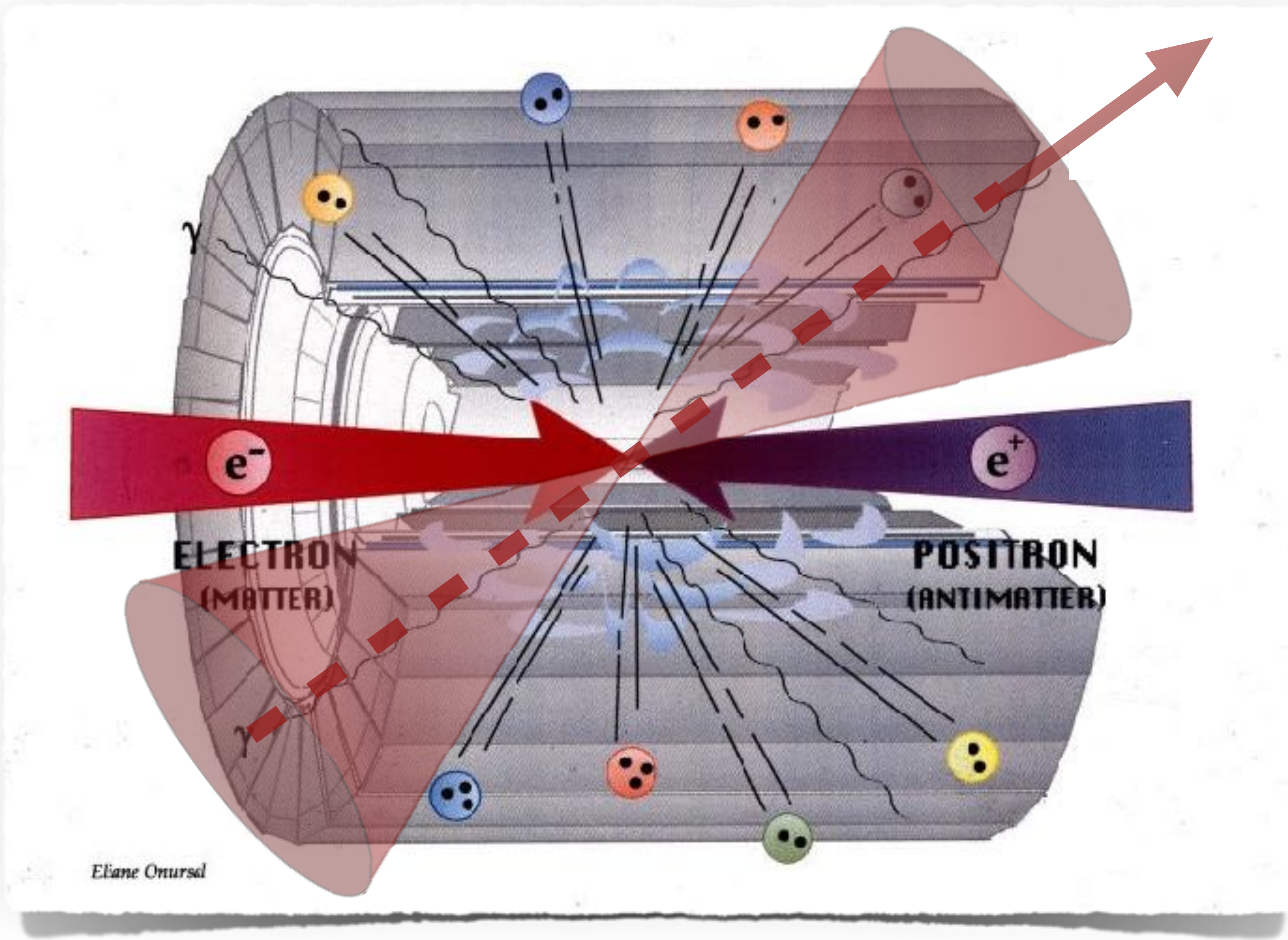
soft function accounts for the soft gluon emissions from final massive states





Non-global Observables

- Insensitive to radiation inside certain region of phase space

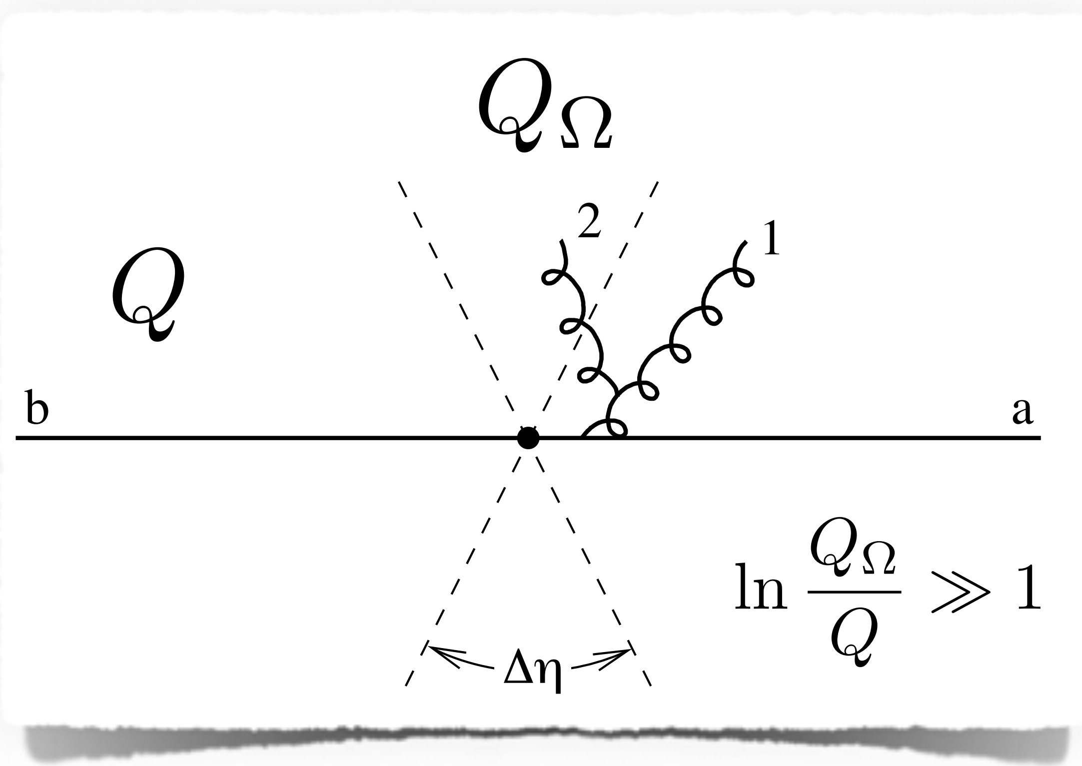


Interjet energy flow

Observables which are insensitive to emissions into certain regions of phase space involve additional NGLs not captured by the usual resummation formula

Exponentiating soft anomalous dimension only resum part of logs:

$$\exp\left[-4\,C_F\Delta\eta\int_{\alpha(Q_\Omega)}^{\alpha(Q)}\frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{2\pi}\right] = 1+4\frac{\alpha_s}{2\pi}C_F\Delta\eta\ln\frac{Q_\Omega}{Q}$$
$$+\left(\frac{\alpha_s}{2\pi}\right)^2\left(8\,C_F^2\Delta\eta^2-\frac{22}{3}\,C_FC_A\Delta\eta+\frac{8}{3}C_FT_Fn_f\Delta\eta\right)\ln^2\frac{Q_\Omega}{Q}$$



Non-global logs:

$$\left(\frac{\alpha_s}{2\pi}\right)^2 C_FC_A\left[-\frac{2\pi^2}{3}+4\,\mathrm{Li}_2\left(e^{-2\Delta\eta}\right)\right]\ln^2\frac{Q_\Omega}{Q}$$

(Dasgupta & Salam 2002)

LL resummation for non-global observables

- The leading logarithms arise from configuration in which the emitted gluons are strongly ordered

$$E_1 \gg E_2 \gg \cdots \gg E_m$$

- In the large-Nc limit, multi-gluon emission amplitudes become simple:

$$N_C^m g_s^{2m} \sum_{(1,\cdots,m)} \frac{p_a \cdot p_b}{(p_a \cdot p_1)(p_1 \cdot p_2) \cdots (p_m \cdot p_b)}$$

- Dasgupta-Salam shower

$$\mathcal{S}(\alpha_s L) \simeq \exp\left(-C_F C_A \frac{\pi^2}{3}\left(\frac{1+(at)^2}{1+(bt)^c}\right)t^2\right) \qquad a = 0.85\, C_A, \qquad b = 0.86\, C_A \qquad c = 1.33$$

(Dasgupta & Salam 2001)

- Banfi-Marchesini-Smye eqation

$$\partial_{\hat{L}} G_{kl}(\hat{L}) = \int \frac{d\Omega(n_j)}{4\pi} W_{kl}^j \left[\Theta_{\rm in}^{n\bar{n}}(j)\, G_{kj}(\hat{L})\, G_{jl}(\hat{L}) - G_{kl}(\hat{L})\right]$$

(Banfi, Marchesini & Smye 2002)

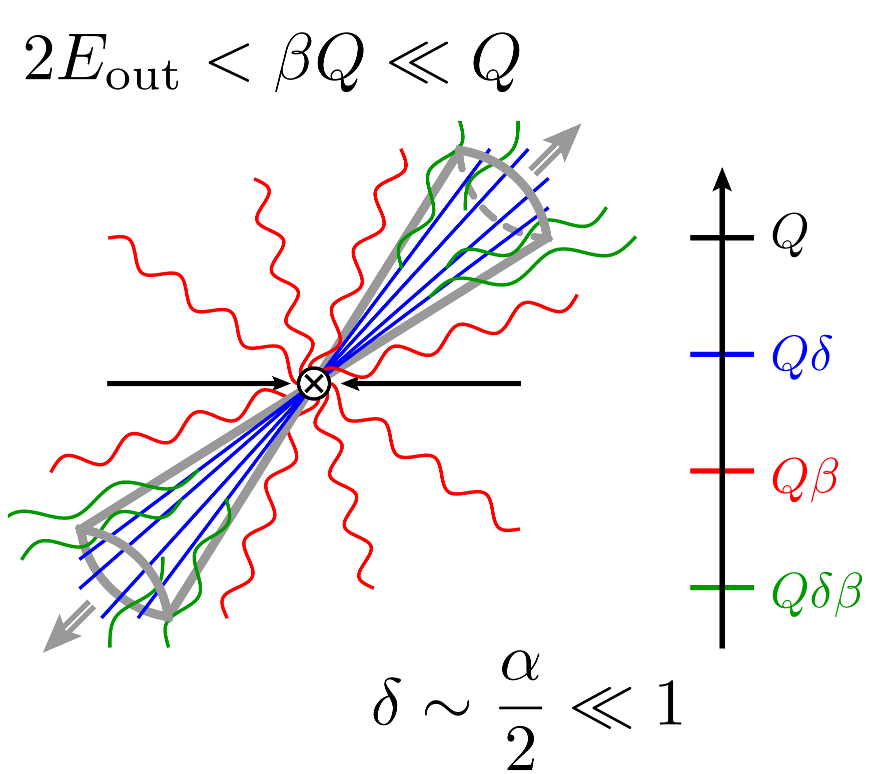
Some recent progress

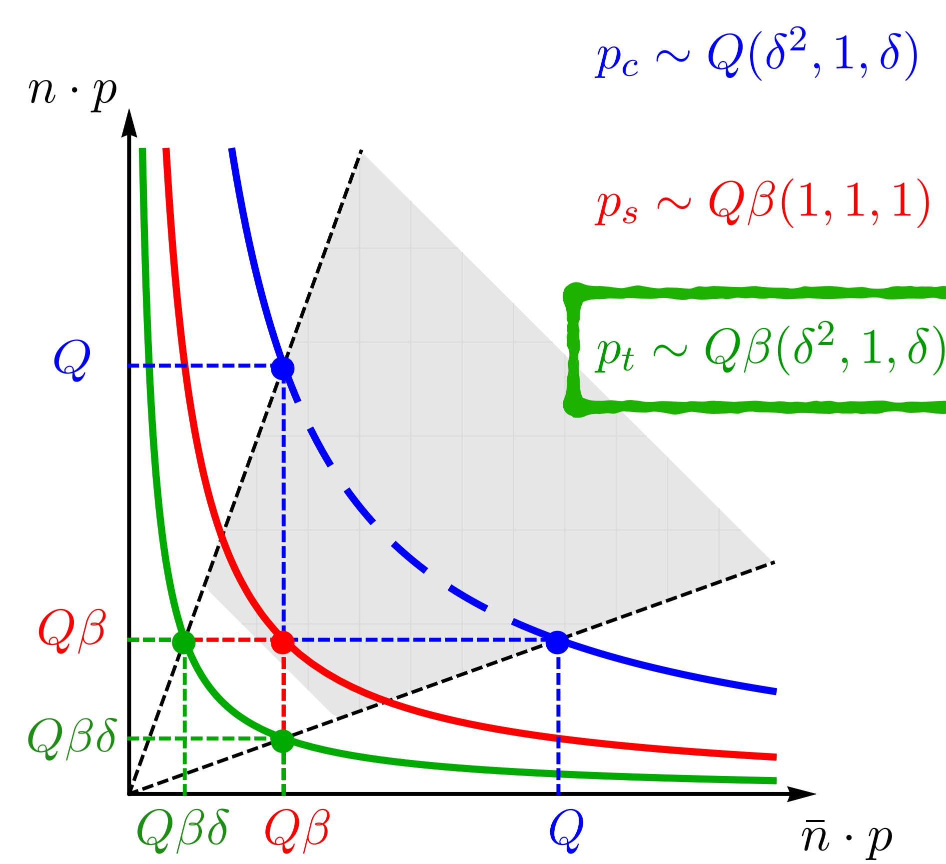
- **Dressed gluon expansion** Larkoski, Moult & Neill ’15 ’16
- **Multi-Wilson-line structure in SCET** Becher, Neubert, Rothen & DYS ’15 ’16
- **Color density matrix** Caron-Huot ’15
- **Collinear logs improved BMS eq** Hatta, Iancu, Mueller, & Triantafyllopoulos ’17
- **Soft (Glauber) gluon evolution at amplitude level, finite Nc** Martínez, Angelis, Forshaw, Plätzer & Seymour ’18
- **Reduced density matrix** Neill & Vaidya ‘18

Effective field theory for jet processes

Becher, Neubert, Rothen & DYS '15 PRL

- A new effective field theory which fully factorizes non-global observables.
- Analysing Sterman-Weinberg jet processes in EFT, we find that in addition to soft and collinear fields their description requires degrees of freedom that are simultaneously soft and collinear to the jets.
- These collinear-soft("coft") particles can resolve individual collinear partons, leading to a complicated **multi-Wilson-line structure**

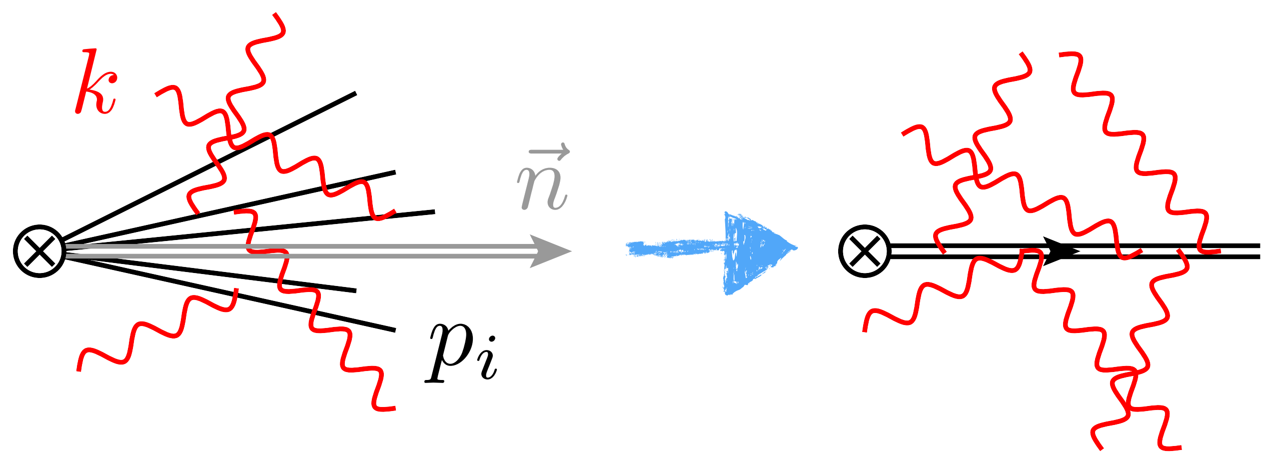




Multi-Wilson-line Structure

Large-angle soft radiation off a jet of collinear particles does not resolve individual energetic patrons

$$\sum_i Q_i \frac{p_i \cdot \epsilon(k)}{p_i \cdot k} \sim Q_{\mathrm{tot}} \frac{n \cdot \epsilon}{n \cdot k}$$



This approximation breaks down for soft radiation collinear to the jet!!!

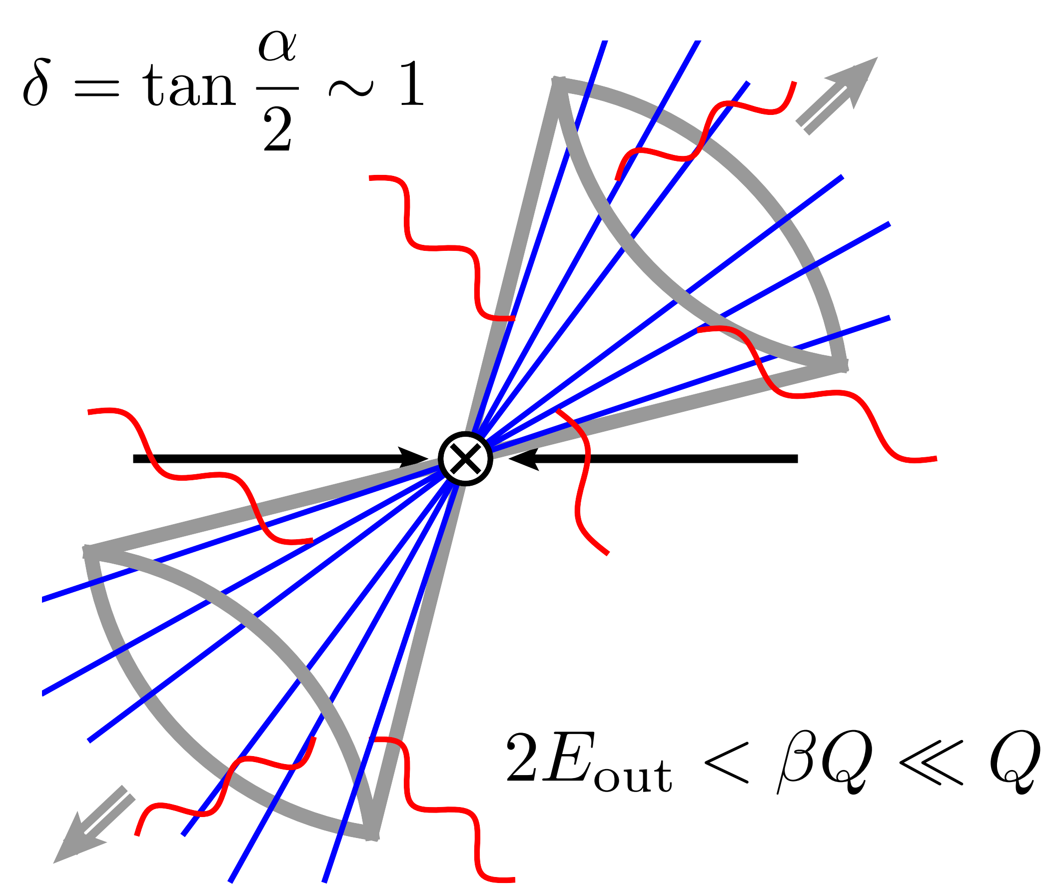
$$k^\mu = \alpha\, n^\mu$$

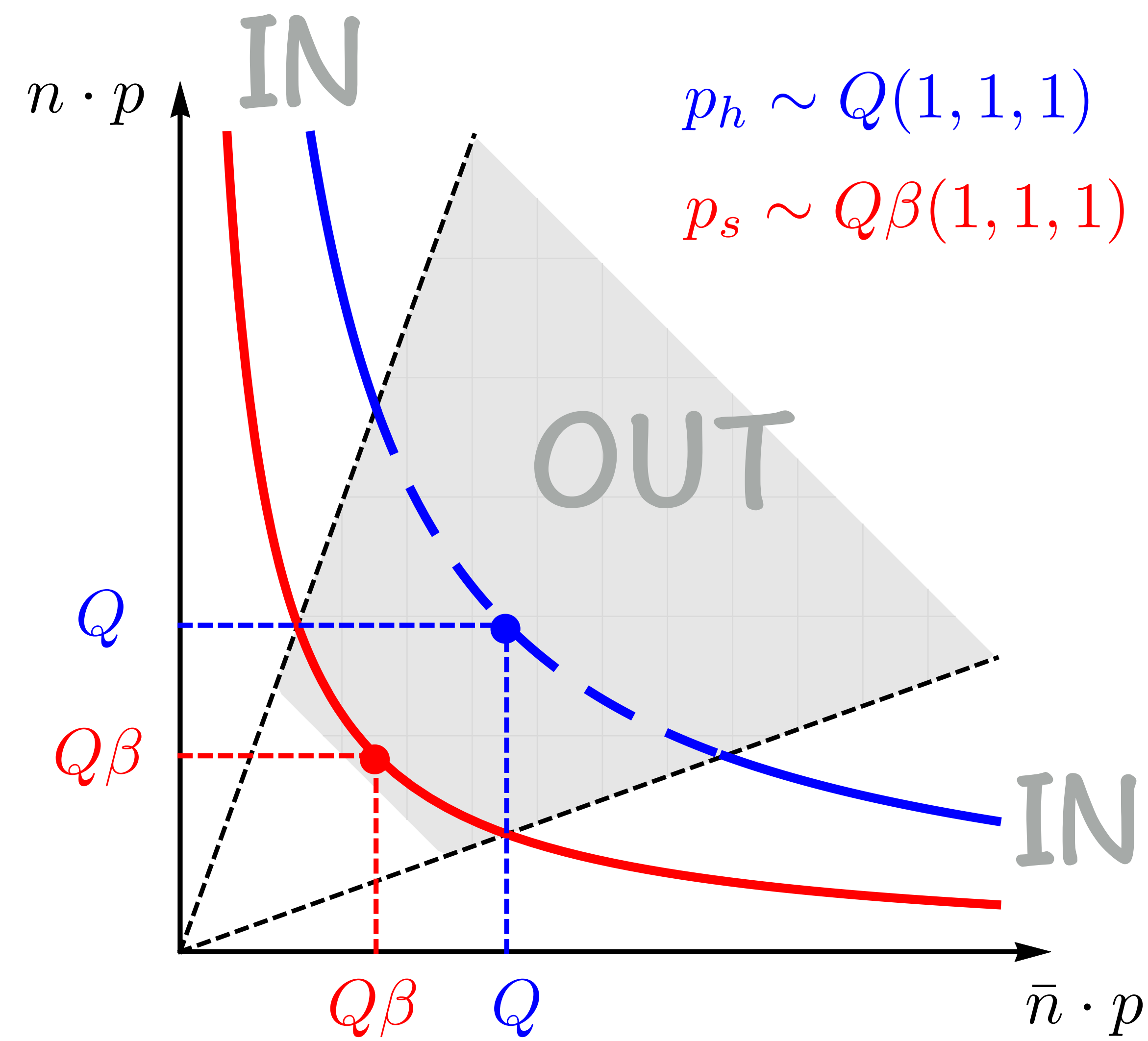
Typically this small region of phase space does not give an $\mathcal{O}(1)$ contribution.

However it does in Non-global observables.

EFT for interjet energy flow

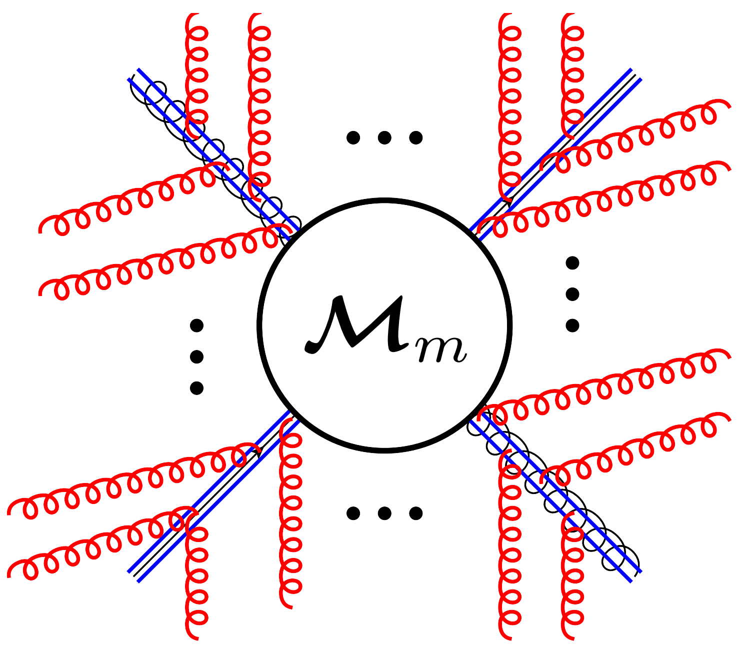
(Becher, Neubert, Rothen & DYS '16; Caron-Huot '15)





Factorization

- The operator for the emission from an amplitude with m hard partons



hard scattering amplitude with m particles (vector in color space)

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\,\ldots\,\boldsymbol{S}_m(n_m)\,|\mathcal{M}_m(\{\underline{p}\})\rangle$$

soft Wilson lines along the directions of the energetic particles (color matrices)

$$\boldsymbol{S}_i(n_i) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n_i\cdot A_s^a(sn_i)\,\boldsymbol{T}_i^a\right)$$

Factorization

- Then the cross section can be written in factorized form as,

$$\sigma(\beta,\delta) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\delta) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q\beta,\delta) \big\rangle$$

- Soft function:

$$\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q\beta,\delta) = \sum\!\!\!\!\!\!\!\int_{X_s} \langle 0|\, \boldsymbol{S}_1^\dagger(n_1)\ldots \boldsymbol{S}_m^\dagger(n_m)\, |X_s\rangle\langle X_s|\, \boldsymbol{S}_1(n_1)\ldots \boldsymbol{S}_m(n_m)\, |0\rangle\, \theta(Q\beta - 2E_{\text{out}})$$

- Hard function: integrating over the energies of the hard particles, while keeping their direction fixed

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\delta) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^{m} \int \frac{dE_i\, E_i^{d-3}}{(2\pi)^{d-2}}\, |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|\, (2\pi)^d\, \delta\Big(Q - \sum_{i=1}^{m} E_i\Big)\, \delta^{(d-1)}(\vec{p}_{\text{tot}})\, \Theta_{\text{in}}^{n\bar{n}}(\{\underline{p}\})$$

- $\otimes$ indicates integration over the direction of the energetic partons

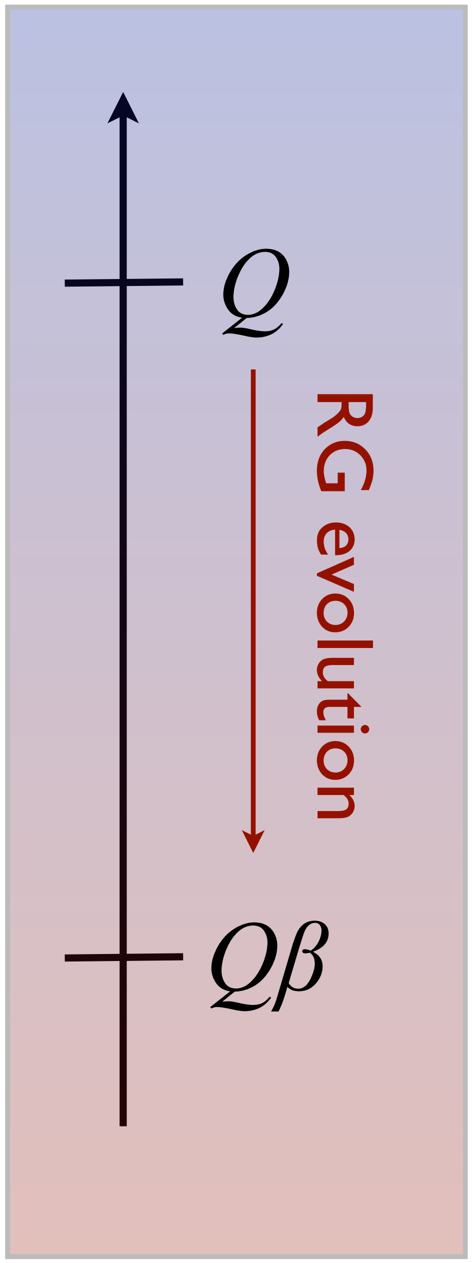
$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\delta) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q\beta,\delta) = \prod_{i=2}^{m} \int \frac{d\Omega(n_i)}{4\pi}\, \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\delta)\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q\beta,\delta)$$

Resummation

$$\sigma(\beta,\delta) = \sum_{l=2}^{\infty} \Big\langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}\}, Q, \delta, \mu_h) \otimes \sum_{m\geq l} \boldsymbol{U}^S_{lm}(\{\underline{n}\}, \delta, \mu_s, \mu_h) \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \delta, \mu_s) \Big\rangle$$

Infinite operators are mixed under RG evolution

- Analytical method fails
- RG evolution = parton shower



e.g. LL evolution

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = \beta Q) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\sigma_{\rm LL}(\delta,\beta) = \sigma_0 \left\langle \boldsymbol{\mathcal{S}}_2(\{n,\bar{n}\}, Q\beta, \delta, \mu_h) \right\rangle = \sigma_0 \sum_{m=2}^{\infty} \left\langle \boldsymbol{U}^S_{2m}(\{\underline{n}\}, \delta, \mu_s, \mu_h) \,\hat{\otimes}\, \boldsymbol{1} \right\rangle$$

Anomalous dimension

One-loop anomalies dimension

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

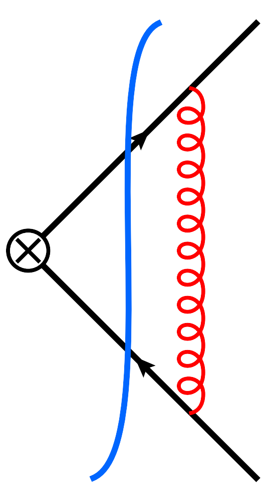
$$\boldsymbol{V}_m = 2\sum_{(ij)} (\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_l)}{4\pi} W_{ij}^l$$
$$- 2\,i\pi \sum_{(ij)} (\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R})\,\Pi_{ij},$$
$$\boldsymbol{R}_m = -4\sum_{(ij)} \boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R}\, W_{ij}^{m+1}\,\Theta_{\text{in}}(n_{m+1})\,.$$

$$W_{ij}^l = \frac{n_i\cdot n_j}{n_i\cdot n_l\, n_j\cdot n_l}$$

- We need to regularize collinear divergences

$$W_{ij}^l \to W_{ij}^l\,\theta(n_l\cdot n_i - \lambda^2)\,\theta(n_l\cdot n_j - \lambda^2)$$

- Imaginary part of V_m from cutting two eikonal propagators (see latter)



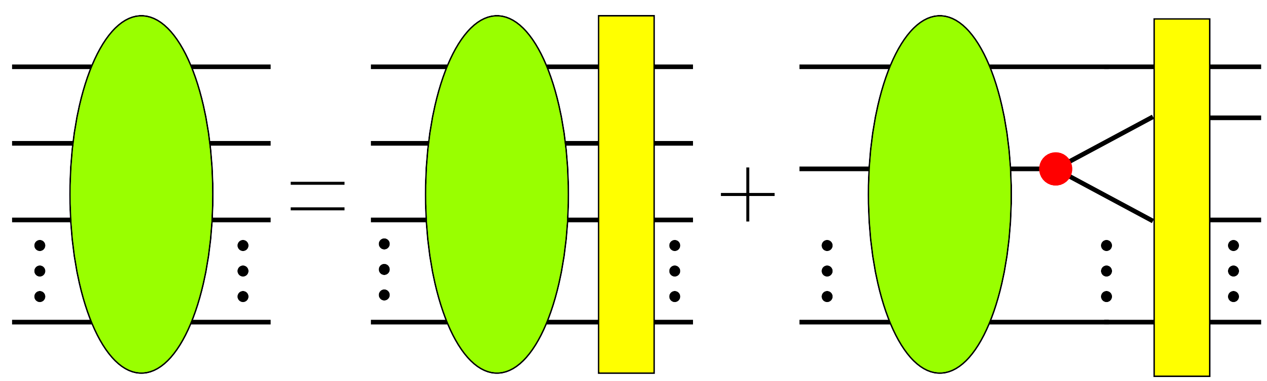
- Two-loop results were calculated by Caron-Huot '15

LL Resummation

RG equation for hard function: $\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\,\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\,\boldsymbol{R}_{m-1}$

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_0)\,e^{(t-t_0)\boldsymbol{V}_m} + \int_{t_0}^{t} dt'\,\boldsymbol{\mathcal{H}}_{m-1}(t')\,\boldsymbol{R}_{m-1}\,e^{(t-t')\boldsymbol{V}_m}$$

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{4\pi}$$



$$\boxed{\sigma_{\rm LL}(\beta,\delta) = \Big\langle \boldsymbol{\mathcal{H}}_2(t) + \int \frac{d\Omega_1}{4\pi}\boldsymbol{\mathcal{H}}_{2+1}(t) + \int \frac{d\Omega_1}{4\pi}\int \frac{d\Omega_2}{4\pi}\boldsymbol{\mathcal{H}}_{2+2}(t) + \dots \Big\rangle}$$

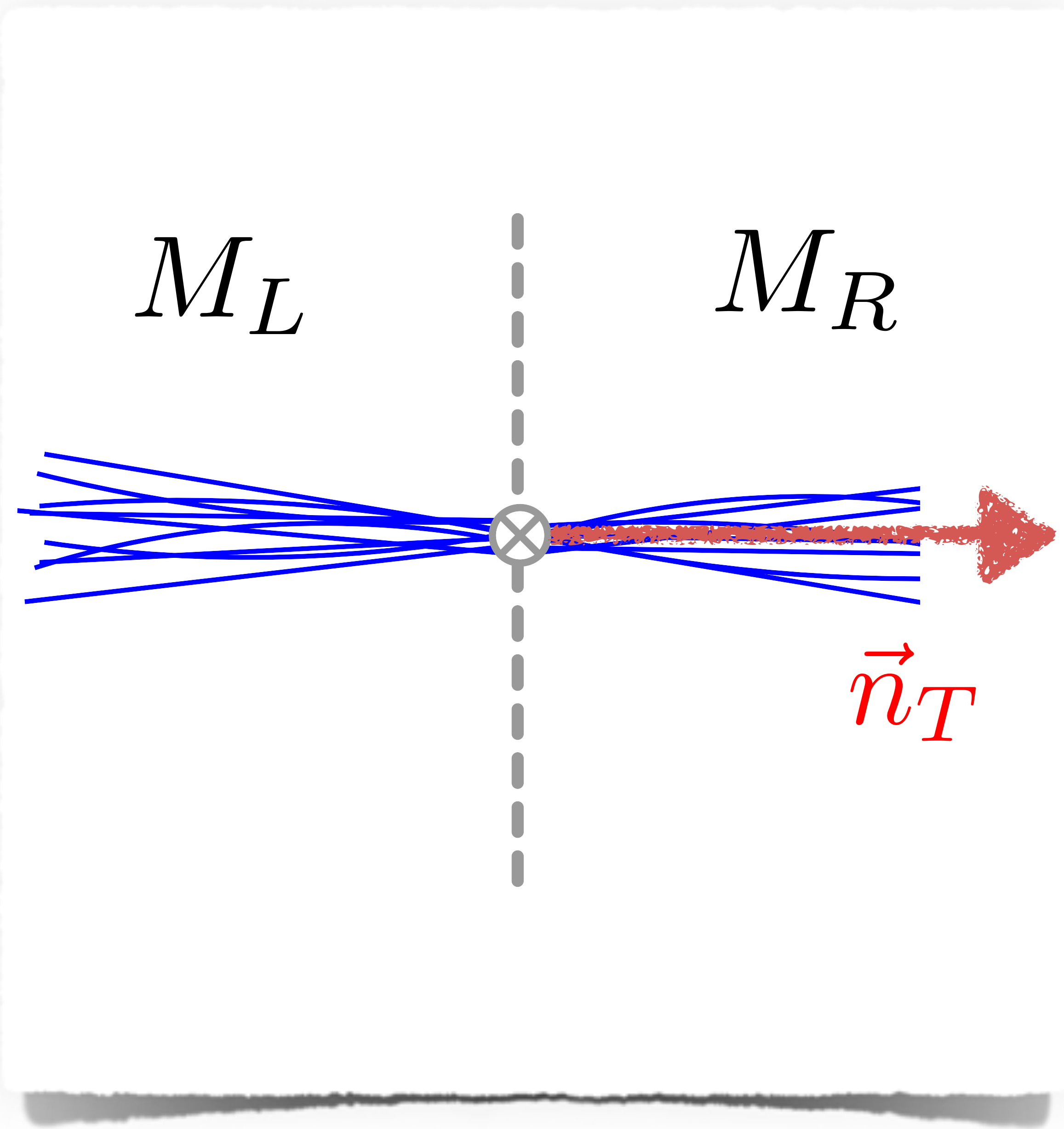
In large Nc limit: $\boldsymbol{T}_i\cdot\boldsymbol{T}_j \to -\frac{N_c}{2}\delta_{i,j\pm1}\,\mathbf{1}$

We re-derive Dasgupta-Salam shower!!!

SCET$_{\mathrm{I}}$: the light-jet mass

Becher, Pacjek & DYS, (JHEP12(2016)018)

Hemisphere mass observables



Heavy-jet mass:

$$\rho_h = \frac{1}{Q^2}\max(M_L^2, M_R^2)$$

Light-jet mass:

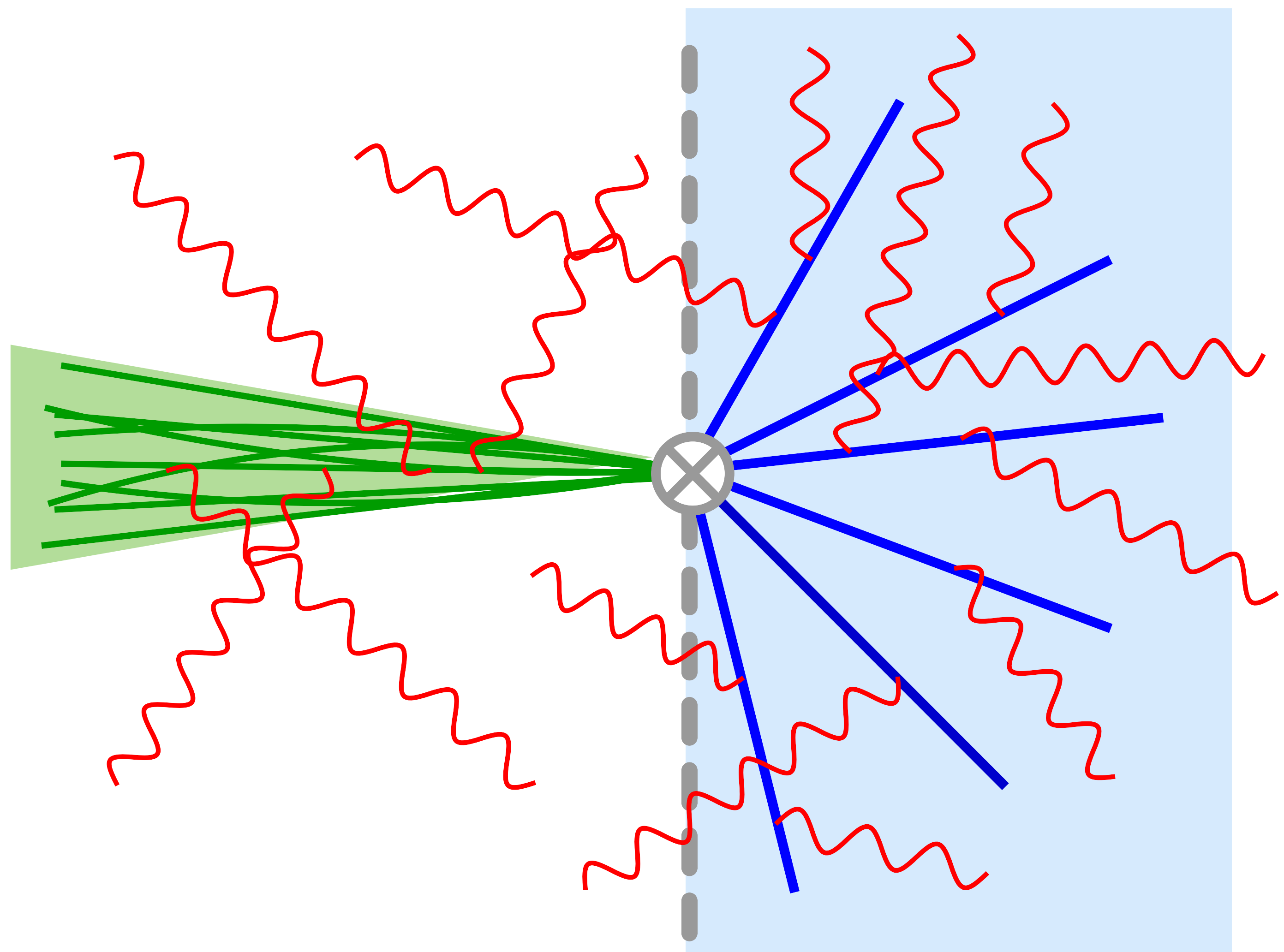
$$\rho_\ell = \frac{1}{Q^2}\min(M_L^2, M_R^2)$$

Heavy-jet mass v.s. Light-jet mass

- Heavy-jet mass: global observable, N^{3}LL accuracy (Chien & Schwartz '13)

- Light-jet mass: non-global observable, NLL accuracy
  - NLL global logs (coherent branching formalism) (Burby & Glover '01)
  - LL non-global logs (Dasgupta & Salam '02)
  - Two-loop hemisphere soft function (Hoang & Kluth '08; Kelley, Schwartz, Schabinger & Zhu '11; Horning, Lee, Stewart, Walsh & Zuberi '11)

Left jet mass

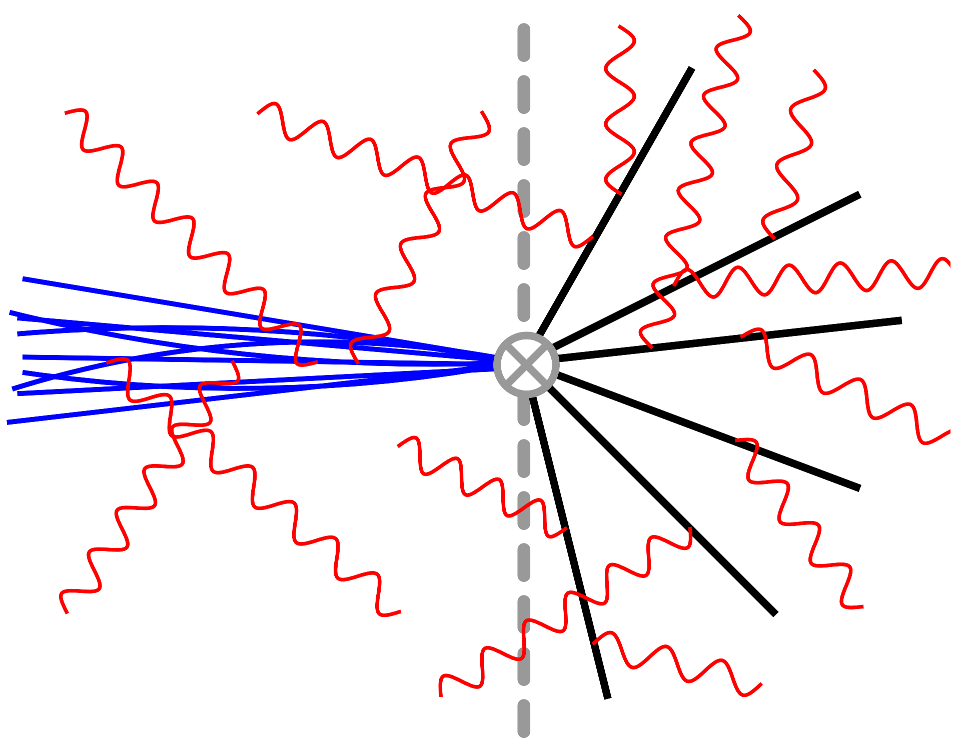


- Right ra
- Radiatio   jet function
- Relevant momentum regions
  - collinear (left hemisphere)
  - hard (right hemishpere)
  - soft: resolve hard partons on the right: multi-Wilson-line operator

Factorization theorem for left-jet mass

$$\frac{d\sigma}{dM_L^2} = \sum_{i=q,\bar{q},g} \int_0^\infty d\omega_L \, J_i(M_L^2 - Q\,\omega_L) \sum_{m=1}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^i(\{\underline{n}\}, Q) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, \omega_L) \rangle$$

M_L << M_R~Q



1. Inclusive jet function:
2. Hard function:
   m hard partons in the right hemisphere, a single parton with flavour i in the left one;
3. Soft function:
   m+1 Wilson lines

Light-jet mass

"Left" and "right" are arbitrary assignments and experiments prefer to measure light-jet mass

$$\rho_\ell = \frac{1}{Q^2}\min(M_L^2, M_R^2)$$

Related to left-jet mass by

$$\frac{d\sigma}{d\rho_\ell} = 2\frac{d\sigma}{d\rho_L} - \frac{d\sigma}{d\rho_h}\bigg|_{\rho_L=\rho_h=\rho_\ell}$$

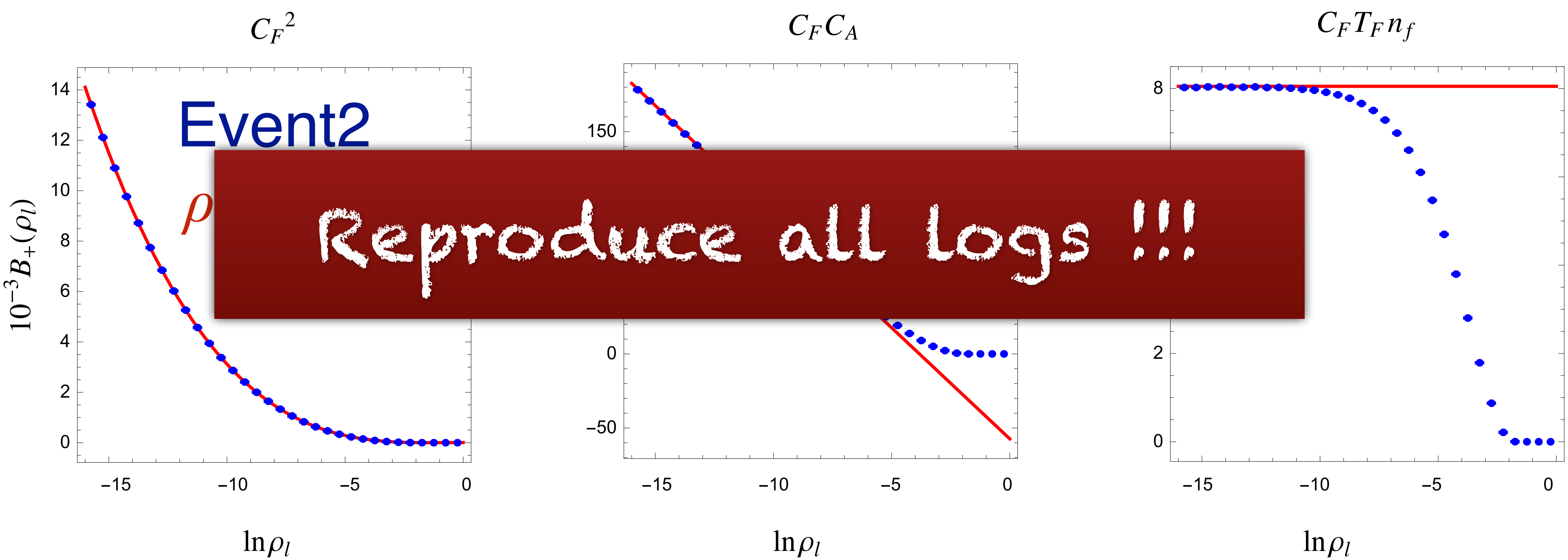
Fixed-order expansion has the form

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\rho_\ell} = \delta(\rho_l)\left\{1 + \left(\frac{\alpha_s}{2\pi}\right)\frac{3C_F}{2} + \left(\frac{\alpha_s}{2\pi}\right)^2 B_\delta\right\} + \left(\frac{\alpha_s}{2\pi}\right)^2\left[\frac{B_+(\rho_l)}{\rho_l}\right]_+ + \cdots$$

Two-loop coefficient

$$
\begin{aligned}
B_+(\rho) = C_F^2 \Bigg[& -4\ln^3\rho - 9\ln^2\rho + \left[-\frac{59}{6} + \frac{4\,\pi^2}{3} + 4\ln^2 2 - \frac{5\ln 3}{2} + 8\,\mathrm{Li}_2\left(-\frac{1}{2}\right)\right]\ln\rho \\
& + \frac{15}{2} + 2\pi^2 + \frac{809\zeta_3}{6} + \frac{88\ln^3 2}{3} + 8\ln 2\ln^2 3 + \frac{5\ln^2 3}{2} - 24\ln^2 2\ln 3 + \frac{27\ln^2 2}{2} \\
& - 28\ln 2\ln 3 + \frac{487\ln 3}{24} - \frac{20}{3}\pi^2\ln 2 - \frac{88\ln 2}{3} + 43\,\mathrm{Li}_2\left(-\frac{1}{2}\right) - 16\,\mathrm{Li}_2\left(-\frac{1}{2}\right)\ln 3 \\
& + 96\,\mathrm{Li}_2\left(-\frac{1}{2}\right)\ln 2 - 8\,\mathrm{Li}_3\left(\frac{3}{4}\right) + 176\,\mathrm{Li}_3\left(-\frac{1}{2}\right) - 8\,I_2 \Bigg] \\
+ C_F C_A \Bigg[& \left[\frac{1}{3} - 2\pi^2 - 4\ln^2 2 + \frac{5\ln 3}{2} - 8\,\mathrm{Li}_2\left(-\frac{1}{2}\right)\right]\ln\rho - \frac{407}{72} - \frac{13\pi^2}{18} - \frac{389\zeta_3}{3} - \frac{8\ln^3 3}{3} \\
& - 52\ln^3 2 - 12\ln 2\ln^2 3 - \frac{15\ln^2 3}{4} + 52\ln^2 2\ln 3 + \frac{43\ln^2 2}{12} - \frac{11}{2}\ln 2\ln 3 \\
& - \frac{917\ln 3}{24} + 6\pi^2\ln 2 + \frac{212\ln 2}{3} + 20\,\mathrm{Li}_3\left(\frac{3}{4}\right) + \frac{235}{6}\mathrm{Li}_2\left(-\frac{1}{2}\right) \\
& + 24\,\mathrm{Li}_2\left(-\frac{1}{2}\right)\ln 3 - 88\,\mathrm{Li}_2\left(-\frac{1}{2}\right)\ln 2 + 16\,\mathrm{Li}_3\left(\frac{1}{3}\right) - 112\,\mathrm{Li}_3\left(-\frac{1}{2}\right) - 8\,I_1 \Bigg] \\
+ C_F T_F n_f \Bigg[& -\frac{13}{9} + \frac{10\,\pi^2}{9} + \frac{4}{3}\ln^2 2 - \frac{5}{6}\ln 3 + \frac{8}{3}\mathrm{Li}_2\left(-\frac{1}{2}\right)\Bigg].
\end{aligned}
$$

Numerical check



NLL resummation

Up to NLL, the factorization theorem simplifies

$$\frac{d\sigma}{dM_L^2} = \sigma_0 \int_0^\infty d\omega_L \, J_q(M_L^2 - Q\,\omega_L, \mu_h) \, \langle \boldsymbol{\mathcal{S}}_1(\{n\}, \omega_L, \mu_h) \rangle$$

with factorisation scale is chosen as $\mu = \mu_h \sim Q$

RG equation for the soft function:

$$\frac{d}{d\ln\mu} \widetilde{\boldsymbol{\mathcal{S}}}_l(\{\underline{n}\}, \tau, \mu) = \sum_{m=l}^{\infty} \boldsymbol{\Gamma}^S_{lm}(\{\underline{n}\}, \tau, \mu) \,\hat{\otimes}\, \widetilde{\boldsymbol{\mathcal{S}}}_m(\{\underline{n}\}, \tau, \mu)$$

Matrix equation but Sudakov piece is diagonal

$$\boldsymbol{\Gamma}^S_{lm}(\{\underline{n}\}, \tau, \mu) = 2\,\Gamma_{\text{cusp}} \ln\left(\frac{\tau}{\mu}\right) \delta_{lm} + \hat{\boldsymbol{\Gamma}}_{lm}(\{\underline{n}\})$$

NLL resummation

Sudakov piece: resummed by standard method

$$\Sigma_q(\rho_L) = \exp\left[2S(\mu_s,\mu_h) - 4S(\mu_j,\mu_h) + 2A_{\gamma^J}(\mu_j,\mu_h)\right] \frac{e^{-\gamma_E \eta}}{\Gamma(\eta+1)} \left(\frac{Q^2\rho_L}{\mu_j^2}\right)^{\eta} \left(\frac{Q\mu_s}{\mu_j^2}\right)^{-\eta_S}$$

$\Sigma_q(\rho_L)$: Jet function in the coherent branching formalism

(Catani & Trentadue 1989)

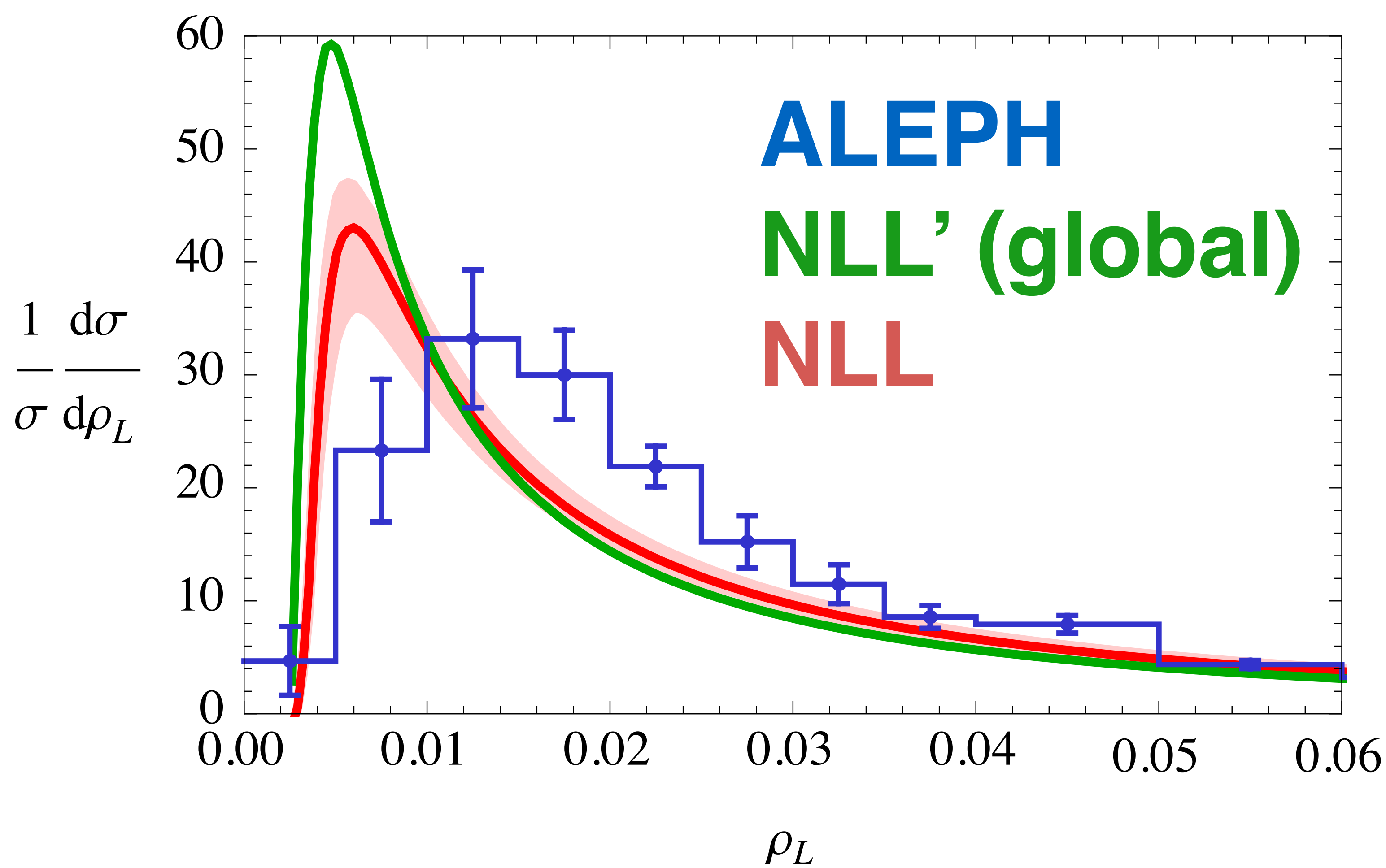
Non-global piece:

$$S_{NG}(\mu_s,\mu_h) = \sum_{m=1}^{\infty} \langle \boldsymbol{U}^S_{1m}(\{\underline{n}\},\mu_s,\mu_h) \,\hat{\otimes}\, \mathbf{1} \rangle$$

In the large Nc limit, the evolution matrix is equivalent to Dasgupta-Salam shower.

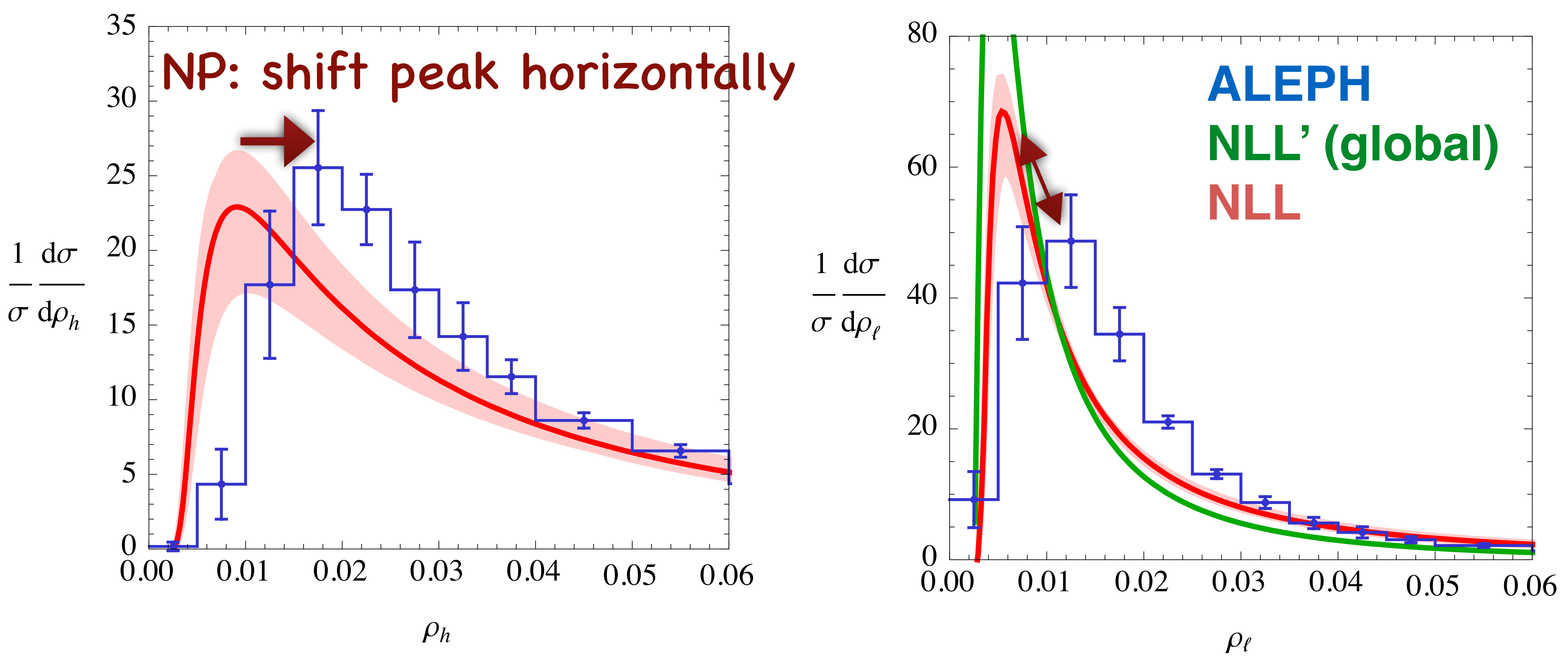
$$S_{\rm NG}(\mu_s,\mu_h) \approx \exp\left(-C_A C_F \frac{\pi^2}{3} u^2 \frac{1+(au)^2}{1+(bu)^c}\right) \qquad u = \frac{1}{\beta_0} \ln \frac{\alpha_s(\mu_s)}{\alpha_s(\mu_h)}$$

Left-jet mass @ NLL



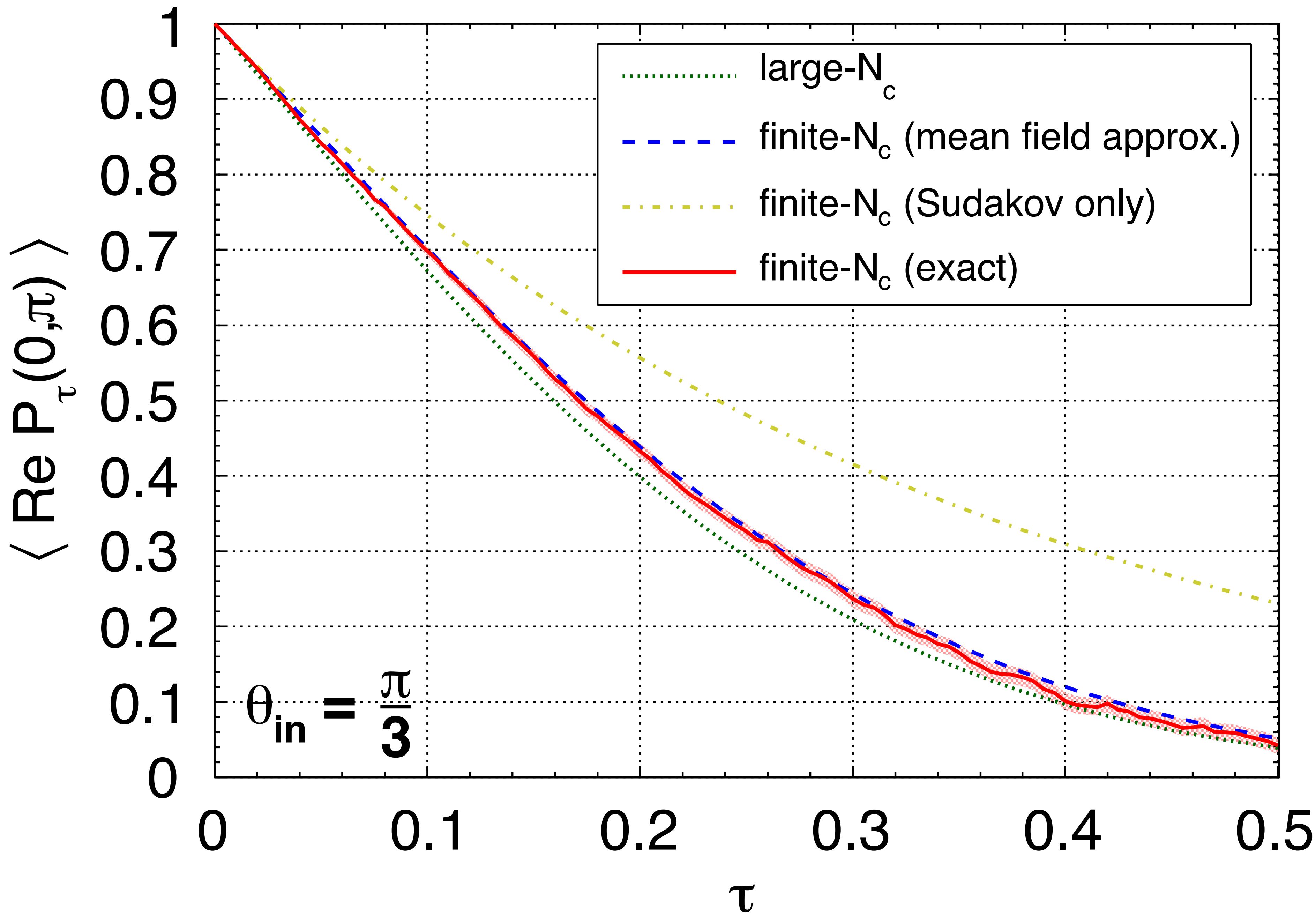
The Non-Global effects are sizeable !!!!

Heavy-jet mass v.s. Light-jet mass



- finite Nc ? N²LL ? Non-perturbative effects ?

Finite Nc effects



Hatta & Ueda, ‘13

SCET$_{\mathrm{II}}$: narrow jet broadening

Becher, Rahn & DYS, (JHEP10(2017)030)

Jet broadening

- Jet broadening probes the transverse momentum of partons inside jet
- In e^+e^- collider, broadening measures the momentum transverse to the thrust axis

$$b_{L(R)} = \frac{1}{2} \sum_{i \in L(R)} |\vec{p}_i^{\perp}| = \frac{1}{2} \sum_{i \in L(R)} |\vec{p}_i \times \vec{n}_T|$$

e.g. total broadening: $b_T = b_L + b_R \,,$

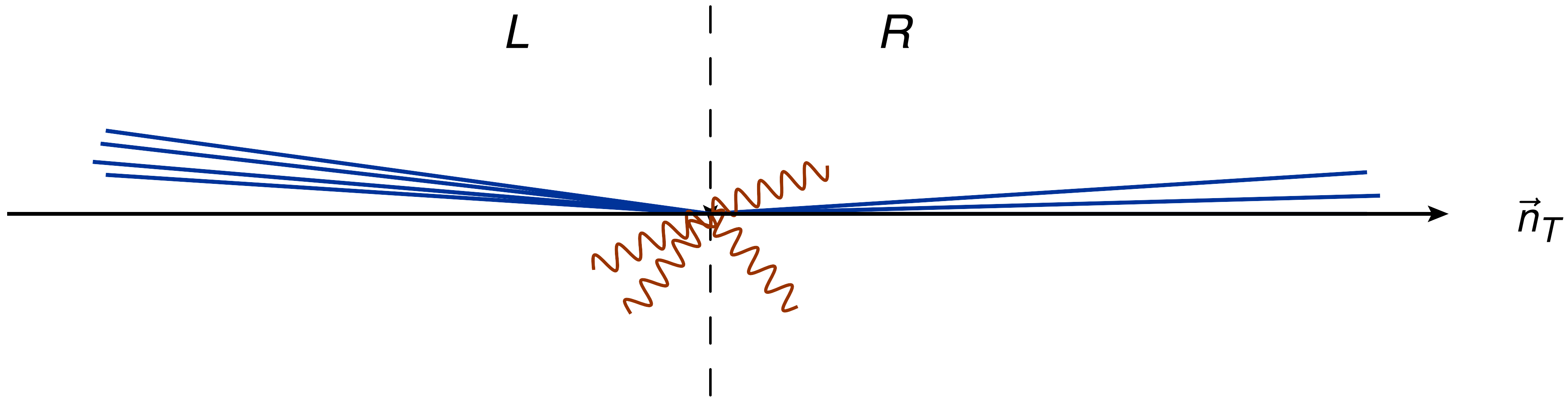
wide broadening: $b_W = \max(b_L, b_R)$

- NLL resummation (Dokshitzer, Lucenti, Marchesini, Salam '98)
- Factorization in SCET (Chiu, Jain, Neill, Rothstein '11, '12; Becher, Bell '11, '12)
- NNLL resummation (Becher, Bell '12; Banfi, McAslan, Monni & Zanderighi '15)

Total(Wide)-broadening

- In the two-jet limit $b_L \sim b_R \ll Q$

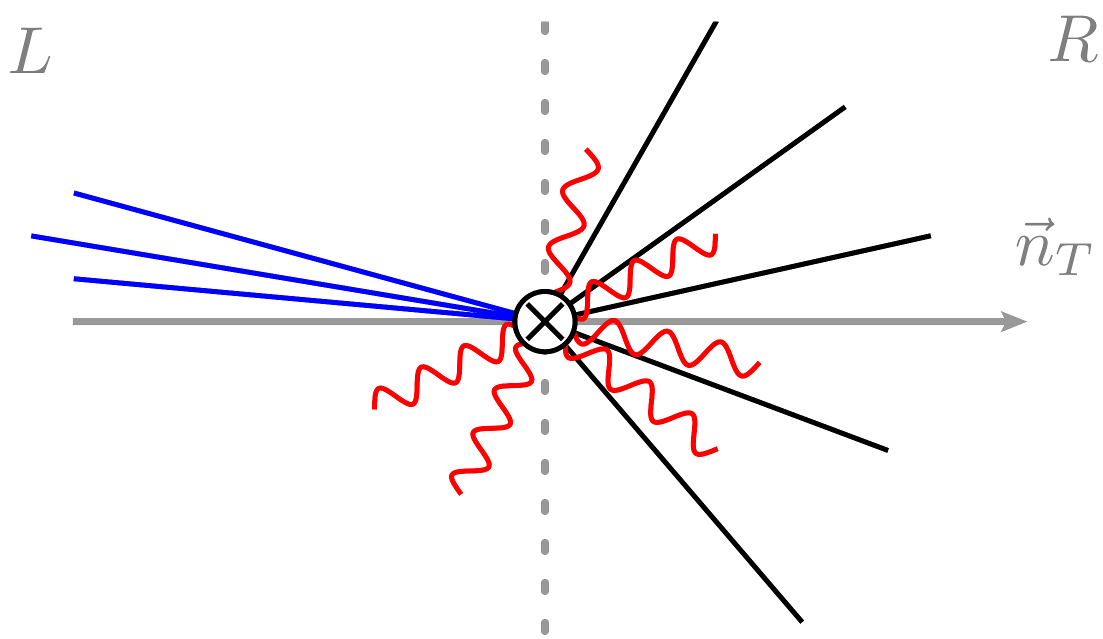
$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_L\,db_R} = H(Q^2,\mu)\int db_L^s \int db_R^s \int d^{d-2}p_L^\perp \int d^{d-2}p_R^\perp$$
$$\times\, \mathcal{J}_L(b_L - b_L^s, p_L^\perp, \mu)\,\mathcal{J}_R(b_R - b_R^s, p_R^\perp, \mu)\,\mathcal{S}(b_L^s, b_R^s, -p_L^\perp, -p_R^\perp, \mu)$$



- relevant low energy modes: $p_c^\perp \sim p_s^\perp \sim b_{L,R}$
- collinear recoils against soft radiation: rapidity logs resummation

Narrow broadening

- Non-global recoil-sensitive(SCET$_{\mathrm{II}}$) observables $b_N = \min(b_L, b_R)$



$$b_L \ll b_R \sim Q$$

- With the rapidity regulator

$$\frac{d\sigma}{db_L} = \sum_{f=q,\bar{q},g} \int db_L^s \int d^{d-2}p_L^\perp \, \mathcal{J}_f(b_L - b_L^s, p_L^\perp) \sum_{m=1}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^f(\{\underline{n}\}, Q) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, b_L^s, -p_L^\perp) \rangle$$

- jet function: same as total broadening
- hard function: same as light-jet mass
- soft function: New

Collinear anomaly

- All-order form of rapidity divergences was derived for total broadening
- Rapidity divergence cancel out between jet and soft function
- Rapidity logs are fully determined by the div. of jet function, the collinear anomaly must be the same as total broadening

$$\frac{d\sigma}{d\tau_L} = \sum_{f=q,\bar{q},g} \int_0^\infty dz_L (Q^2 \tau_L^2)^{-F_B^f(\tau_L, z_L, \mu)} \frac{z_L}{(1+z_L^2)^{3/2}} \sum_{m=1}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^f(\{\underline{n}\}, Q) \otimes \overline{\boldsymbol{\mathcal{W}}}_m^f(\{\underline{n}\}, \tau_L, z_L) \rangle$$

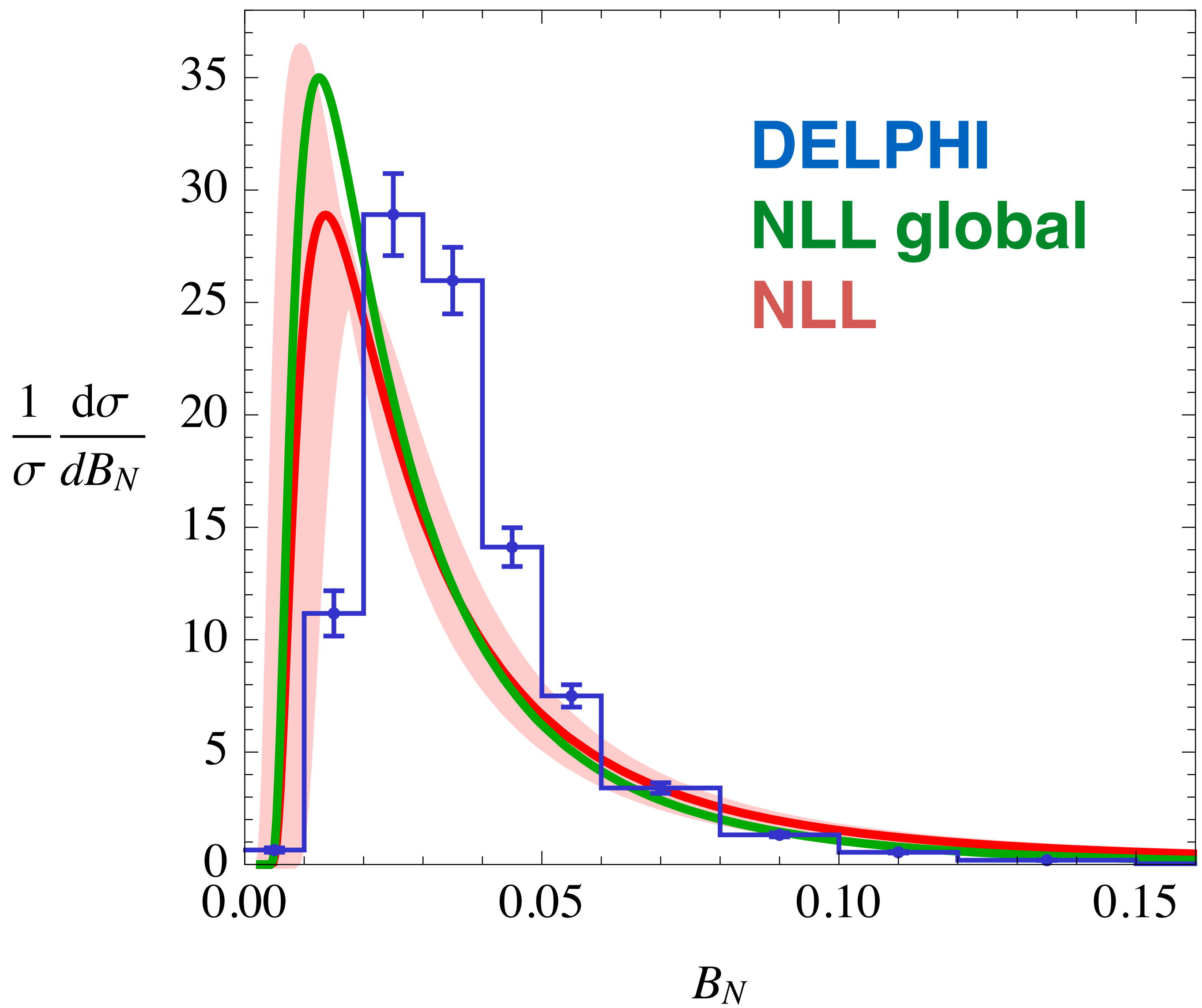
combination of jet and soft function

Rapidity logs are resummed by collinear anomaly factor or rapidity-RG(Chiu, Jain, Neill, Rothstein 11, 12)

Some manipulations:

- Laplace transform $b_L \to \tau_L$
- Fourier transform $p_L^\perp \to x_L^\perp$
- dimensionless $z_L = \dfrac{2|x_L^\perp|}{\tau_L}$

NLL results



- For recoil-sensitive observables dominant non-perturbative are non-perturbative corrections to the anomaly coefficients. (Becher & Bell '13)

$$B_N \to B_N - \frac{\mathcal{A}}{2}\ln\frac{1}{B_N}$$

with $\mathcal{A} \approx 0.3$ GeV extracted from thrust which implies shifts of $\Delta B_N \approx 0.007$ near peak

Collinear limit and NGLs

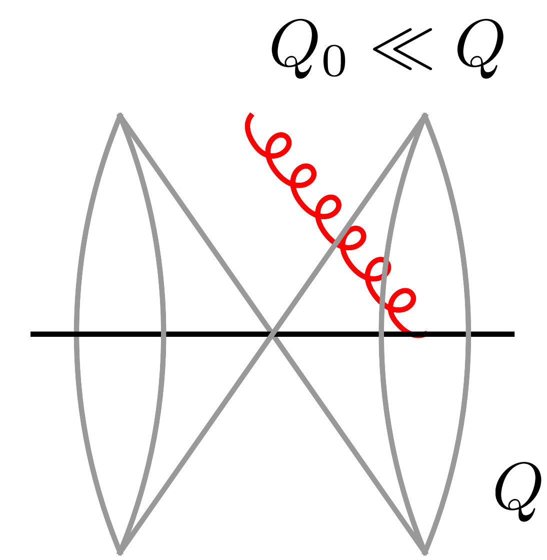
- Interjet energy flow
  - Soft radiations from two Wilson lines (global)

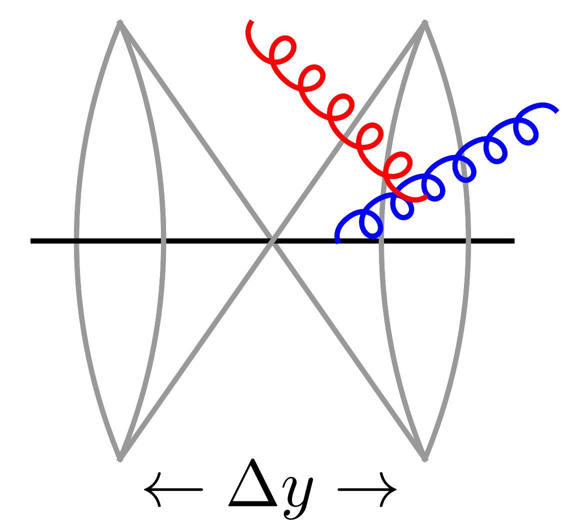
$$\frac{\sigma_{\rm GL}^{\rm LL}}{\sigma_0} = \exp\left[-8\, C_F \Delta y\, t\right] \qquad\qquad t = \int_{\alpha(Q_0)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\frac{\alpha}{4\pi}$$

  - Leading NGLs at two-loops

$$\frac{\sigma_{\rm NGL}^{\rm LL}}{\sigma_0} = 4\, C_F C_A \left[-\frac{2\pi^2}{3} + 4\,{\rm Li}_2\left(e^{-2\Delta y}\right)\right] t^2$$

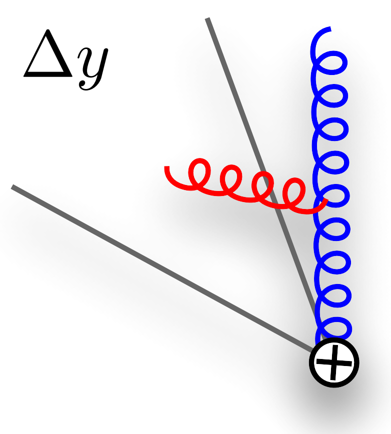
  - Large gap limit: $\Delta y \to \infty$
    - NGL: coft mode, jet radius resummation Becher, Neubert, Rothen & DYS `15; Chien, Hornig, Lee `15
  - Narrow gap limit: $\Delta y \to 0$
    - Collinear enhanced power corrections

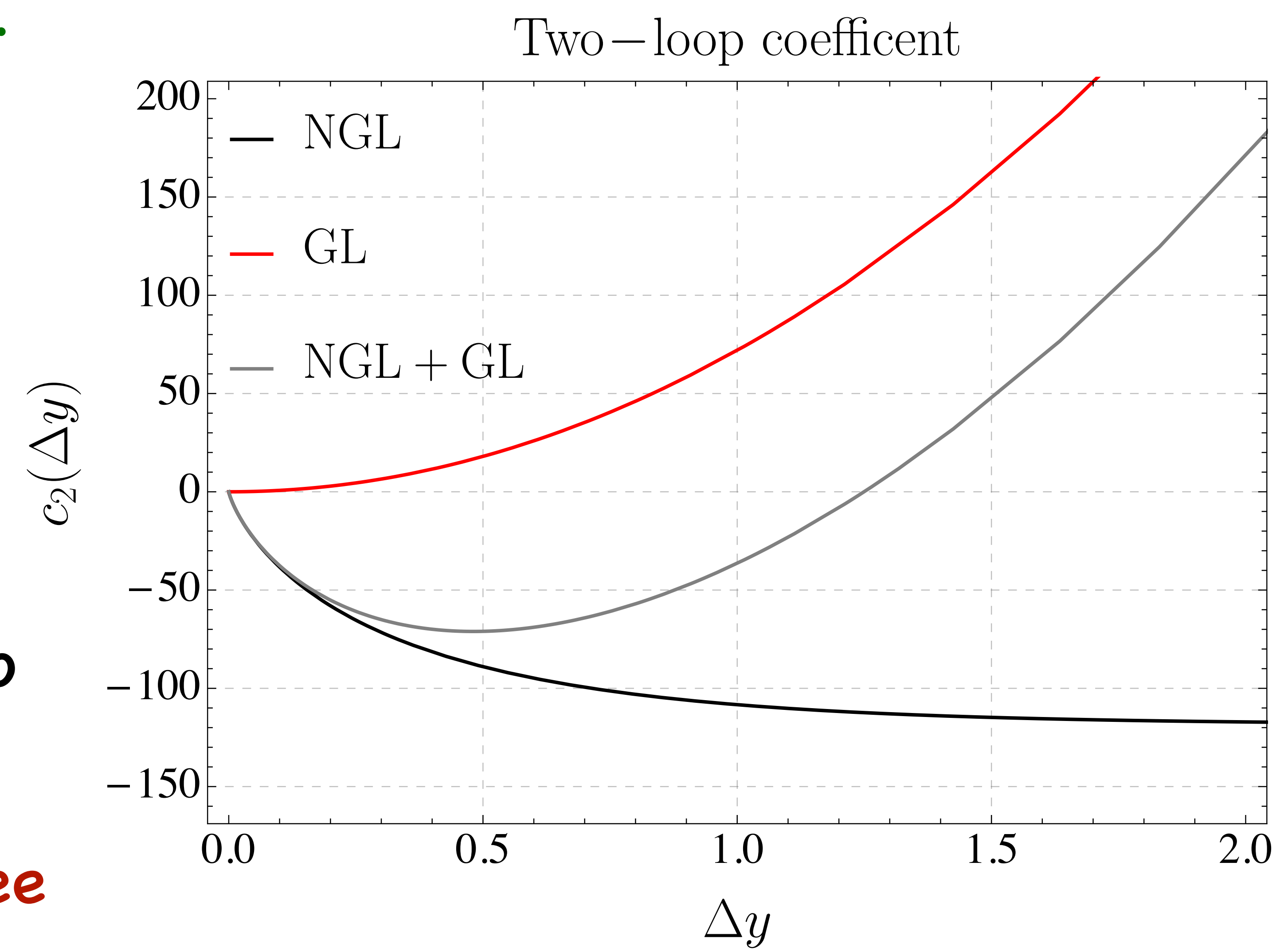




Leading log at two loop

- In narrow gap limit: $$\frac{\sigma^{\mathrm{LL}}_{\mathrm{NGL}}}{\sigma_0} = 4\,C_F C_A\Big[8\,\Delta y\big(\ln(2\Delta y)-1\big) - 4\,\Delta y^2 + \ldots\Big]t^2$$
- Collinear enhancement from boundary region (Hatta, et.al. '17)





- Power correction, interesting to study in SCET framework
- An example: photon isolation (see latter)

Automated resummation for Non-global observables

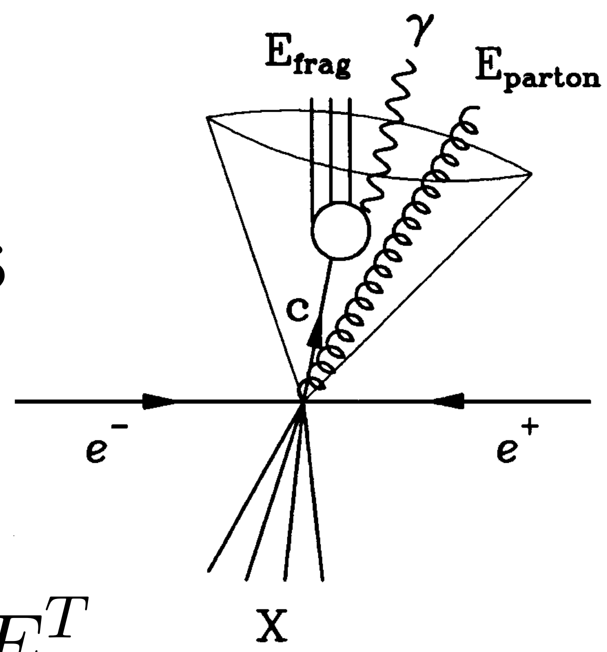
(Balsiger, Becher, DYS, 1803.07045)

$$d\sigma_{\rm LL}(Q,Q_0) = \sum_{m=k}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_k(\{\underline{n}\},Q,\mu_h) \otimes \boldsymbol{U}_{km}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1} \big\rangle$$

- Use Madgraph5_aMC@NLO generator
  - event file with directions and large-N_c color connections of hard partons
  - provides lowest multiplicity hard function for given process
- Run our shower on each event to generate additional partons and write result back into event file
- Analyze events, according to cuts on hard partons, obtain resummed cross section with hard cuts and veto scale

Isolated photon production

- Experiments use isolation cone to reduce photon from hard scattering from photons due to hadron decays such as $\pi^0 \to \gamma\gamma$.



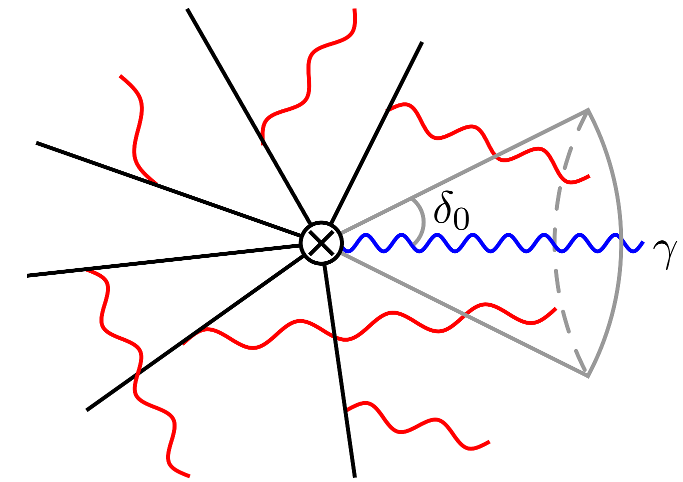
- ATLAS '16 imposes $E^T_{\rm iso} = 4.8\,{\rm GeV} + 0.0042\,E^T_\gamma$ on hadronic energy inside cone.

- Large logs of $\epsilon_\gamma = E^T_\gamma / E^T_{\rm iso}$

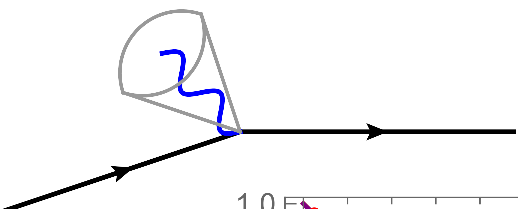
- GLs: $(\alpha_s R^2 \ln \epsilon_\gamma)^n$ NGLs: $R^2 \times \alpha_s^n \ln^n \epsilon_\gamma \ln^{n-1} R$

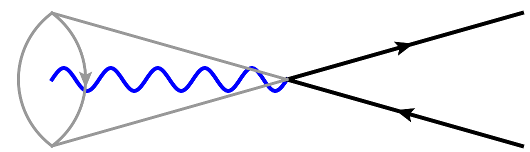
Effect of isolation cut at lepton collider

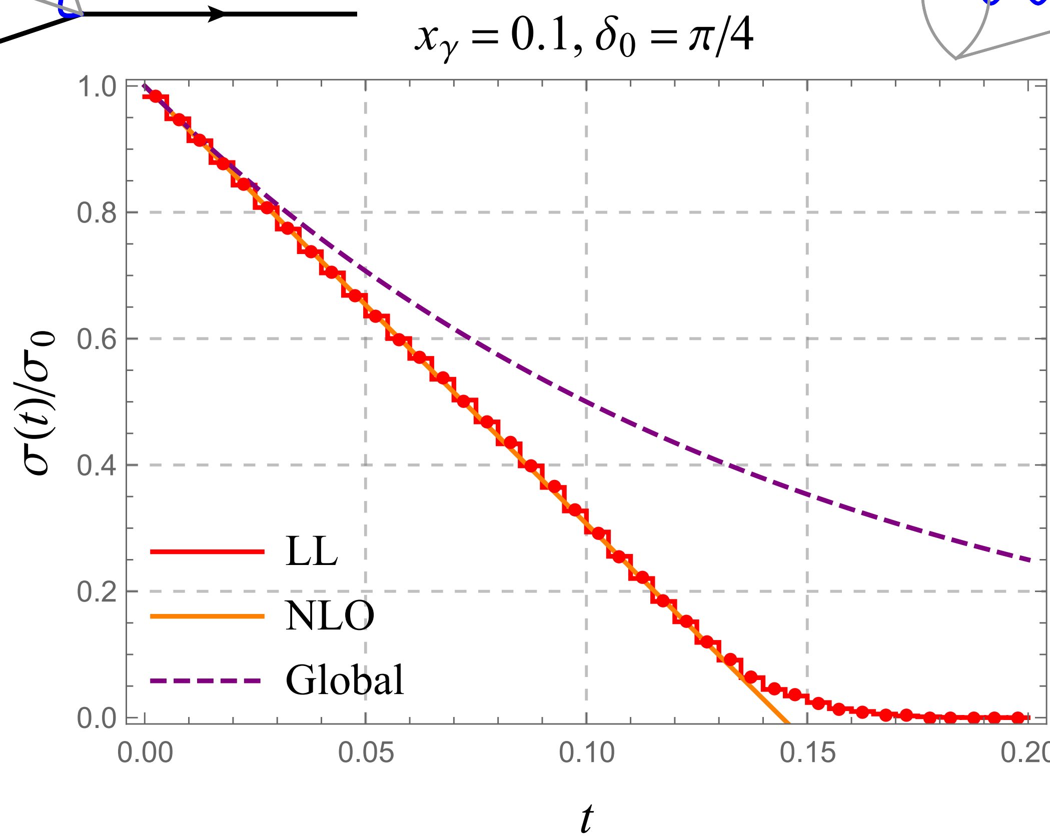
$$\frac{\mathrm{d}\sigma(\epsilon_\gamma,\delta_0)}{\mathrm{d}E_\gamma} = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_{\gamma+m}\left(\{\underline{n}\}, E_\gamma, Q, \delta_0\right) \otimes \boldsymbol{\mathcal{S}}_m\left(\{\underline{n}\}, \epsilon_\gamma\, E_\gamma, \delta_0\right)\rangle$$

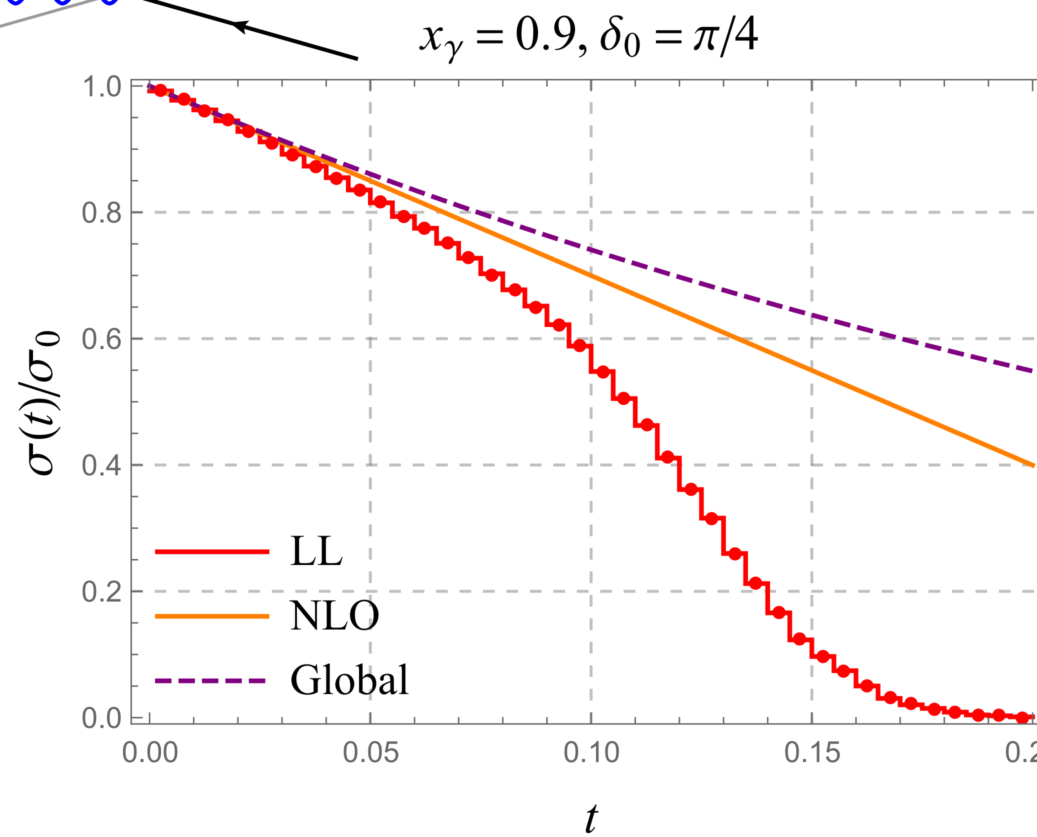


$E_{\rm in} < E_{\rm iso} = \epsilon_\gamma\, E_\gamma$



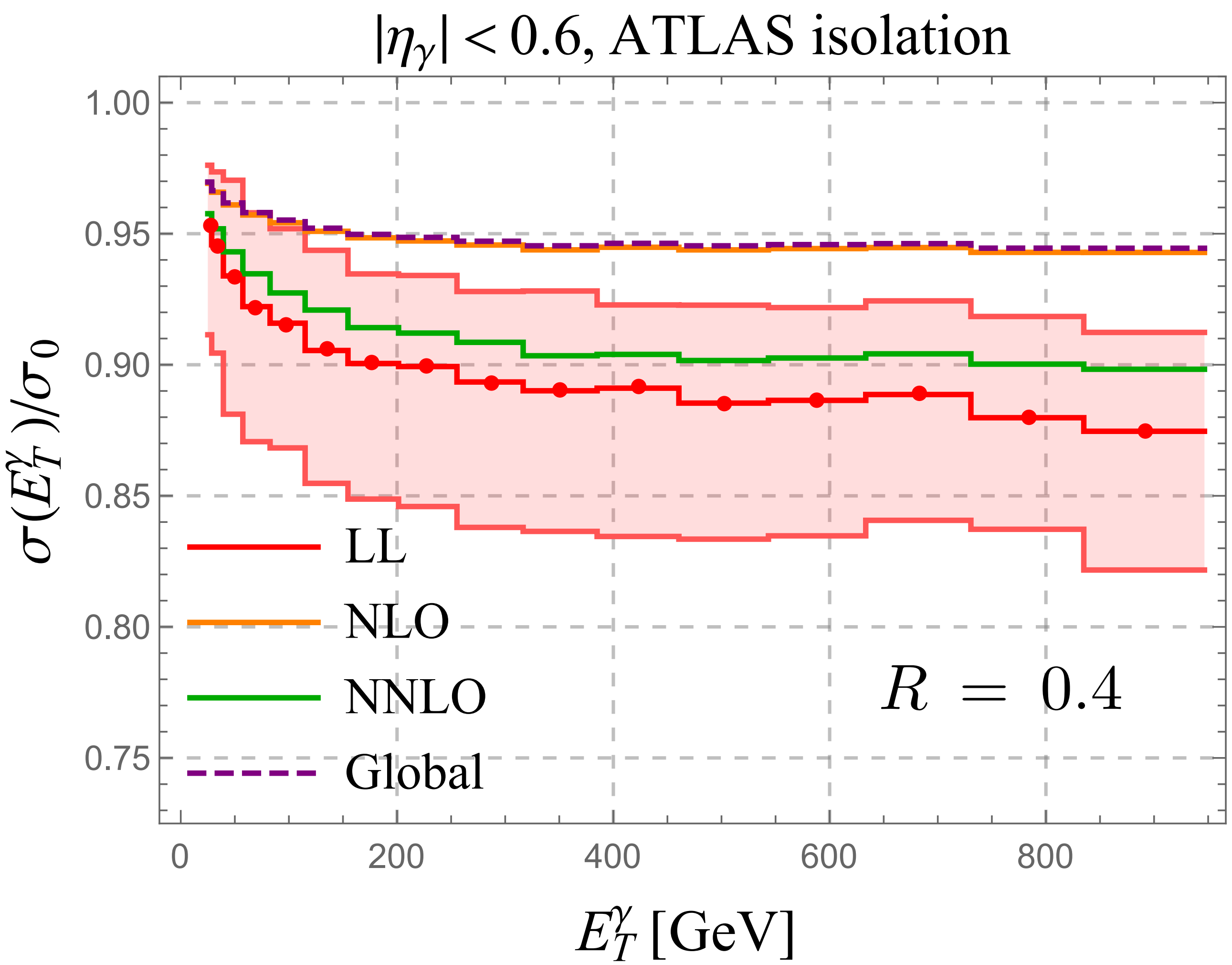






Sizable NGLs corrections

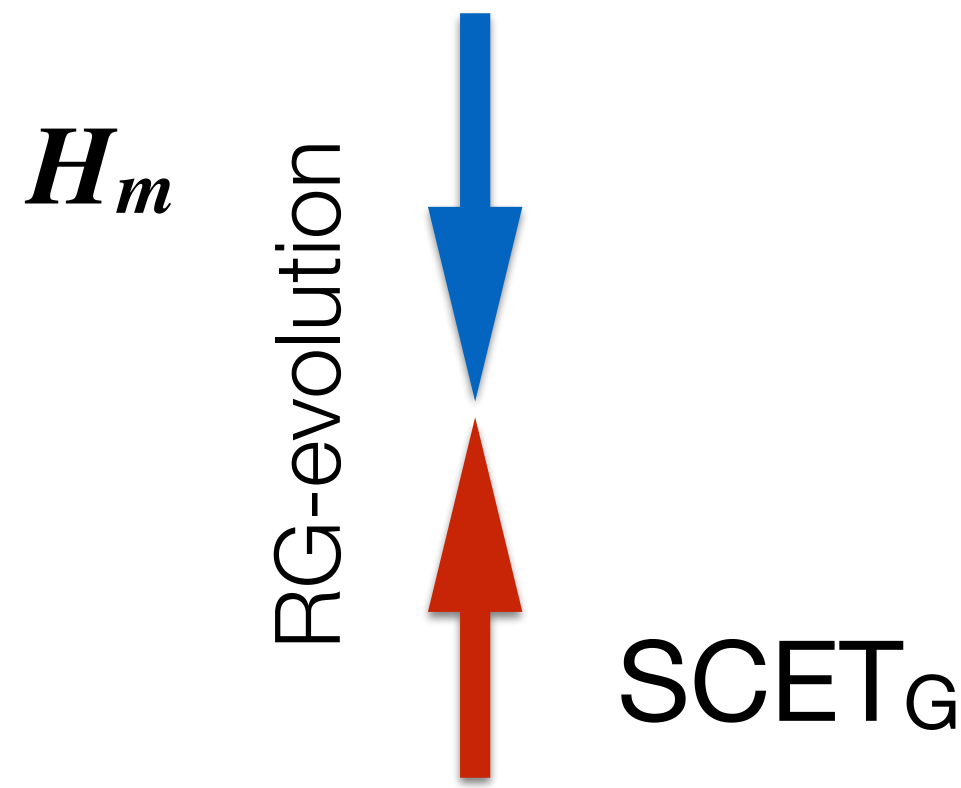
Effects on γ isolation at LHC



- NLO: ~5% reduction, NNLO ~10%, resummed ~ 12%
- NGL dominates over global contribution: naive exponentiation (dashed) not appropriate!

Effective theory of Glauber

The Glauber effects discussed so far are part of the hard anomalous dimension



RG evolution must match up with low-energy theory: SCET + Glauber gluons (Rothstein & Stewart '16)

Conclusion and outlook

- For non-global observables, we obtained a parton shower from effective field theory
  - first-principles derivation of shower, based on RG evolution
  - flexible implementation of shower using MG5_aMC@NLO
  - To resum NLLs, one should include higher-order corrections to the anomalous dimension matrix and matching coefficients
  - when the veto region is small, NGLs are enhanced due to dependence on the size of the veto region
- (Finite N_c) + Glauber + non-global = super-leading log
  - interesting to understand in EFT framework

Thank you