Precision p_T Spectrum of the Higgs Boson

by Iain Stewart

arXiv: 1805.00736

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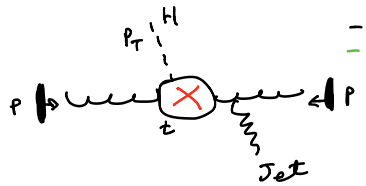
NNLO + N^{3}LL = NNLO Jet + SCET



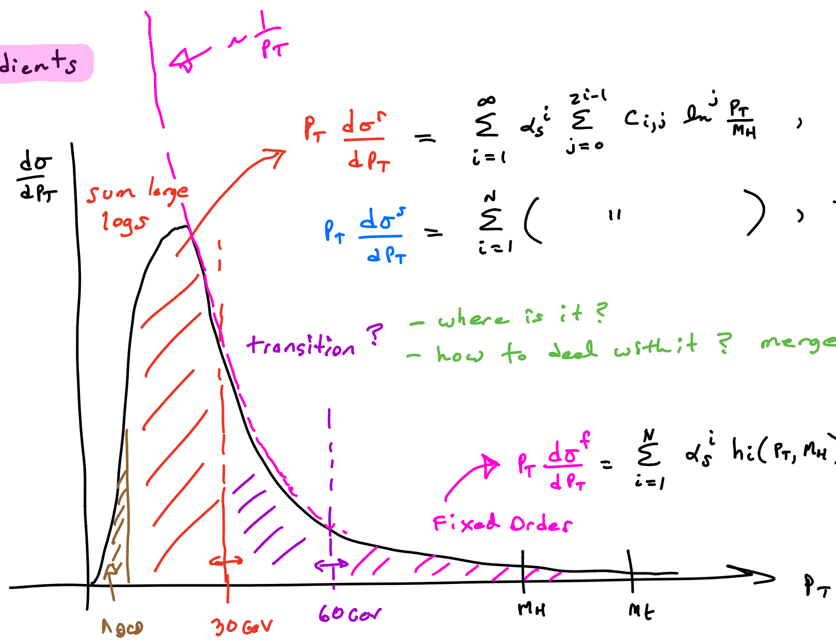
- GGH ok $p_T < m_t$
- mult. m_t dep.

gluon fusion

Motivation

- Key Higgs observable, next after $\sigma = \int dp_T^2 \frac{d\sigma}{dp_T^2}$
- reweigh MC for expt
- new physics sensitive
  [eg. Yotam, HuaXing, J. Zupan & Yukawa's (u & d)
  eg. light BSM through ratios
  eg. boosted Higgs & modified couplings]
- large QCD corrections eg. $\frac{\sigma_{NLO}}{\sigma_{LO}} = 2.3$ ($\frac{35}{15}$), $\frac{\sigma_{NNLO}}{\sigma_{NLO}} = 1.2$ ($\frac{43}{35}$), $\frac{\sigma_{N^3LO}}{\sigma_{NNLO}} = 1.02$ ($44/43$)
  Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2015)

Ingredients



(LL, NLL etc below)

$$p_T \frac{d\sigma^r}{dp_T} = \sum_{i=1}^{\infty} \alpha_s^i \sum_{j=0}^{2i-1} c_{ij} \ln^j \frac{p_T}{m_H}$$, singular terms $p_T \ll m_H$, all orders in α_s

$$p_T \frac{d\sigma^s}{dp_T} = \sum_{i=1}^{N} \Big(\quad " \quad \Big)$$, truncated to α_s^N

transition ? – where is it ? – how to deal with it ? merge smoothly

$$p_T \frac{d\sigma^f}{dp_T} = \sum_{i=1}^{N} \alpha_s^i\, h_i(p_T, m_H) = \underset{LO}{\alpha_s h_1} + \underset{NLO}{\alpha_s^2 h_2} + \underset{NNLO}{\alpha_s^3 h_3}$$

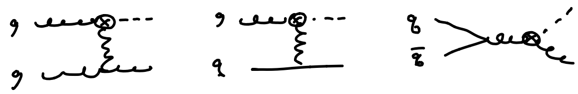
remove double counting

$$p_T \frac{d\sigma}{dp_T} = p_T \frac{d\sigma^r}{dp_T} + \left(p_T \frac{d\sigma^f}{dp_T} - p_T \frac{d\sigma^s}{dp_T} \right)$$

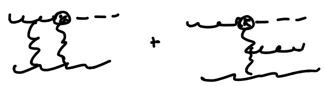
"nonsingular" $\equiv p_T \frac{d\sigma^n}{dp_T} \sim \mathcal{O}(p_T^2)$ for $p_T \ll m_H$

Fixed order

LO



NLO



NNLO

IR poles: $\frac{1}{\epsilon^4} + \frac{1}{\epsilon^3} + \cdots$, $\int \frac{1}{\epsilon^2} + \cdots = \frac{1}{\epsilon^4} + \frac{1}{\epsilon^3}$, $\iint = \frac{1}{\epsilon^4} + \frac{1}{\epsilon^3} + \cdots$ ← numeric!

need $\mathcal{O}(\epsilon^0)$

complicated ↘ ↙ choose wisely!

Subtractions: $\lim_{\delta \to 0} \int_\delta \frac{dx}{x} \left[f(x) - E(x) \right] + \int_\delta \frac{dx}{x} E(x) + f(0) \ln \delta$

(good $\delta \to 0$ limit) (↑ analytic, cancel $\ln \delta$)

many x's

$$\frac{d\sigma^{NNLO}}{dp_T} = \int_{\Phi_3} \left(d\sigma^{RR} - d\sigma^{S} \right) + \int_{\Phi_2} \left(d\sigma^{RV} - d\sigma^{T} \right) + \int_{\Phi_1} \left(d\sigma^{VV} - d\sigma^{U} \right)$$

← right behavior in various limits

collinear, soft, c+s, ...

non-trivial!

- Antenna Subt. NNLO JET : 2016 (Higgs) Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier
- Sector Improved Decomposition Subt. 2015 Boughezal, Caola, Melnikov, Petriello, Schulze
- N-Jettiness (Global) Subt. 2015 Boughezal, Focke, Giele, Liu, Petriello

$\delta \equiv$ resolution $\neq 0$ → numerics worse at small p_T

→ ~ millions of CPU hours $p_T > 1\,GeV$ (1 yr on 1000 cores)

$$\frac{d\sigma}{dp_T^2} = \pi\sigma_0 \int dx_a\, dx_b\, \delta\!\left(x_a x_b - \frac{M_H^2}{E_{cm}^2}\right) \int d^2b\, e^{i \vec{p}_\perp\cdot \vec{b}}\, W(x_a, x_b, M_H, \vec{b})$$

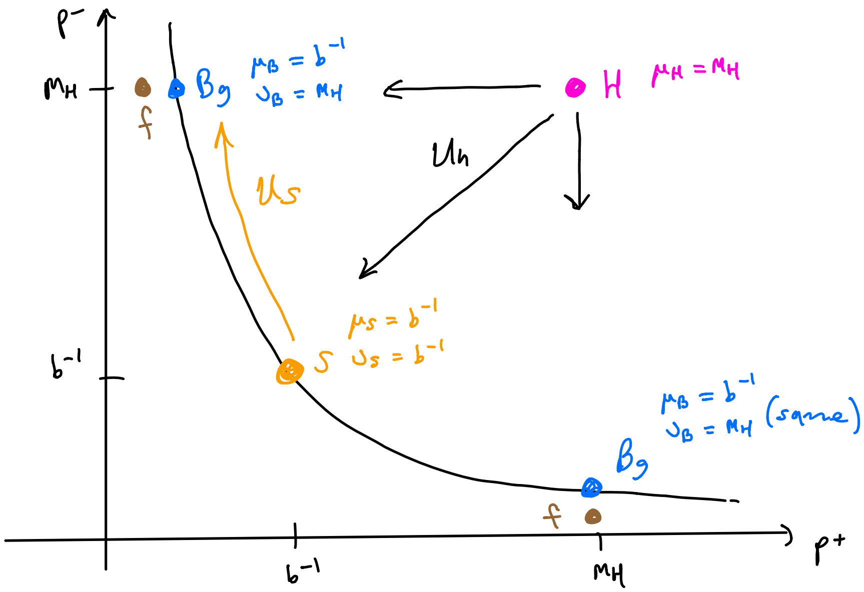
← independent of scales to order $N^k LL$ one is working

$$W = H(M_H,\mu_H)\, U_h(\mu_h,\mu_B)\, S_\perp(\vec{b},\mu_S,\nu_S)\, U_S(b,\mu_B,\mu_S;\nu_B,\nu_S)\, B_g(x_a,\vec{b},M_H,\mu_B,\nu_B)\, B_g(x_b,\vec{b},M_H,\mu_B,\nu_B)$$

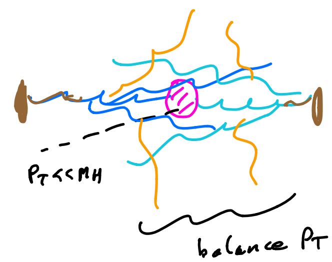
↑ hard (H) ↑ soft ($S_\perp$) ↑ beam fns (B_g)

$$B_g = \sum_i \int_x^1 \frac{dz}{z}\, \mathcal{I}_{gi}\!\left(\frac{x}{z},\vec{b},M_H,\mu_B,\nu_B\right) f_i(z,\mu_B)$$

↑ PDFs (f_i)



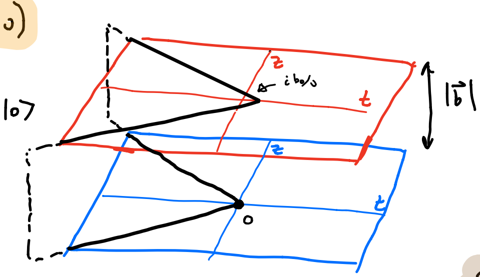
μ & ν are inv. mass & "rapidity" scales



Definitions [field thy defns on side board]

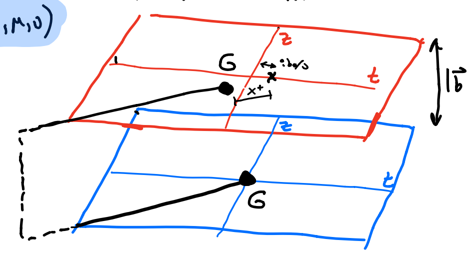
$S(b_\perp,\mu,\nu)$

⟨0| operator |0⟩
↑ Wilson Lines

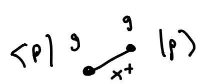


$B_g(x,\vec{b},M_H,\mu,\nu)$

⟨p| operator |p⟩ ← operator with $G^{\mu\nu}$'s on ends



$f_g(z,\mu)$



Log Resummation Orders

$$\ln W = L\sum_{k=1}^{\infty}(\alpha_s L)^k + \sum_k (\alpha_s L)^k + \alpha_s \sum_k (\alpha_s L)^k + \alpha_s^2 \sum_k (\alpha_s L)^k$$

(LL) (NLL) (NNLL) (N^3LL)

$L = \ln(b\, M_H)$ needed for all singular L terms at NNLO ↗ (N^3LL)

- H, B_g, B_g, S boundary conditions @ 3-loops except B_g (only logs known @ 3-loops)
- U_H, U_S solve RGEs

$$\mu \, d/d\mu \; U_H = \left(\Gamma_{cusp} \ln \frac{m_H}{\mu} + \gamma_V \right) U_H$$

$$\mu \, d/d\mu \; U_S = \left[\Gamma_{cusp} \ln \frac{\mu}{\nu} - \gamma_S \right] U_S$$ → write U_S ?

$$\nu \, d/d\nu \; U_S = \left[\int_{\mu}^{1/b} \frac{d\mu'}{\mu'} \Gamma_{cusp} + \gamma_r \right] U_S$$

↳ rapidity anom. dim. @ 3-loops 2016 MIT — Li, Zhu, (Neill)

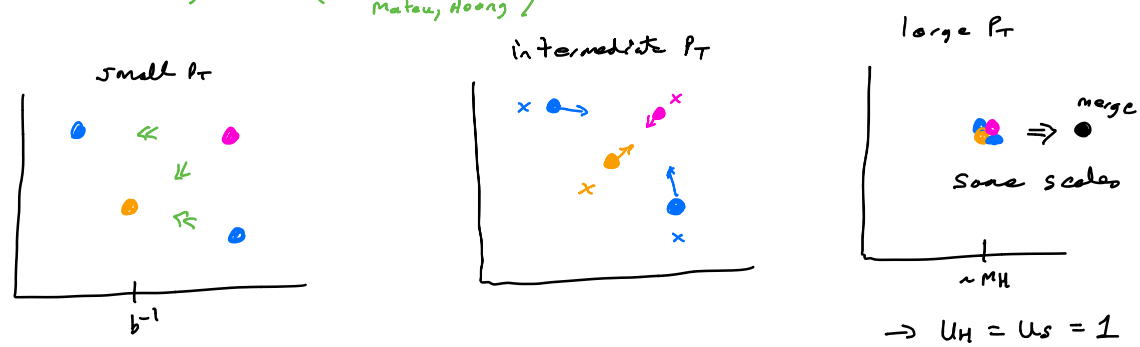
p_T N^3LL : MC for emissions — Monni, Re, Torrielli (2016)

p_T N^3LL + NNLO ; no improvement at small p_T — Bizon, Monni, Rottoli, Torrielli (2017)

Factorization ⇒ simple analytic $d\sigma^S/dp_T \big|^{N=3}$ ⇒ use to check / improve numerics @ NNLO

⇒ key to obtaining high precision results at small p_T

Transition (smooth) — profile functions (SS, Ligeti, Tackmann & FS, Abbate, Fickinger, Mateu, Hoang) — $\mu_S = \mu_S(b, p_T) = \mu_B$, $\nu_S = \nu_S(b, p_T)$, etc. — Neill, Vaidya, Rothstein



→ $U_H = U_S = 1$

$$\frac{d\sigma^r}{dp_T} = \frac{d\sigma^s}{dp_T}$$

$$\Rightarrow \frac{d\sigma}{dp_T} = \frac{d\sigma^f}{dp_T} \checkmark$$

- vary functional form
- vary boundary scales

} perturbative uncertainty

Go to Slides

Plots to Show

- earlier results NNLO by 3 groups
- earlier results N^3LL by Pier et. al.
- $d\sigma^s/dp_T$ vs $d\sigma^f/dp_T$ (Fig. 1)
- Fig. 2, discuss transition
- Money Plot, Fig. 3, $\pm 6\%$ for $p_T < 30$ GeV

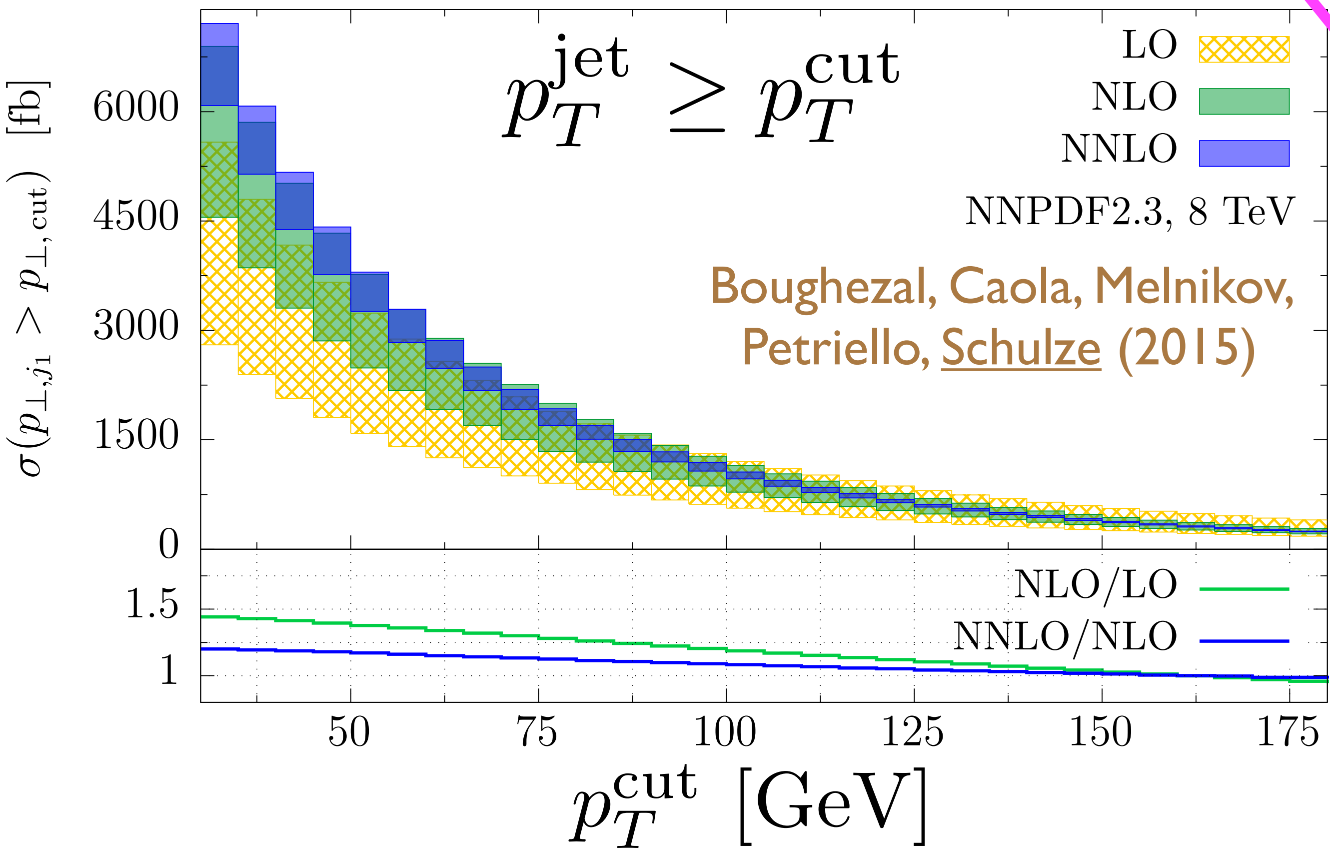
Precision Transverse Momentum Spectrum of the Higgs Boson

Iain Stewart
MIT

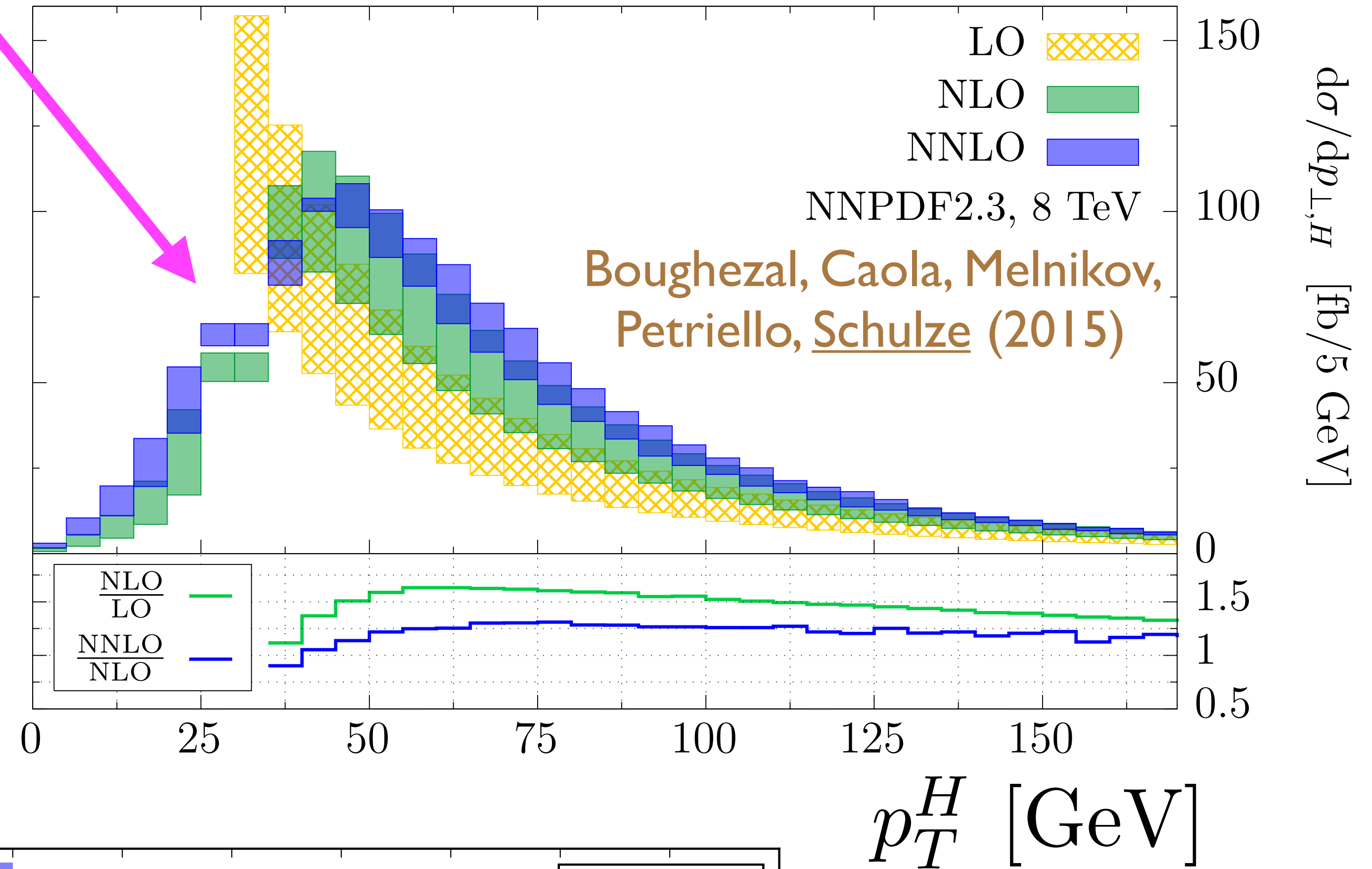
Plots for Vienna Blackboard Talk
(May 2018)

Earlier Results at NNLO

Sector Decomp.

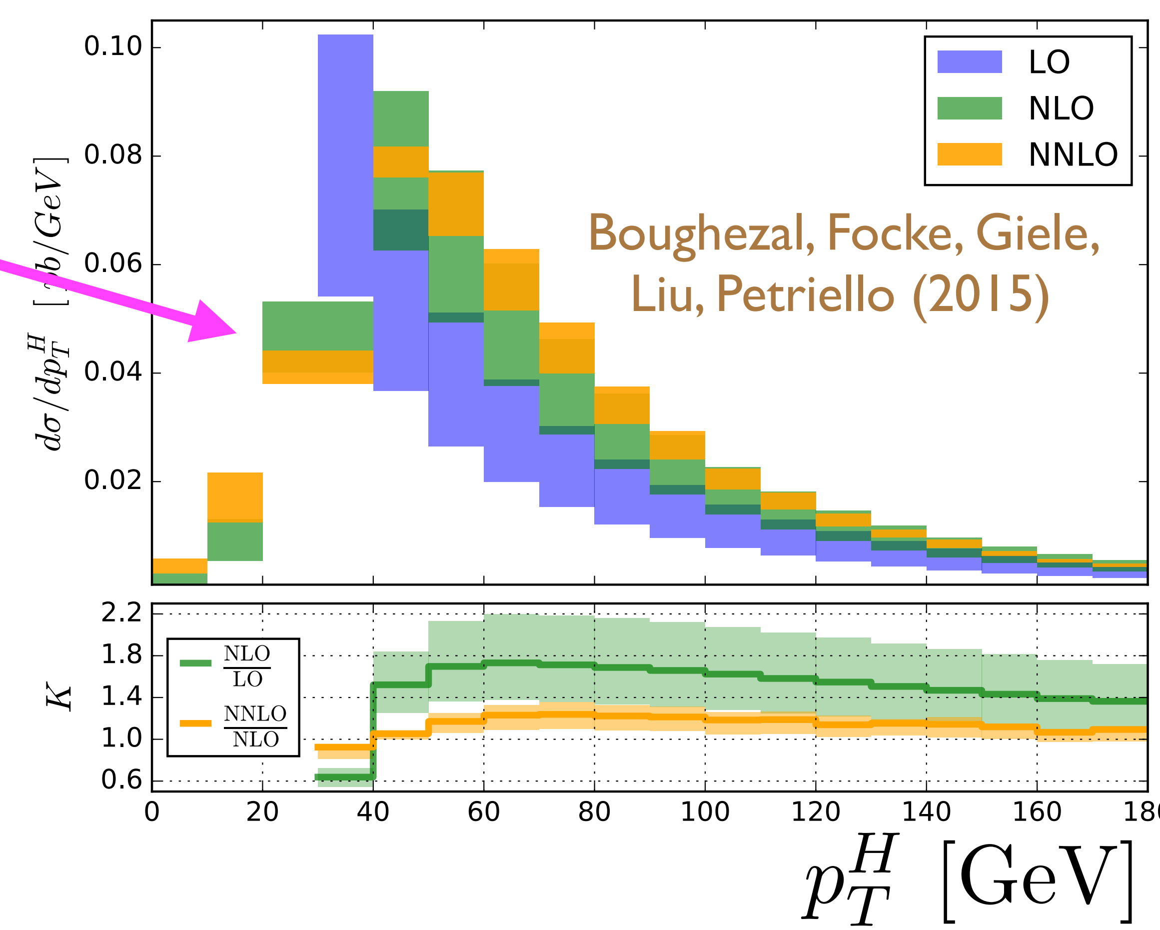


$p_T^{\text{jet}} \geq 30$ GeV



N-Jettiness Subtractions

$p_T^{\text{jet}} \geq 30$ GeV

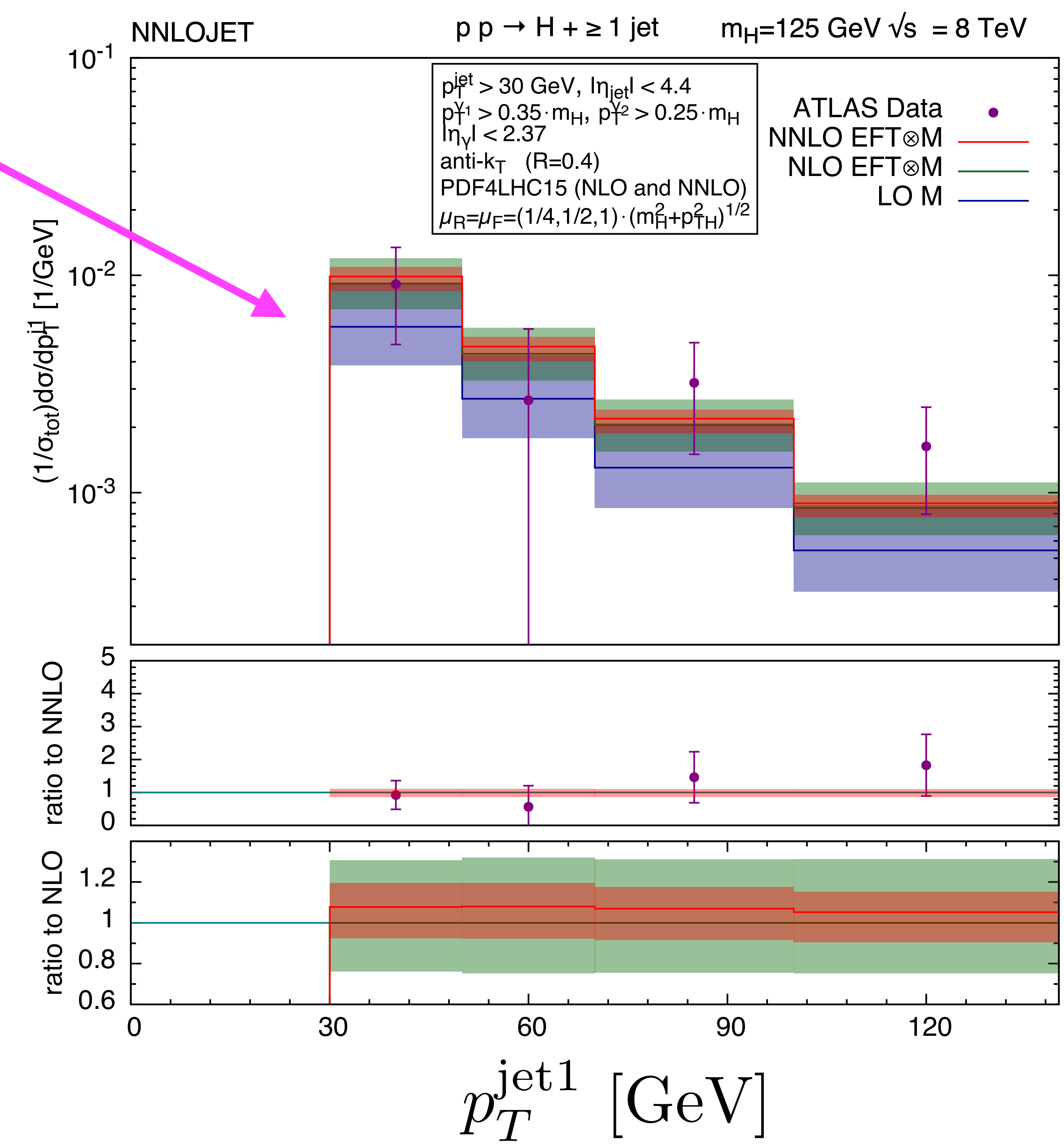


Earlier Results at NNLO

Antenna Subtractions (NNLOJET)

Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier (2016)

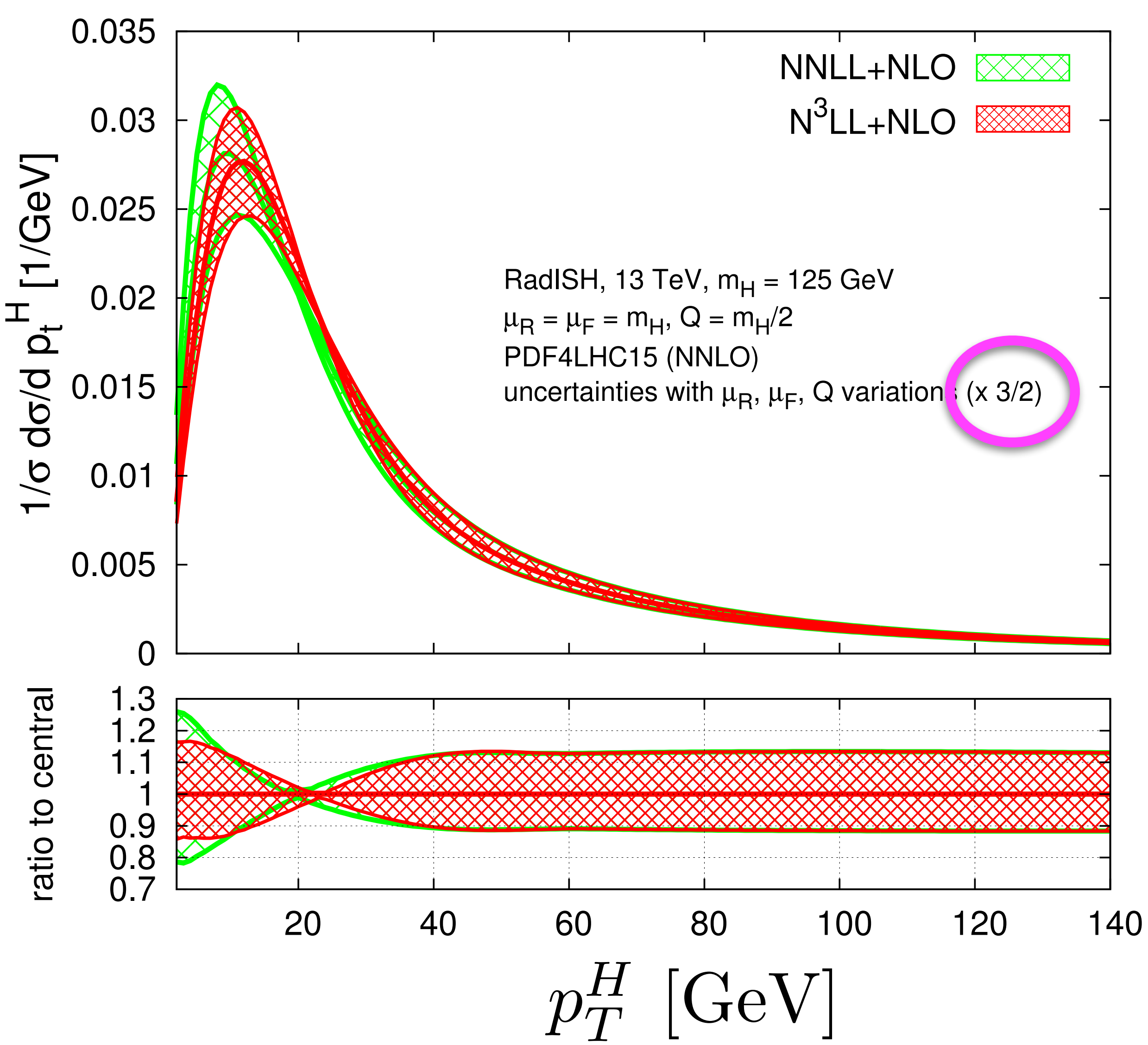
Predictions above 30 GeV



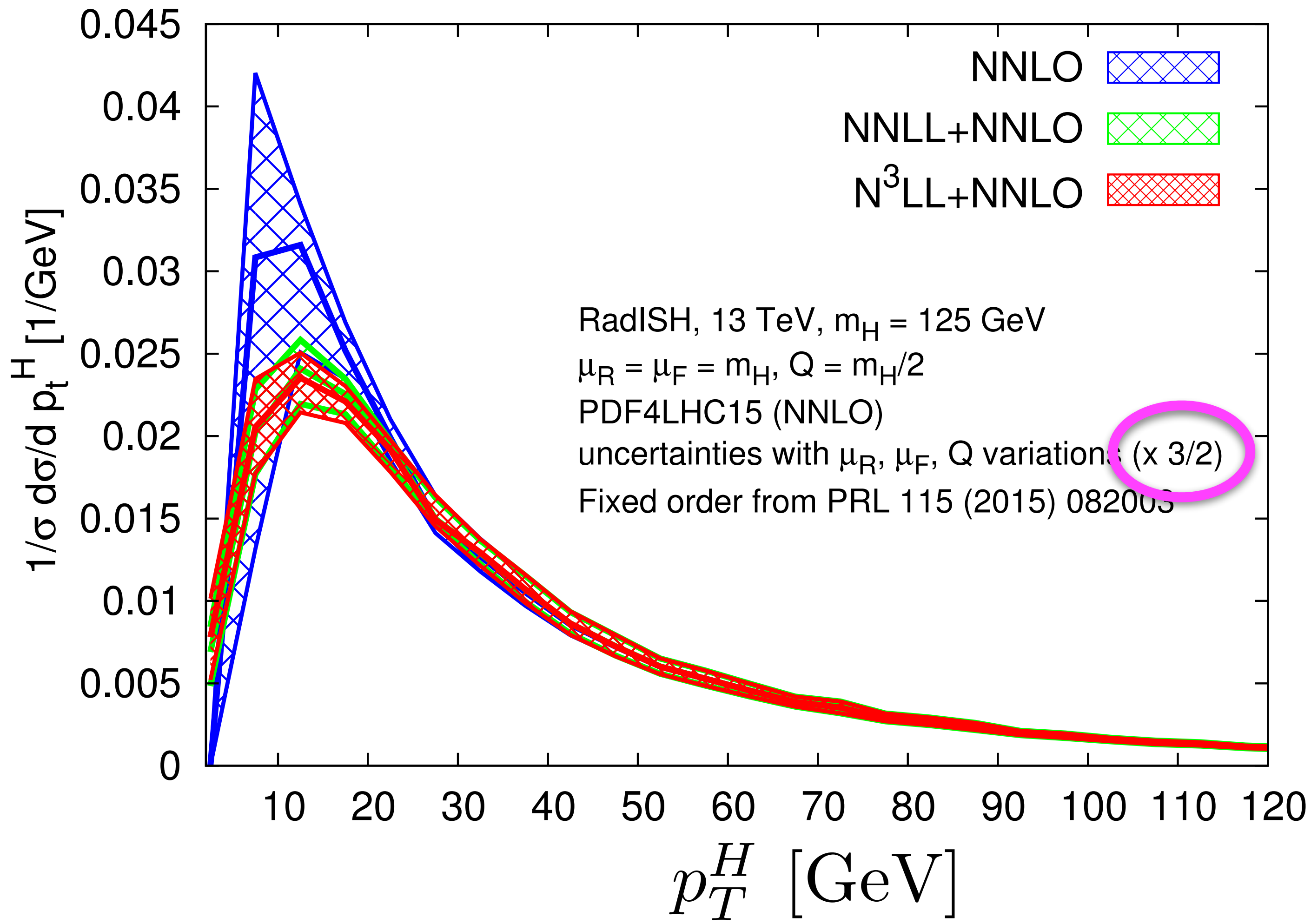
Earlier Result at N³LL+NNLO

Bizon, Monni, Re, Rottoli, Torrielli (2017) [using MC integrations with ARES type method]

with NLO



with NNLO from sector decomp.

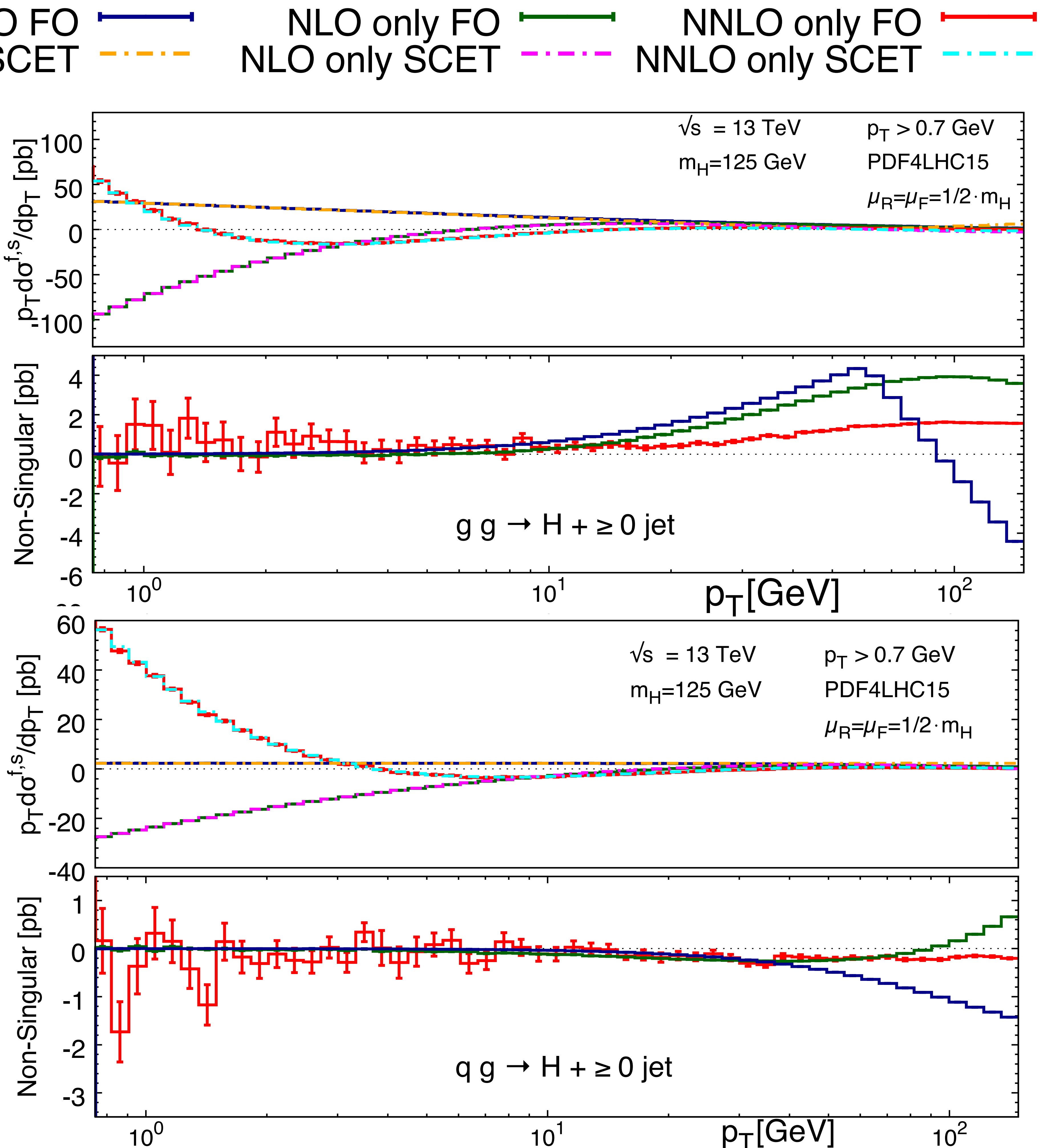


Note: Resummation uncertainties use variation by factor of 3/2 here (not a factor of 2)

Our Results

Chen, Gehrmann, Glover, Huss, Li, Neill, Schulze, IS, Zhu (arXiv:1805.00736)

NNLO Fixed Order
vs
Singular SCET

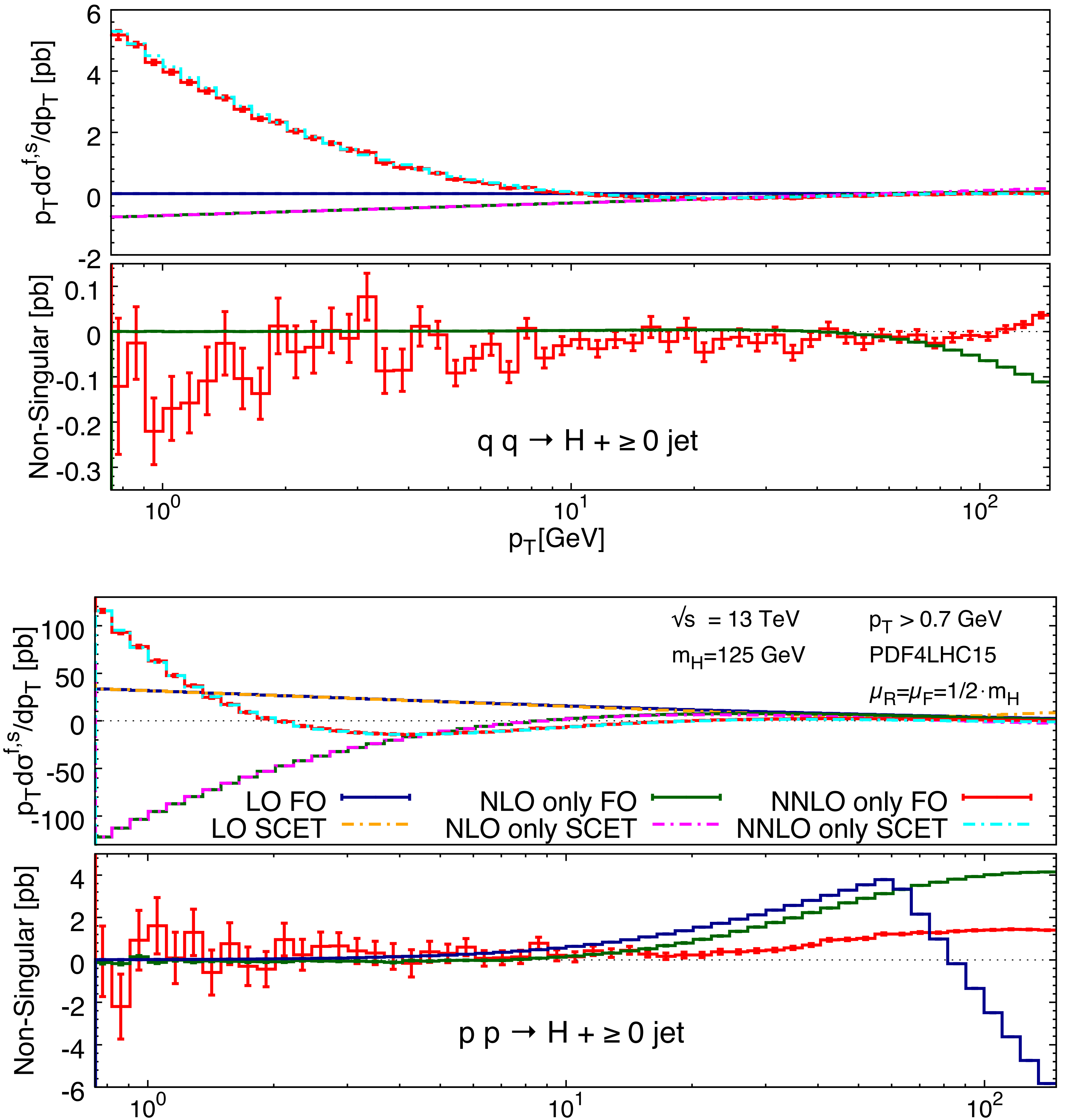


non-singular:

$p_T d\sigma^{\rm n}/dp_T \;\sim\; \mathcal{O}(p_T^2)$

for $p_T \ll m_H$

NNLO Fixed Order vs Singular SCET



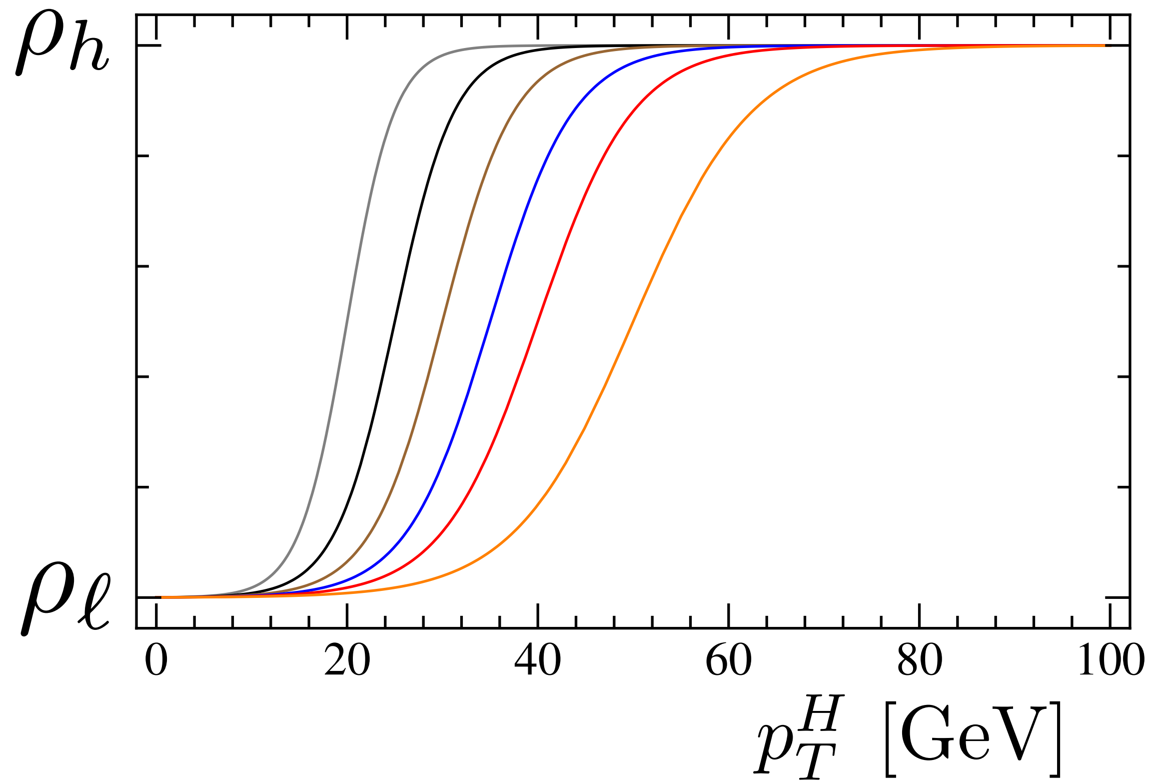
non-singular:

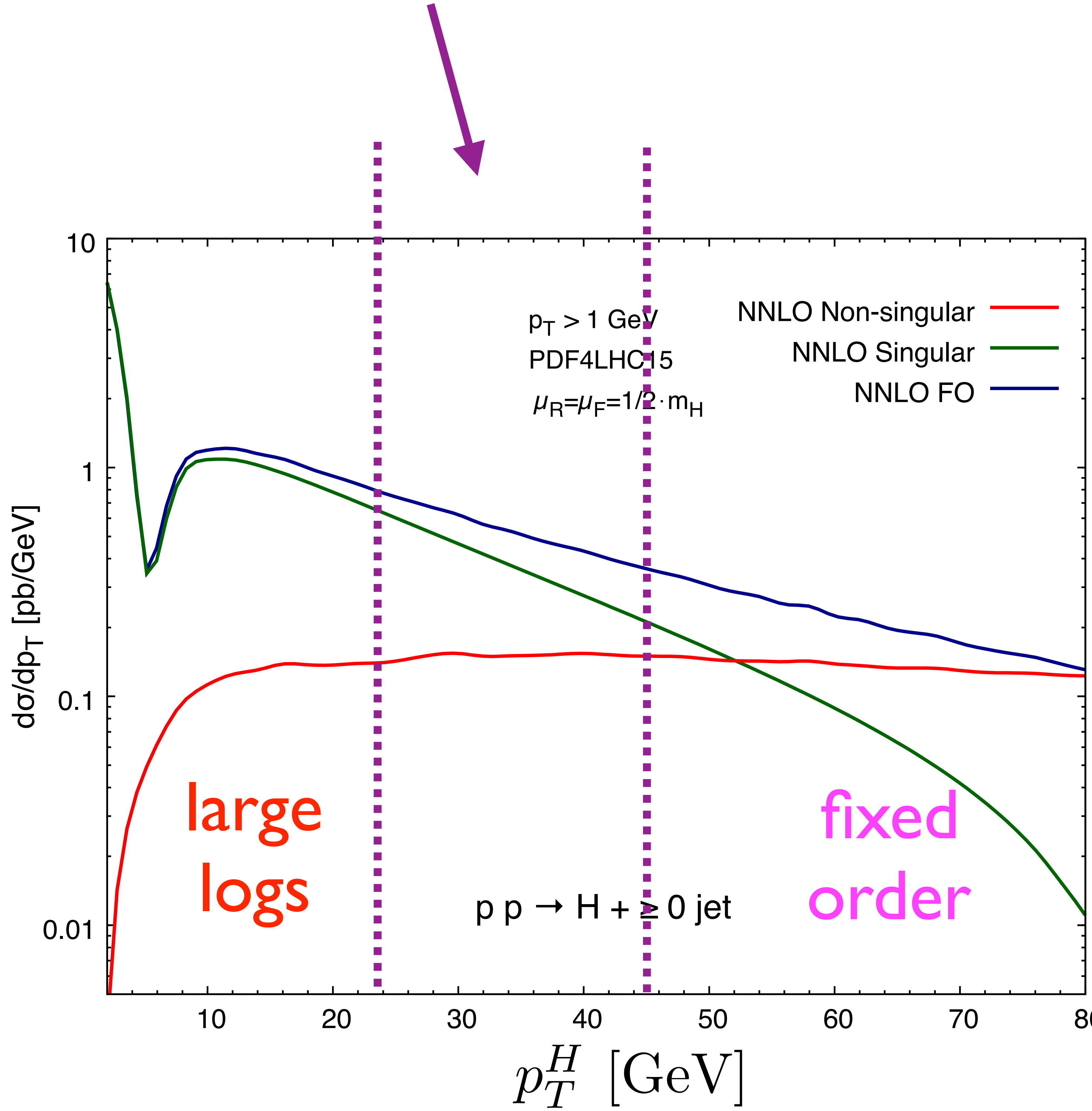
$p_T d\sigma^{\rm n}/dp_T \ \sim\ \mathcal{O}(p_T^2)$

for $p_T \ll m_H$

Compare Singular & Nonsingular to determine the Transition Region

vary the profiles:





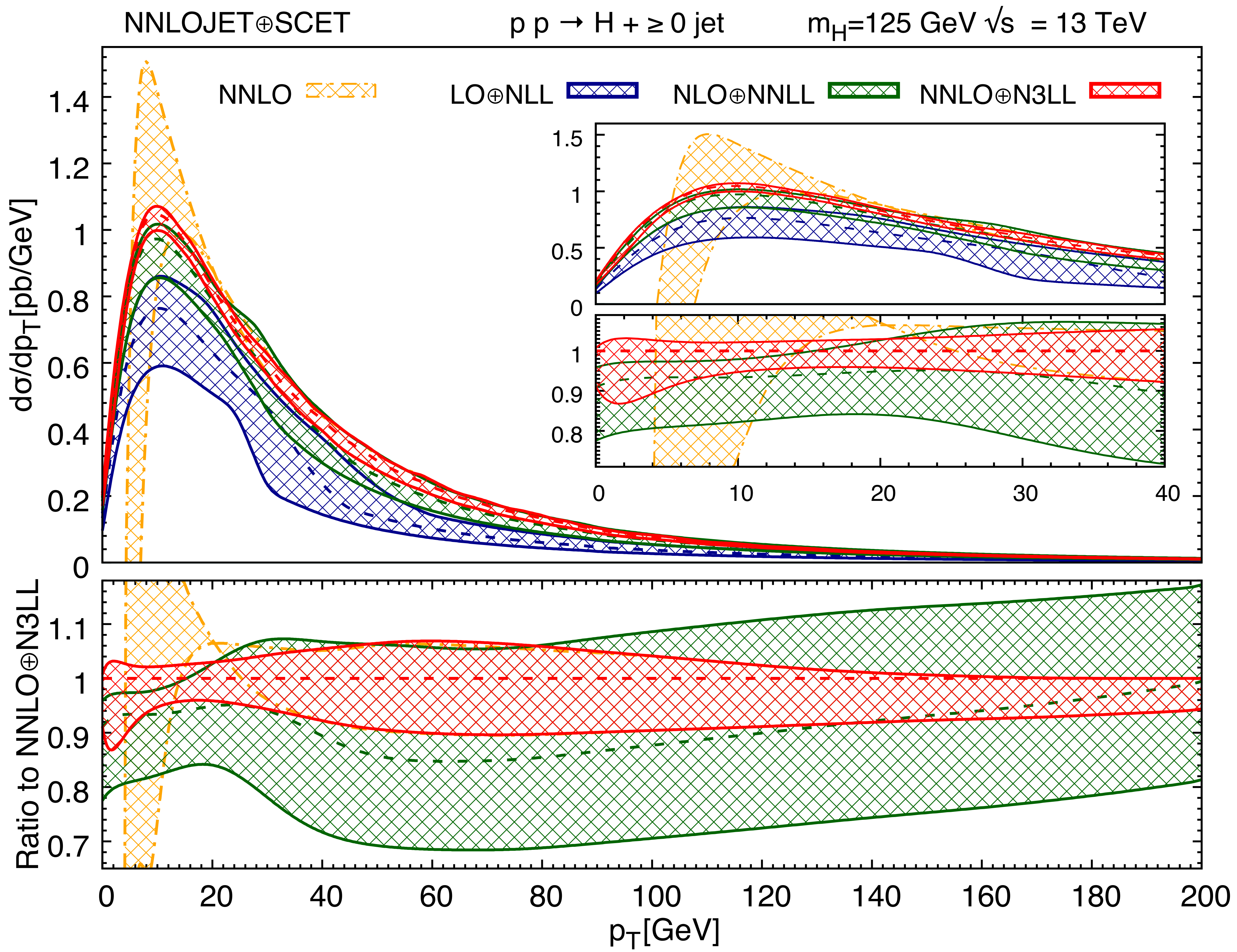
vary endpoints ρ_h & ρ_ℓ by factors of 2

egs. $\rho_\ell(\mu_S) \in \left[\frac{1}{2}b^{-1}, 2b^{-1}\right]$

$\rho_h(\mu_S) \in \left[\frac{1}{4}m_H, m_H\right]$

Our Money Plot

Chen, Gehrmann, Glover, Huss,
Li, Neill, Schulze, IS, Zhu (2018)



The End

Backup

Soft Function Relations

Rapidity regulated TMD S

$S(b_\perp, \mu, \nu)$

singular
=regulator

$\tau = \frac{1}{\nu} \to 0$

Threshold Soft Function

$S_{\rm thr}(\tau, \mu)$

$b_\perp \to 0$ smooth

$$S(\vec{b}_\perp, \tau, \mu) = \frac{1}{C} \sum_{X_s} {\rm tr} \langle 0 | T\{S^\dagger_{\bar n}(0) S_n(0)\} \exp\Big[-\mathcal{P}^0 b_0 \tau - i \vec{b}_\perp \cdot \vec{\mathcal{P}}_\perp \Big] |X_s\rangle\langle X_s| \bar{T}\{S^\dagger_n(0) S_{\bar n}(0)\}|0\rangle_{\rm ren}$$

$b^+ = b^- = i b_0 \tau$

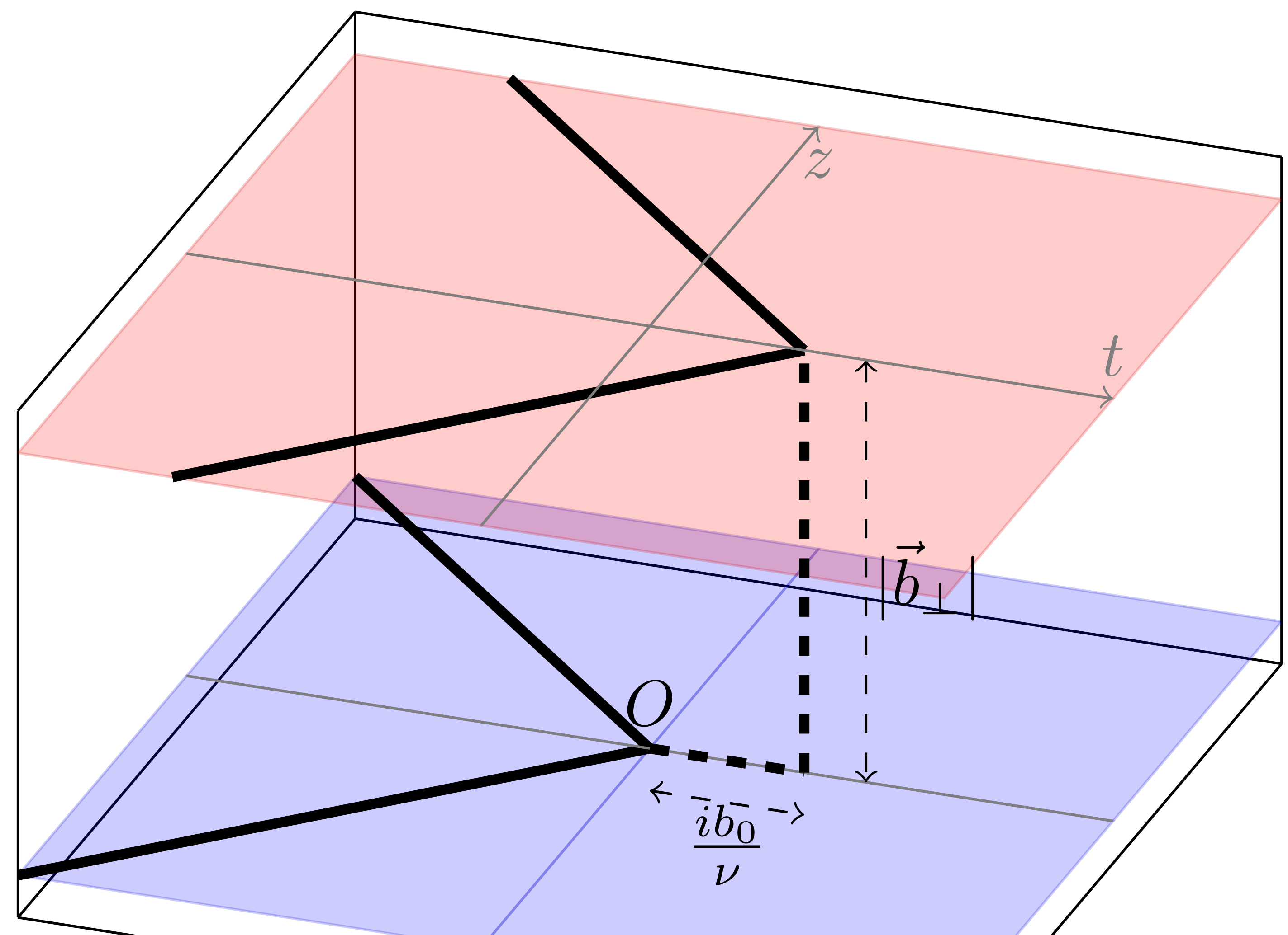
identical to

Fully Differential S

$S_{\rm F.D.}(b^+ b^-, \vec{b}_\perp, \mu)$

Two-loop result

Y. Li, Mantry, Petriello (2011)



Same for Beam Function

Rapidity regulated TMD B

$$B_{q/N}(x, Q, \vec{b}_\perp, \mu, \nu)$$

$$\tau = \frac{1}{\nu} \to 0$$

singular
=regulator

$$B_{q/N}(z, Q, \vec{b}, \mu, \nu) = \int dx^+ e^{izp^- x^+/2} \Big\langle p \Big| (\bar{\psi}_n W_n)\Big(x^+ + \frac{ib_0}{\nu}, \frac{ib_0}{\nu}, \vec{b}_\perp\Big) \frac{\bar{n}\!\!\!/}{2} \cdots (W_n^\dagger \psi_n)(0) \Big| p \Big\rangle$$

0-bin
& ren

Fully Differential B