α_s from non-strange hadronic τ decays

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$$\delta\alpha_s(M_Z) \simeq \left(\frac{\alpha_s(M_Z)}{\alpha_s(m_\tau)}\right)^2 \delta\alpha(m_\tau)$$

Papers

- O. Cata, M. Golterman and S. Peris, JHEP **0508**, 076 (2005)

- O. Cata, M. Golterman and S. Peris, Phys. Rev.**D77**, 093006 (2008)

-O. Cata, M. Golterman and S. Peris, Phys. Rev.**D79**, 053002 (2009).

- D. Boito, O. Cata, M. Golterman, M. Jamin, K. Maltman, J. Osborne and S. Peris, Phys. Rev. **D84**, 113006 (2011)

- D. Boito, M. Golterman, M. Jamin, A. Mahdavi, K. Maltman, J. Osborne and S. Peris, Phys. Rev. **D85**, 093015 (2012)

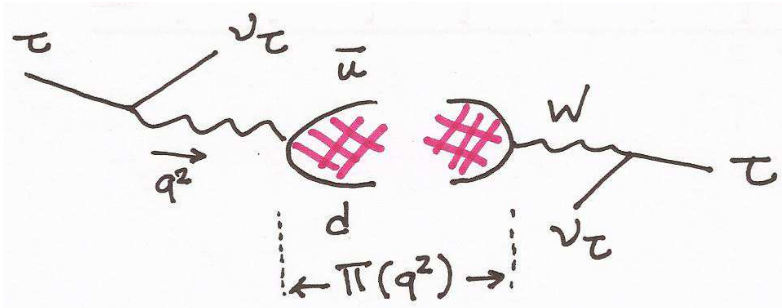
- D. Boito, M. Golterman, K. Maltman, J. Osborne and S. Peris, Phys. Rev. **D91**, no. 3, 034003 (2015)

- S. Peris, D. Boito, M. Golterman and K. Maltman, Mod. Phys. Lett. **A31**, no. 30, 1630031 (2016)

- D. Boito, M. Golterman, K. Maltman and S. Peris, Phys. Rev. **D95**, no. 3, 034024 (2017)

- D. Boito, I. Caprini, M. Golterman, K. Maltman and S. Peris, arXiv:1711.10316 [hep-ph], to appear in Phys. Rev. **D**.

QCD in τ decay

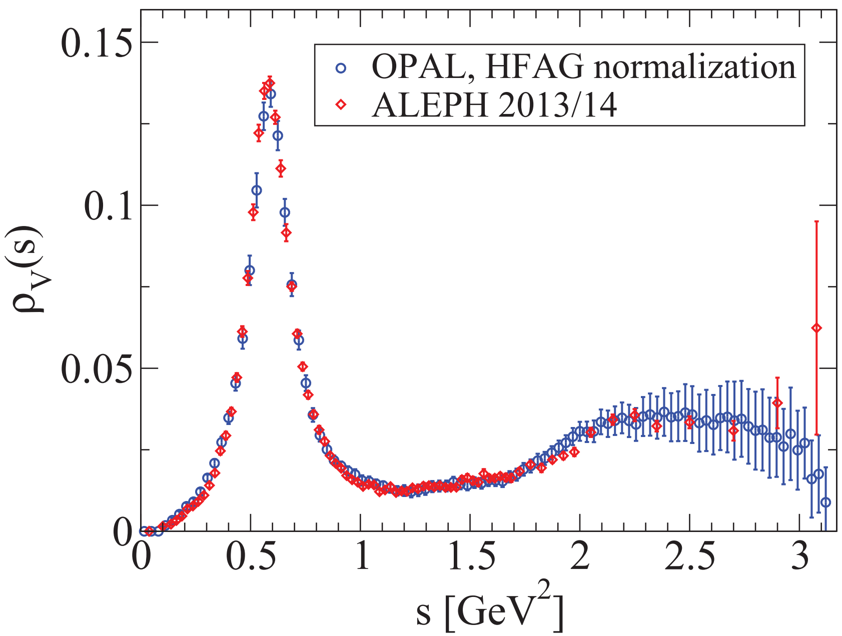


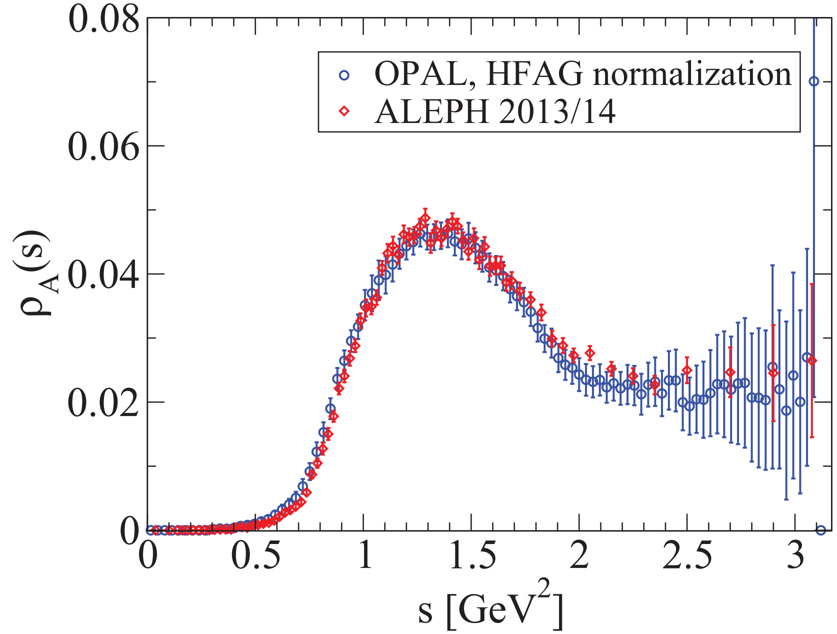
$$w_T(s;s_0) = \left(1+2\frac{s}{s_0}\right)\underbrace{\left(1-\frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

$$w_L(s;s_0) = 2\left(\frac{s}{s_0}\right)\underbrace{\left(1-\frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

$$\boxed{s_0 = m_\tau^2} \qquad \rho_{V,A} = \frac{1}{\pi}\mathrm{Im}\Pi_{V,A}$$

$$\frac{\Gamma(\tau\to\nu_\tau \mathrm{H}_{ud}(\gamma))}{\Gamma[\tau\to\nu_\tau e\bar{\nu}_e(\gamma)]} = 12\pi^2|V_{ud}|^2 S_{EW}\int_0^{s_0}\frac{ds}{s_0}\left[w_T(s;s_0)\rho^{(1+0)}_{V+A}(s) - w_L(s;s_0)\rho^{(0)}_A(s)\right]$$



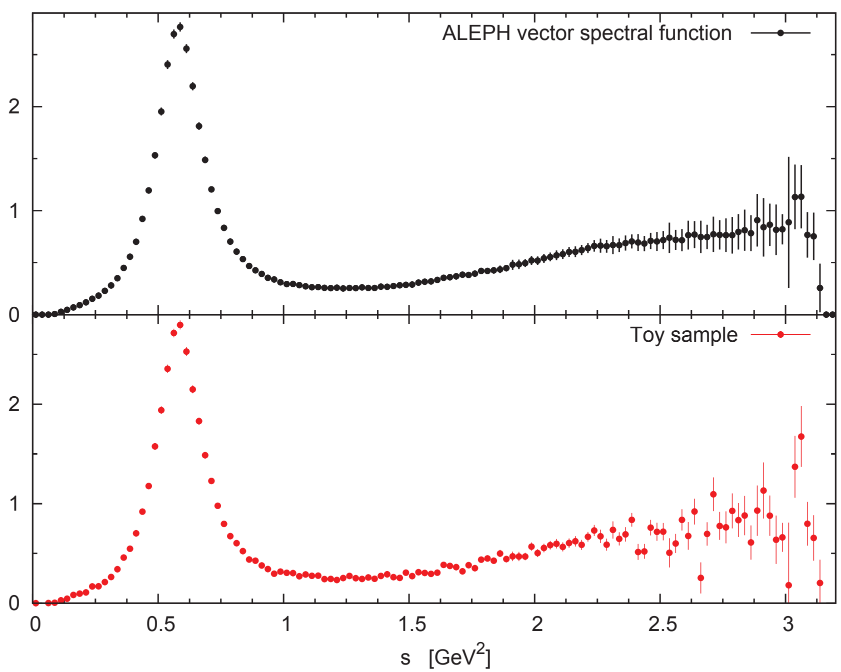


Since 2014: New correlation matrices.

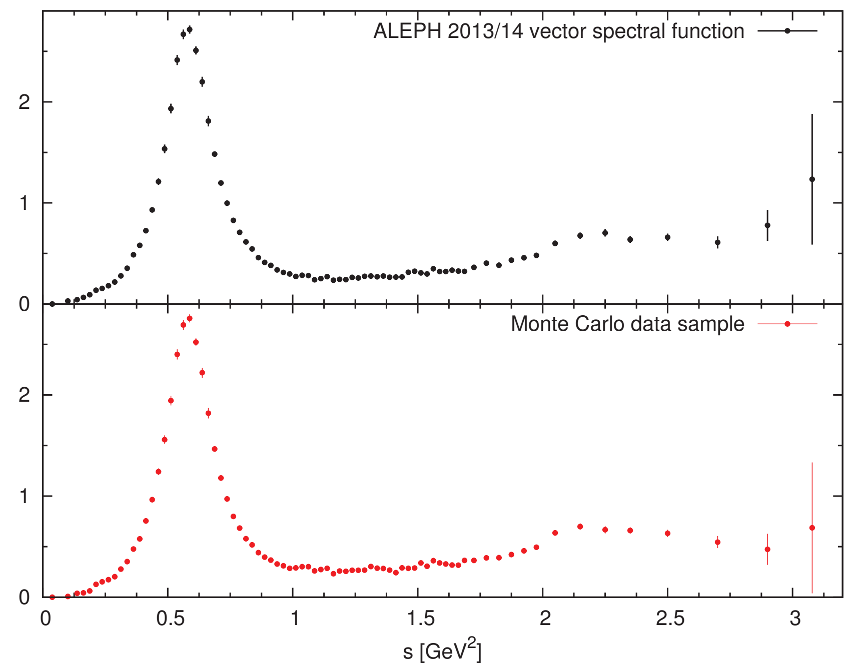
Boito et al. '11
Davier et al. '14

-Create pseudo-data:

2011



2014



WRONG!

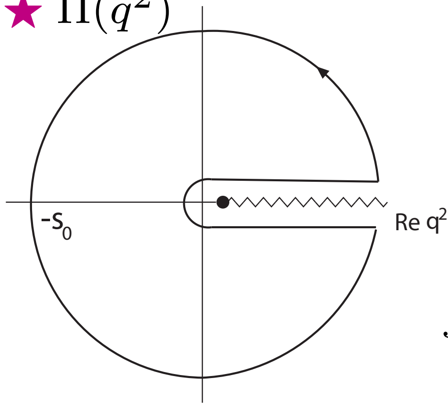
OK!

$t < 2014$ Aleph correlations underestimated.

Theoretical Foundations (I)

Shankar '77; Braaten-Narison-Pich '92

★ $\Pi(q^2)$



"Cauchy's Theorem" $\left(z = q^2 \ ; \ \rho(t) = \frac{1}{\pi}\mathrm{Im}\Pi \ ; \ w_n = \text{polynomial}\right)$:

$$\int_0^{s_0} dt \, w_n(t) \underbrace{\rho(t)}_{exp.} = \frac{1}{2i\pi} \oint_{|z|=s_0} dz \, w_n(z) \, \Pi(z)$$

$$= \frac{1}{2i\pi} \oint_{|z|=s_0} dz \, w_n(z) \left[\underbrace{\Pi_{\mathrm{OPE}}(z)}_{\mathcal{O}(\alpha_s^4)} + \underbrace{\Pi(z) - \Pi_{\mathrm{OPE}}(z)}_{\Pi_{DV}(z)} \right]$$

★ $\Pi_{DV} \to 0 \Longleftrightarrow \Pi_{\mathrm{OPE}} \to \Pi$.

(Cata-Golterman-S.P. '05)

However,

- Π_{OPE} expected asymptotic : $\Pi_{DV}(z) \to 0, \ z \to \infty$.
- OPE no good on the Minkowski axis (spect. fnct. shows oscillations)

$\Longrightarrow$ use polynomial $w_n(s_0) = 0$ ($\equiv$ "pinching")

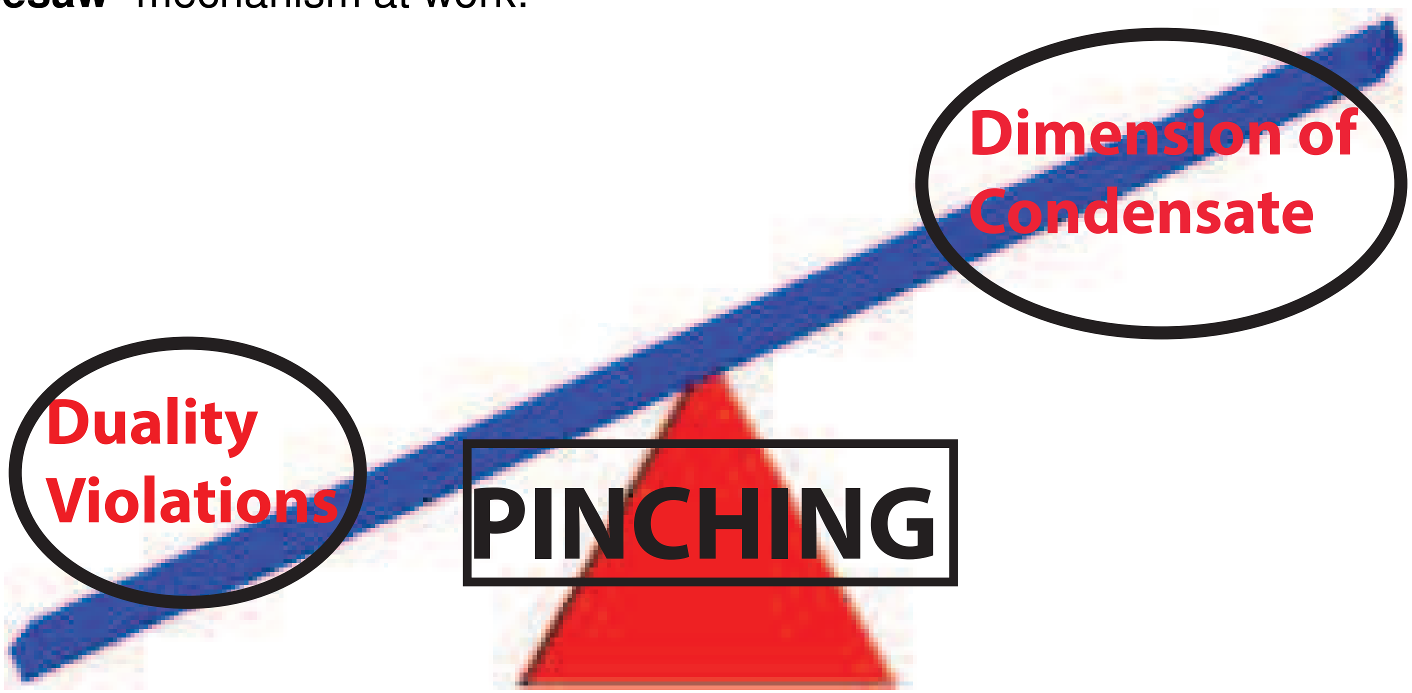
Main Theoretical Message:

(Maltman-Yavin ’08, Boito et al. ’11)

★ No free lunch: with pinching one has a **price to pay**:

It is not possible to simultaneously **suppress DVs** and **condensates**.

★ “**Seesaw**” mechanism at work:

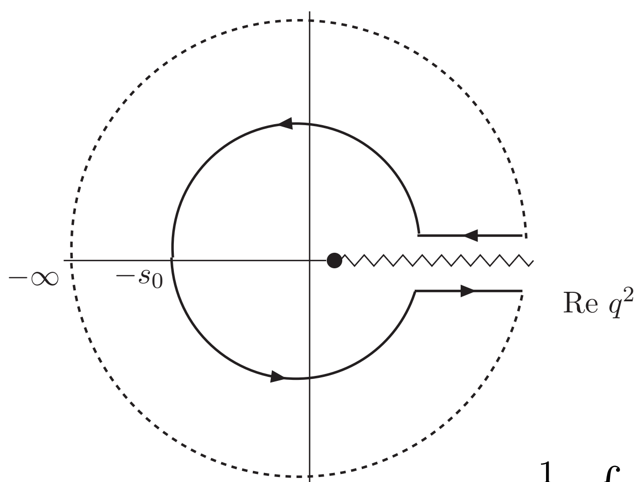


Theoretical Foundations (II)

- Need a better control of systematic error

$\Longrightarrow$ need **quantitative** knowledge of DVs.

- $\Pi_{DV}(s) \to 0$, as $s \to \infty$. Then:



(Cata-Golterman-S.P. '05)

$$-\frac{1}{2\pi i} \oint_{|z|=s_0} dz\, w(z)\, \Pi_{DV}(z) = -\underbrace{\int_{s_0}^{\infty} ds}_{\text{extrapolation!}} \; w(s) \; \frac{1}{\pi} \mathrm{Im}\Pi_{DV}(s)$$

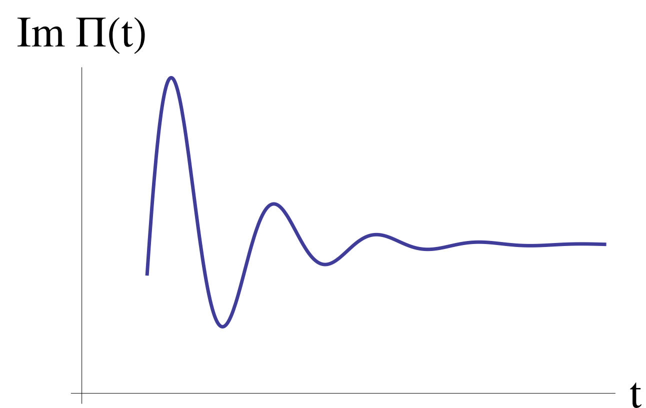
Theoretical Foundations (III)

Cata,Golterman, S.P. '05, '08

★ A phenomenological **educated guess:** (See Regge/Asymptotic Theory theoretical discussion later on...)

-For s large enough:

$$\frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s) \quad \simeq \quad \mathrm{e}^{-\delta} \quad \underbrace{\mathrm{e}^{-\gamma s}}_{\text{asymp. exp.}} \quad \underbrace{\sin(\alpha+\beta s)}_{\text{oscillations}}$$



(asymp. exp. $\Leftrightarrow$ recall renormalons $\sim \mathrm{e}^{-\gamma/\alpha_s}$)

independently for V and A (i.e. 8 DV parameters in total).

• Assuming no DVs $\equiv$ assuming $\mathrm{e}^{-\delta} = 0$ (unlike data, which is not flat).

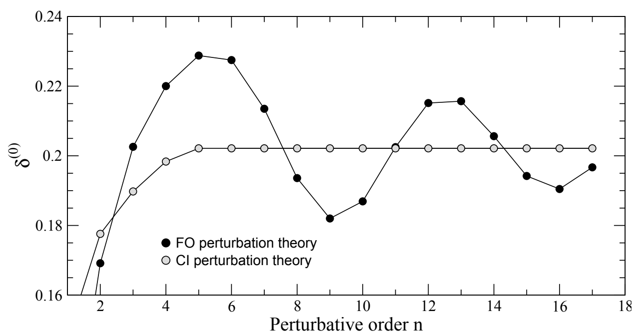
Pert. Theory: CIPT vs. FOPT

Caprini-Fischer '09; Menke '09; Descotes-Malaescu '09; Cvetic '10,...

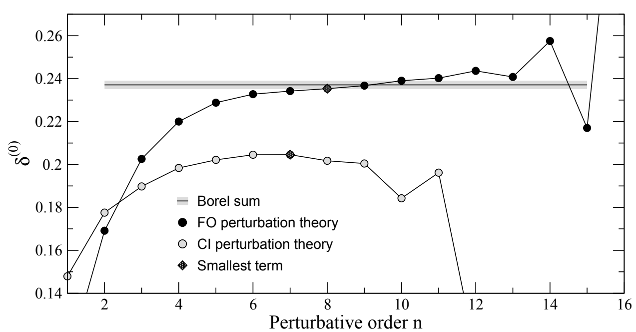
★ Partial integration: $\oint dz\, w_n(z)\, \Pi(z) = \oint dz\, \widetilde{w}_n(z) \underbrace{\left(-z\frac{d}{dz}\Pi(z)\right)}_{\text{Adler's D(z)}}$

★ $\mathrm{D}(z) = \frac{1}{4\pi^2}\sum_{n,k} c_{n,k}\ \underline{\alpha_s(\mu)^n\ \log^k\left(-z/\mu^2\right)}$

★ Since $z = s_0\, \mathrm{e}^{i\phi}$, two (extreme) choices:



- CIPT: $\mu^2 = -z$. Good *if* Adler function series stops at finite order.
  LeDiberder-Pich '92, Pivovarov '92



- FOPT: $\mu^2 = s_0$. Good *if* Adler function series grows factorially (renormalons).
  Beneke-Jamin '08

For many years thought to be the dominant source of error...

A Change of Strategy (I)

Old Strategy: (LeDiberder-Pich '92)

- Use 5 pinched weights

$$w_{kl}(y) = (1-y)^2(1+2y)(1-y)^k y^l \quad , \quad y = s/s_0, \quad s_0 = m_\tau^2 \text{ (only)}$$

with $(k,l) = \{(0,0),(1,0),(1,1),(1,2),(1,3)\}$.

- Fit to extract 4 param. : α_s and $C_{D=4,6,8}$.
- Assume (unknown) OPE condensates $C_{D=10,12,14,16} = 0$. (∼ OPE convergent)
- Assume (unknown) Duality Violations =0.
- May use V and A, but assume $V + A$ more reliable.

(Davier et al. '14)

$$\begin{aligned}
\langle \frac{\alpha_s}{\pi} GG \rangle \quad = \quad & (-0.5 \pm 0.3) \times 10^{-2}\ \text{GeV}^4\ , \qquad \chi^2 = 0.43,\ p = 51\% \qquad V\ , \\
& (-3.4 \pm 0.4) \times 10^{-2}\ \text{GeV}^4\ , \qquad \chi^2 = 3.4,\ p = 7\% \qquad A\ , \\
& (-2.0 \pm 0.3) \times 10^{-2}\ \text{GeV}^4\ , \qquad \chi^2 = 1.1,\ p = 29\% \qquad V + A\ .
\end{aligned}$$

- Check Weinberg sum rules.

A Change of Strategy (II)

New Strategy (Boito et al. '11 and '12):

- Do not use $w(y)$ with a term linear in y. (Beneke et al. '13)
- Do not assume any condensate is zero. (Let the data speak.)
- Do not assume that Duality Violations are zero. (Let the data speak.)

  For $s \geq s_{min}$ (Regge/asymp. series assumption; see later on):

$$\rho_{DV}^{V,A}(s) = \mathrm{e}^{-\delta_{V,A} - \gamma_{V,A}\, s} \sin\left(\alpha_{V,A} + \beta_{V,A}\, s\right)$$

  c.f. old strategy model assumption: $\mathrm{e}^{-\delta_{V,A}} = 0$.

- Fit to $\alpha_s, C_{D=6,8}$ and DV parameters with 3 weights:

$$w_0 = 1, w_2 = 1 - y^2 \quad \text{and} \quad w_3 = (1-y)^2(1+2y)$$

  Use all data for $s_0 \geq s_{min}$, (s_{min} to be determined by the fit as well).

- Use V and A. Check spectral functions.
- Check Weinberg sum rules.

We did lots of other fits as well...

Fits :

- V channel, $w_0 = 1$.
- V and A channels, $w_0 = 1$.
- V channel, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V and A channels, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V channel, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1 - y)^2(1 + 2y)$.
- V and A channels, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1 - y)^2(1 + 2y)$.

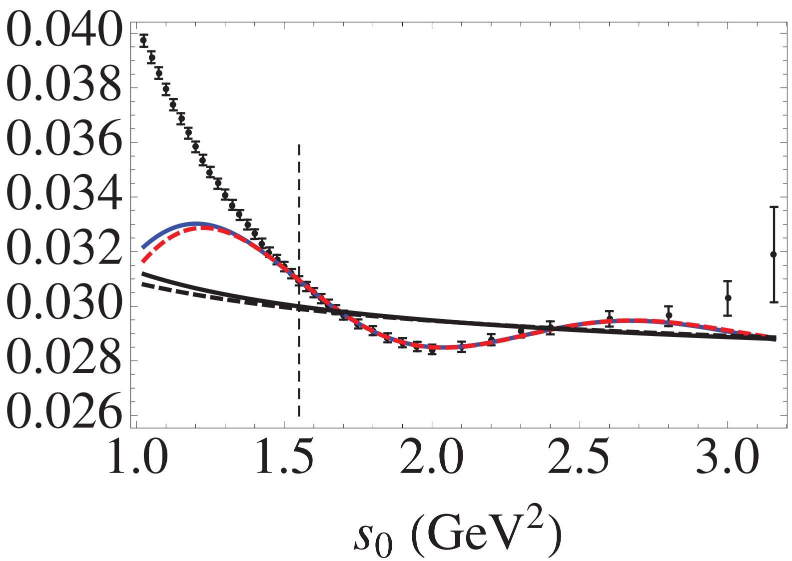
Consistent results in all cases.

Example: Fit to $w_0 = 1$, V channel (I).

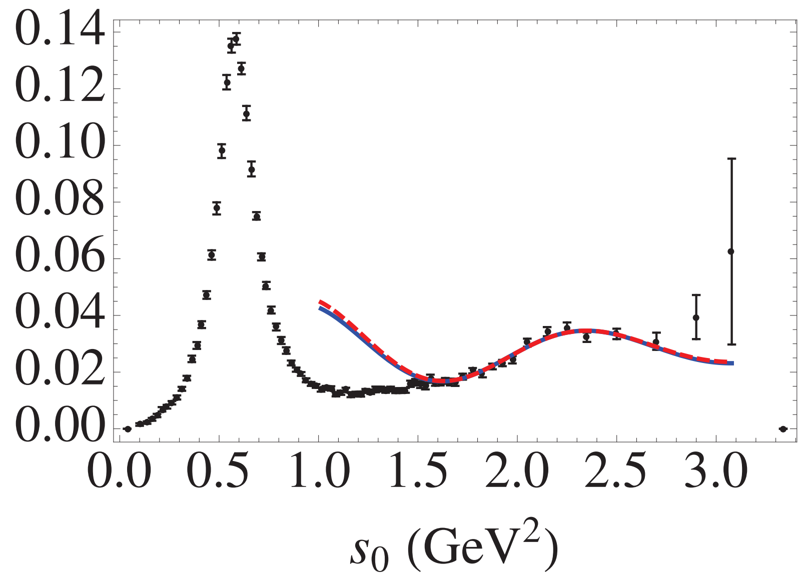
$s_{min} = 1.55\ \mathrm{GeV}^2$, $\chi^2/dof = 24.5/16$ $(p = 8\%)$ (This is FOPT, CIPT similar)

<u>curves</u>: red=CIPT blue =FOPT black =no DV

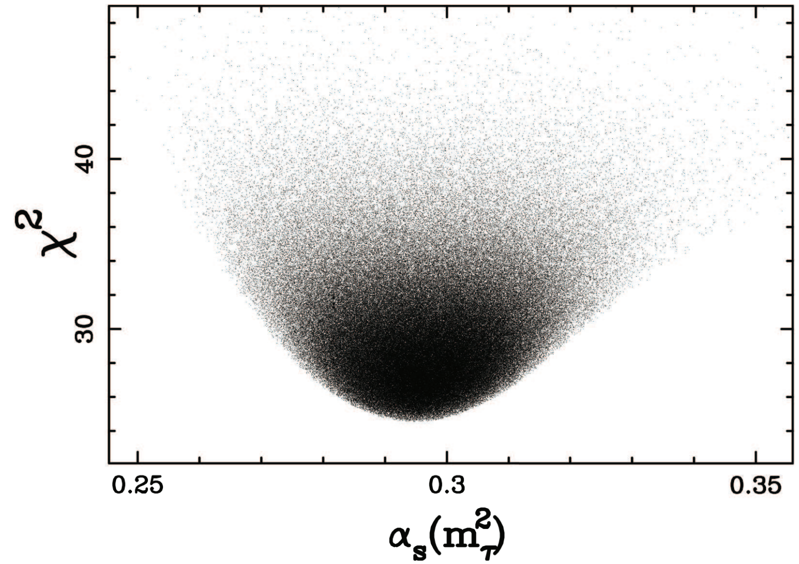
$w_0 = 1$ spectral integral

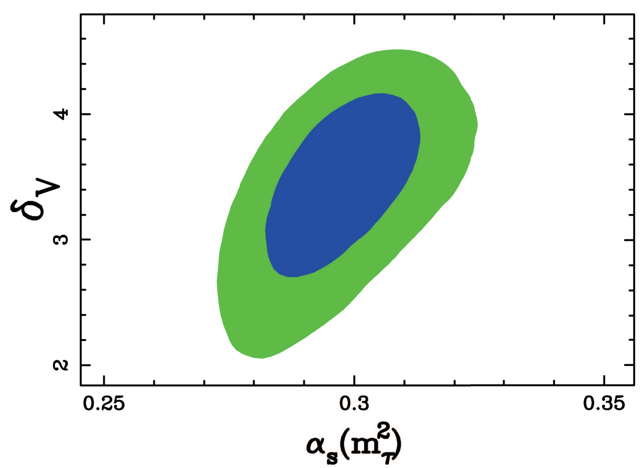


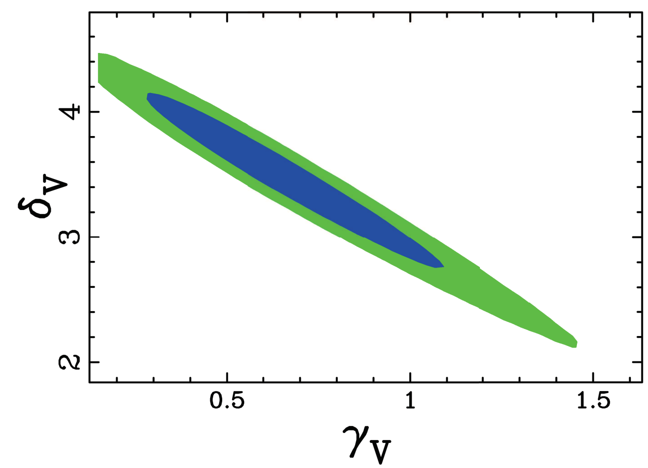
ALEPH's V spectrum



Example: Fit to $w_0 = 1$, V channel (II).







(68% and 95% contour plots), FOPT. Clearly $DVs \neq 0$.

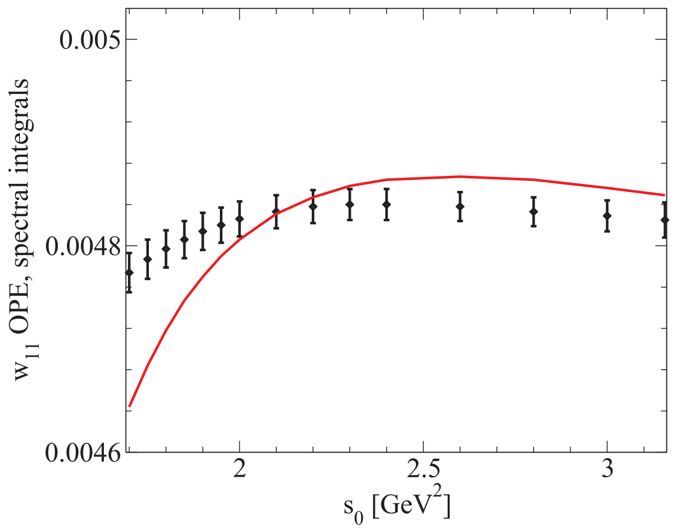
Tests: w_{11}, w_{12}, w_{13}

Looking only at $s = m_\tau^2$ potentially misleading. (Maltman-Yavin '08).

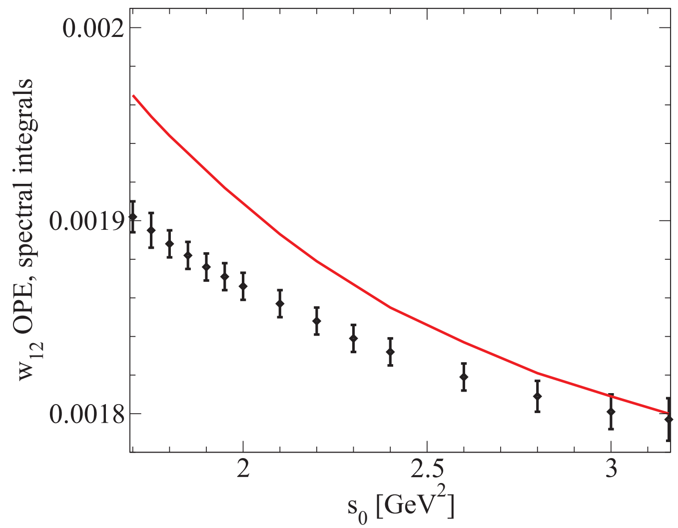
(Davier et al. '14)

CIPT, V+A:

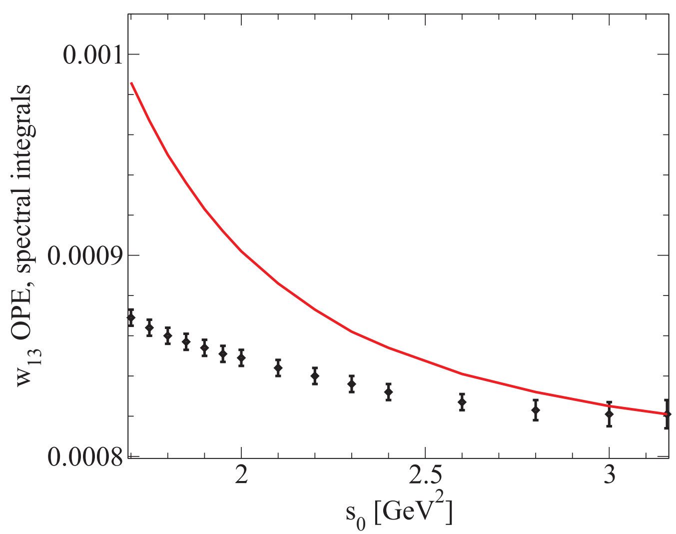
w_{11}



w_{12}



w_{13}



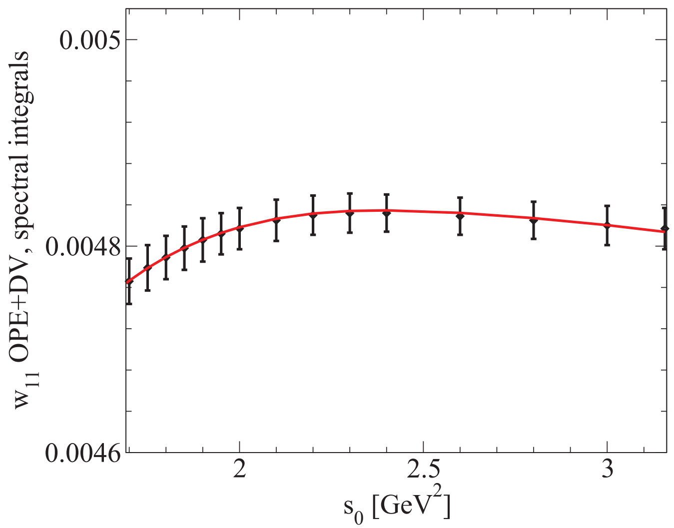
Tests: w_{11}, w_{12}, w_{13}

$D > 8$ condensates vital !

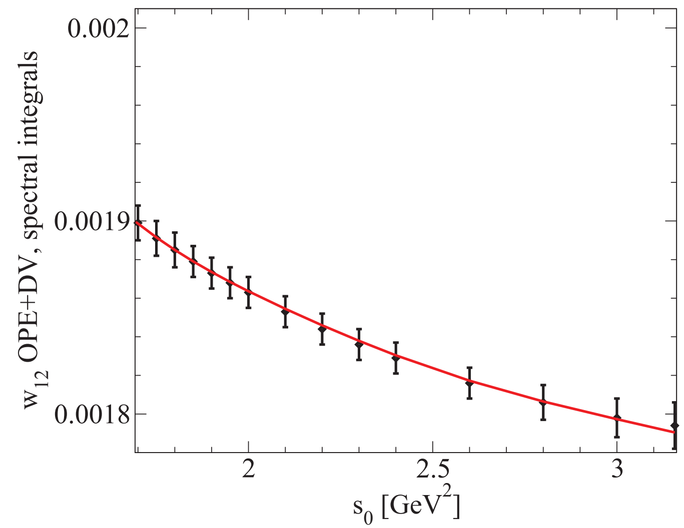
(Boito et al. '15)

CIPT, V+A:

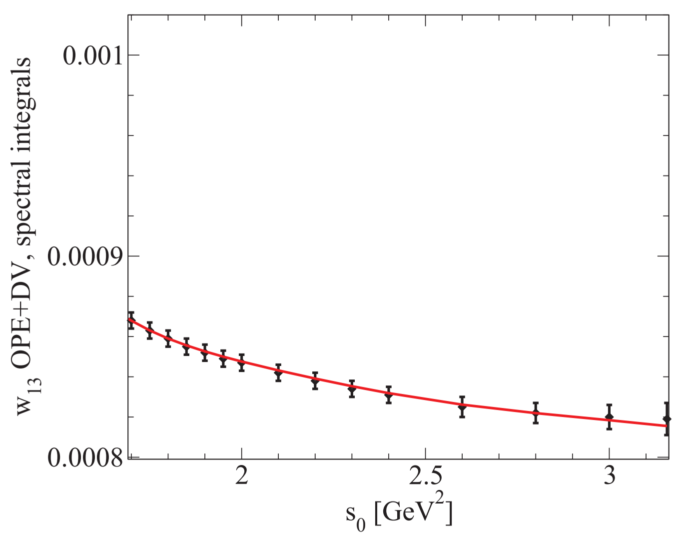
w_{11}



w_{12}



w_{13}



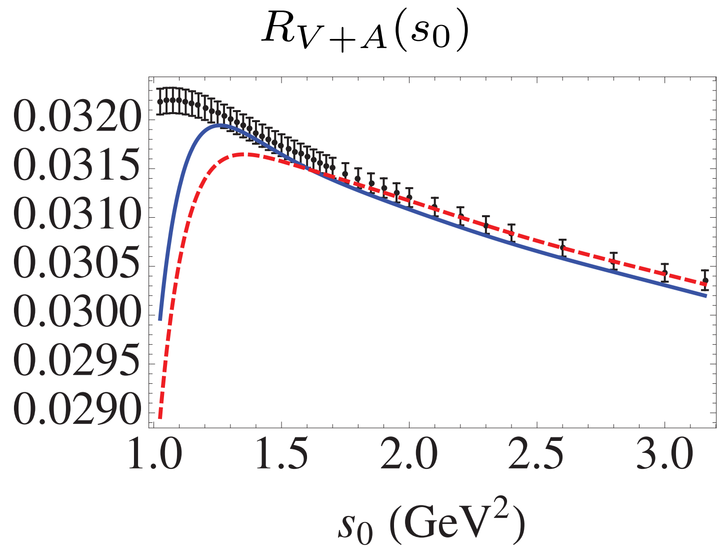
Not part of our fit !

Used $s_{min} = 1.55\ \mathrm{GeV}^2$
$w = 1, 1 - y^2, w_\tau$

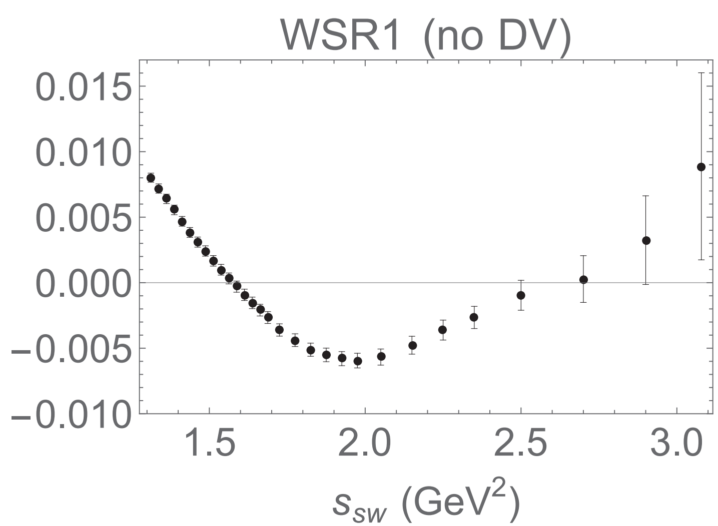
.

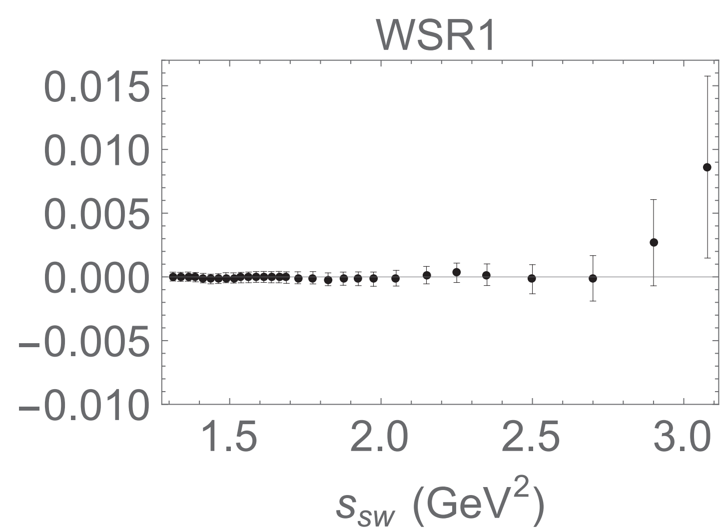
Classic Tests

red=CIPT   blue =FOPT   (Boito et al. '15).



Weinberg sum rule: $\int_0^\infty ds \left(\rho_V^{(1)}(s) - \rho_A^{(1)}(s)\right) - 2f_\pi^2 = 0$





Results

Aleph:

(FOPT) $\alpha_s(m_\tau) = 0.296 \pm 0.010 \longrightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014$

(CIPT) $\alpha_s(m_\tau) = 0.310 \pm 0.014 \longrightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019$

- N.B. "Old Strategy" produces a shift, i.e.

  $\alpha_s(m_\tau) \sim +0.03$ higher, (and $\sim$ half errors) (Davier et al. '14)

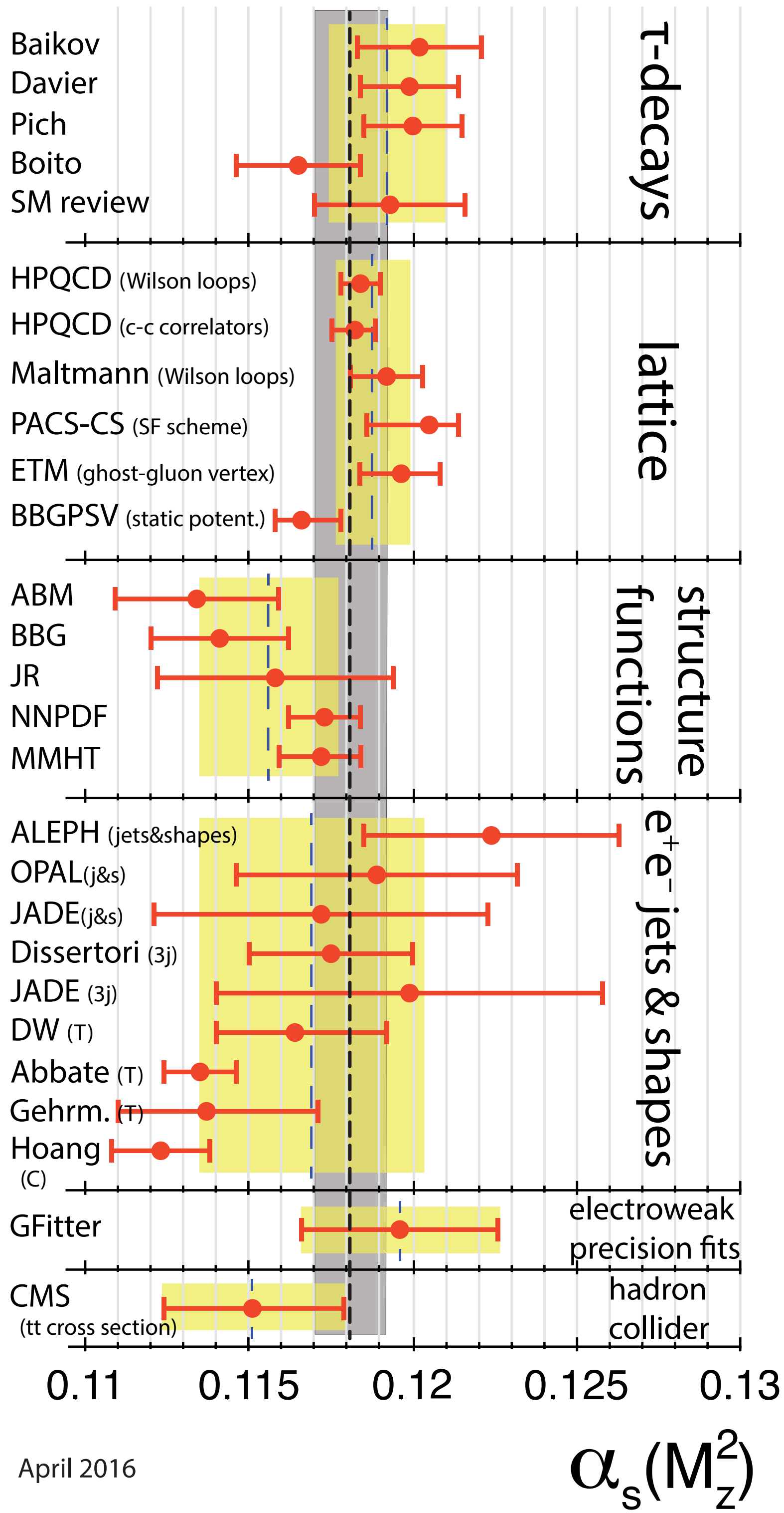
- Using **Aleph + Opal** data, we get:

$\alpha_s(m_Z) = 0.1165 \pm 0.0012$ (FOPT) $\alpha_s(m_Z) = 0.1185 \pm 0.0015$ (CIPT)

(Current PDG 2017 world average: $\alpha_s(m_Z) = 0.1181 \pm 0.0011$)

.

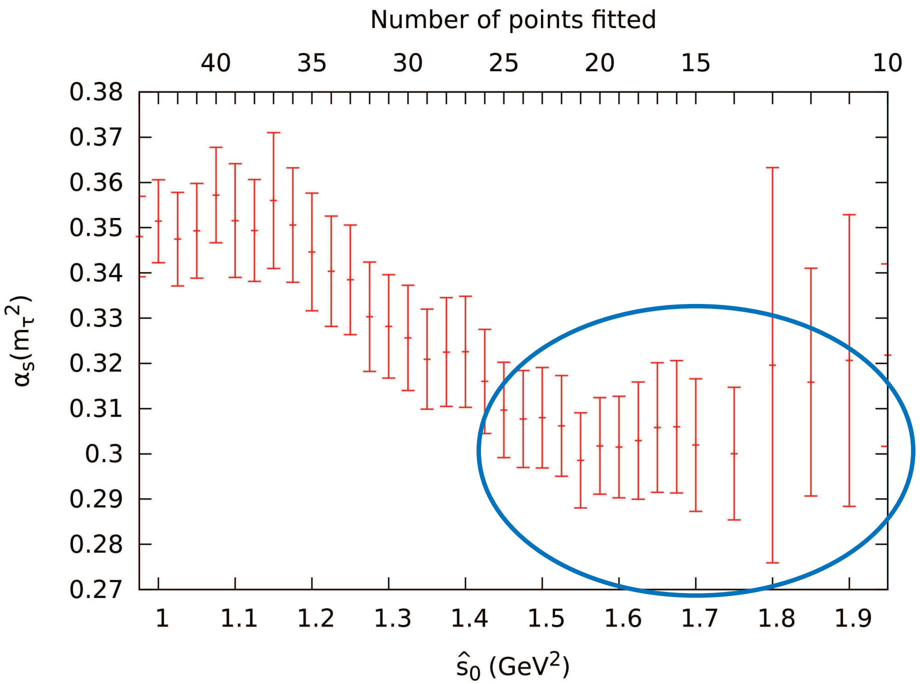
α_s *Overview*



Pich and Rodriguez-Sanchez '16

- New reanalysis with the Old Strategy $\Rightarrow$ same conclusions (not a surprise).
  (and same s_0 dependence problems)
  All criticisms have been addressed in Boito et al. Phys. Rev. **D95** (2017) 034024.

- Example: when DVs are included they obtain (FOPT)



$\Leftarrow$ Stability as $\hat{s}_0 = s_{min}$ increases,
as expected for the DV ansatz.
(So this is actually nice...)

<u>Question of Principle:</u>

If modeling DVs is useful for determining condensates in $V - A$

(as, e.g., in Glez-Alonso, Pich, Rguez-Sanchez '15,'16)

why wouldn't it be useful for α_s in $V + A$ as well ?

Towards a Theory of DVs ? (I)

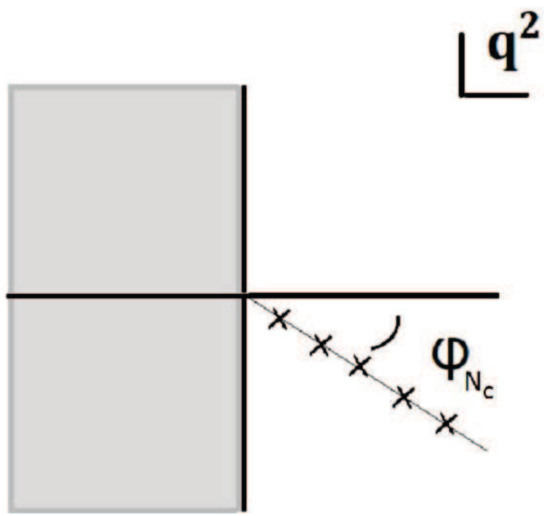
Boito, Caprini, Golterman, Maltman, SP '17

-A theory of DVs requires NP input.

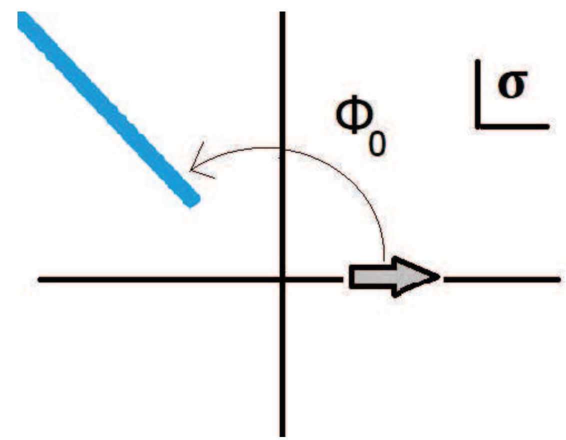
-Let's start with (exact !) Disp. Rel. of Adler's function ($q^2 < 0$, Euclidean):

$$\mathcal{A}(q^2) = -q^2 \int_0^\infty d\sigma\ \mathrm{e}^{\sigma q^2}\ \sigma \mathcal{B}(\sigma) \quad , \quad \mathcal{B}(\sigma) = \int_0^\infty dt\ \rho(t)\ \mathrm{e}^{-\sigma t}$$

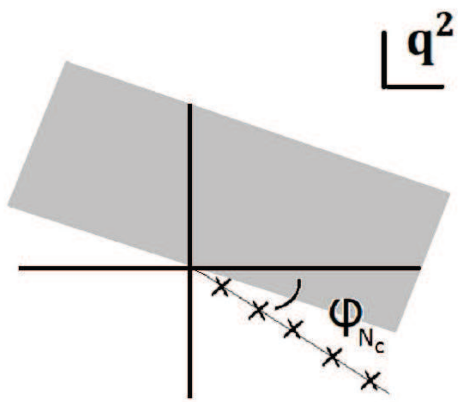
$\sigma q^2 = (\sigma > 0, q^2 < 0) = (\sigma < 0, q^2 > 0)$, i.e. rotate $\sigma \Leftrightarrow q^2$ analytic continuation.



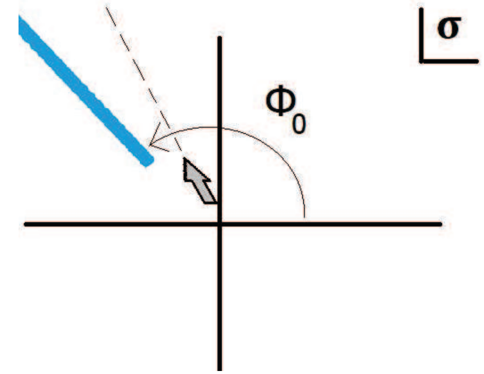
$\Leftrightarrow$

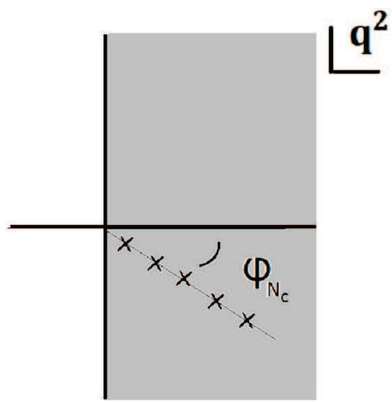


Towards a Theory of DVs ? (II)

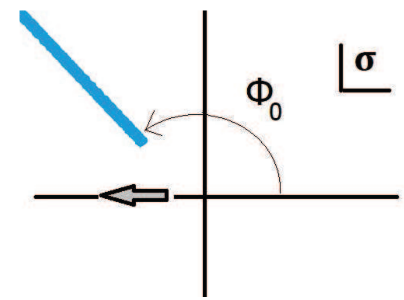


$\Leftrightarrow$

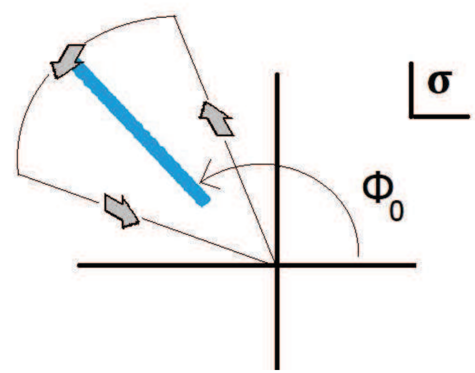




$\Leftrightarrow$



$\Pi_{DV}(q^2) \quad \sim$



Towards a Theory of DVs ? (III)

- Asymptotic Regge-like spectrum ($\Lambda_{QCD} = 1$, Regge slope) for $N_c = \infty$ in chiral limit:

-Masses

$$M^2(n) = n + b \log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right) \quad , \quad n \gg 1 \; .$$

-Decay constants

$$F(n) = 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right) \quad , \quad n \gg 1 \; .$$

- When $N_c = 3$ ($d = 2$ QCD, strings, phenomenology):

Blok et al. '98; Shifman et al. '08; Masjuan et al. '12

$$\varphi_{N_c} = -\frac{\Gamma}{M} \sim -\frac{a}{N_c} + \ldots$$

we have a branch point in σ plane at

$$\sigma_0 = 2\pi \; \mathrm{e}^{i\Phi_0} + \ldots \quad , \quad \Phi_0 = \frac{a}{N_c} + \frac{\pi}{2} + \ldots$$

Boito et al. '17

producing ($\mathrm{Re}\; q^2 > 0 \; , \; \mathrm{Im} q^2 > 0$)

$$\Pi_{DV}(q^2) \sim \mathrm{e}^{-2\pi q^2 \frac{a}{N_c}} \; \mathrm{e}^{i 2\pi (q^2 - c - b \log q^2)} \left(1 + \mathcal{O}\left(\frac{1}{N_c}; \frac{1}{q^2}; \frac{1}{\log q^2}\right)\right)$$

Towards a Theory of DVs ? (IV)

Agreement DVs in tau decay ⇔ Regge spectrum.

Fits of meson spectrum to radial trajectories:

Anisovich et al. '00; Klempt et al. '12; Masjuan et al. '12;

$$\Lambda^2 = 1.35(4)\ \mathrm{GeV}^2 \qquad , \qquad \frac{\Gamma}{M} = 0.12(8) \simeq \frac{a}{N_c}$$

leading to

$$\beta_V = \frac{2\pi}{\Lambda^2} = 4.7(2)\ \mathrm{GeV}^2 \qquad , \qquad \gamma_V = \frac{2\pi}{\Lambda^2}\frac{a}{N_c} = 0.6(4)\mathrm{GeV}^{-2}$$

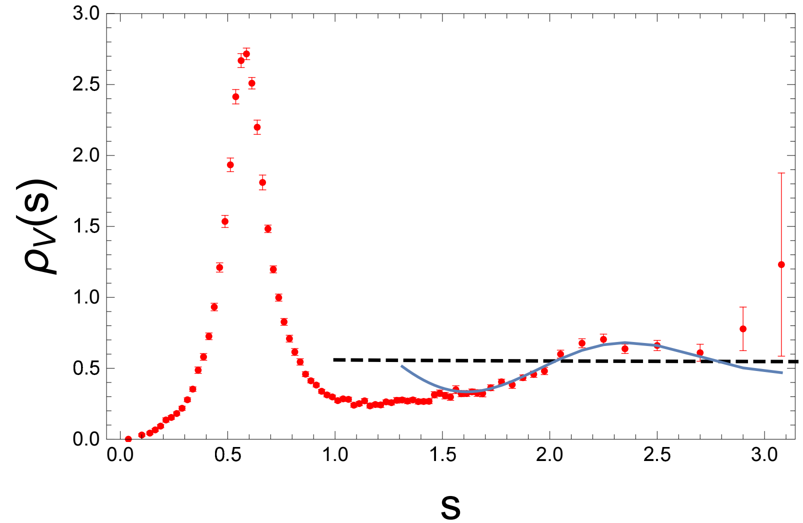
to be compared with results from fits to τ decay:

$$\beta_V = 4.2(5)\ \mathrm{GeV}^2 \qquad , \qquad \gamma_V = 0.7(3)\ \mathrm{GeV}^{-2}$$

coincidence ?,... Notice the 2π.

Conclusions and Outlook

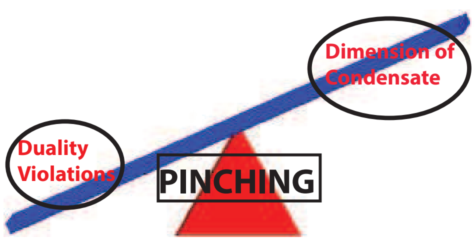
- DVs are clearly visible in the data.



(DVs are not a question of principle, they exist in practice.)

- Pinching does not allow a simultaneous reduction of DVs **and** higher-dim condensates

(unlike what has been assumed so far in the “Old Strategy" Method).



This introduced an unquantified systematic error.

Conclusions and Outlook (II)

• The new strategy using DV's passes all known tests, experimental and theoretical, performing better than the "Old Strategy".

• New theoretical framework to analyze DVs, based on analytical properties in the Borel-Laplace variable σ.

So far, analytic result for Π_{DV} based on asymptotic Regge espectrum. We think that this result is general, though (at least in the chiral limit).

Generalization to heavy quarks ?

•Better data (Babar and Belle ?) would help significantly.

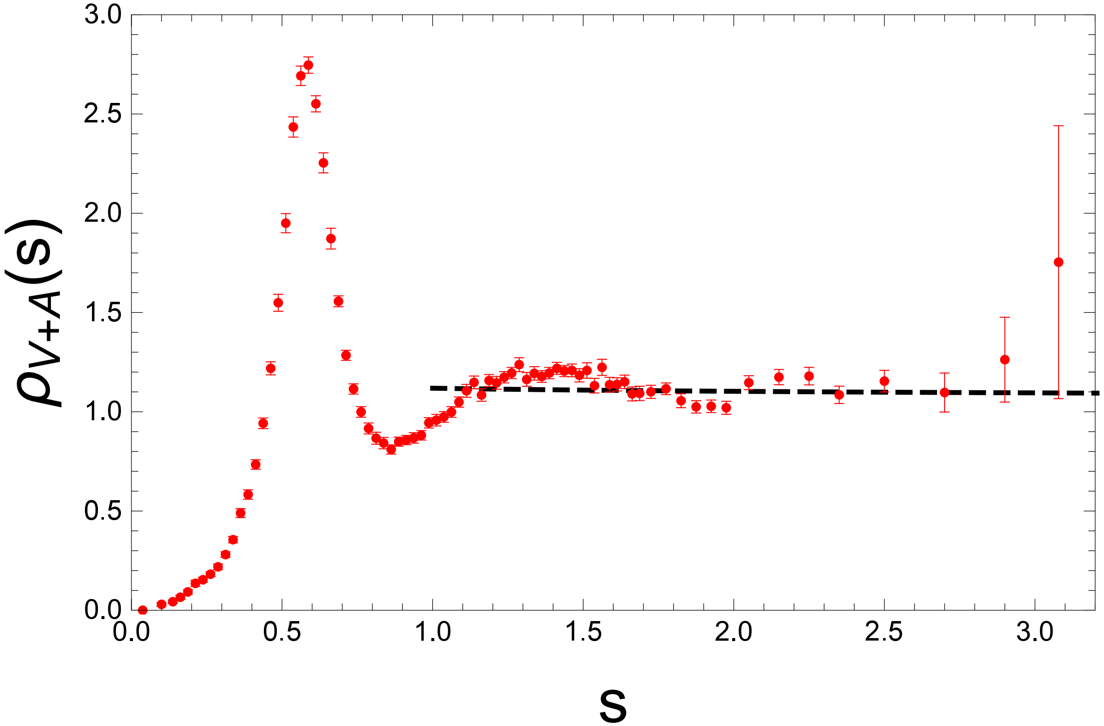
We are currently working on the analysis using e^+e^- data...

THANK YOU !

BACK-UP SLIDES

Spectral Functions (I)

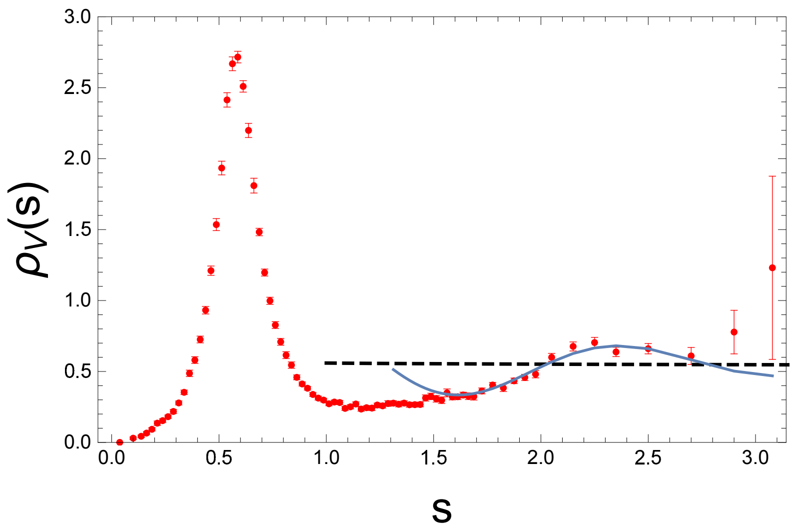
Sometimes $V + A$ is presented as "evidence" that DVs $\simeq 0$ at large s:

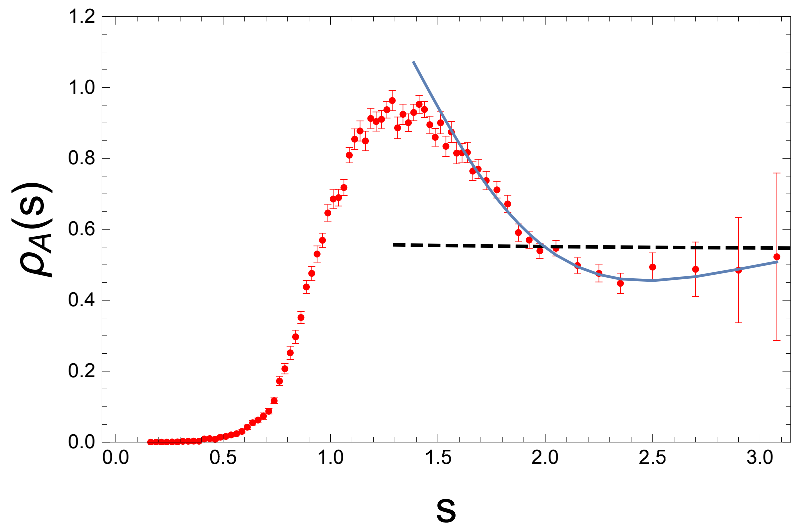


However...

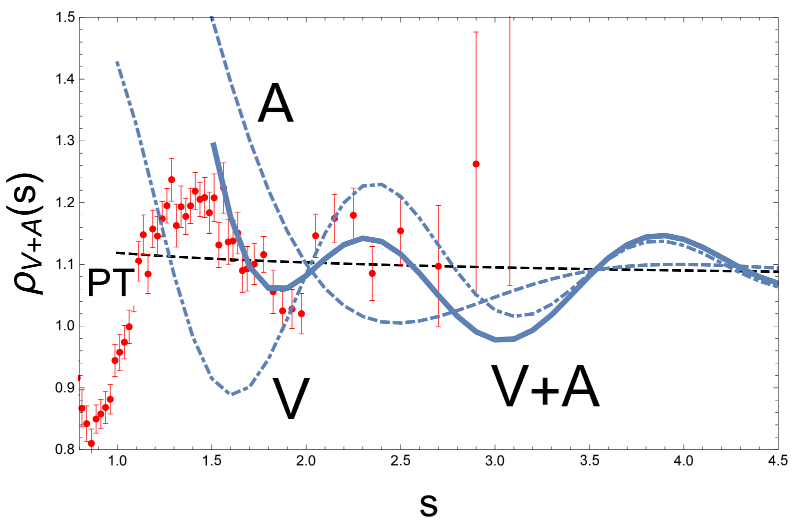
Spectral Functions (II)

A closer look reveals a (partial) fortuitous cancellation for $2\ \mathrm{GeV}^2 \lesssim s \lesssim 2.8\ \mathrm{GeV}^2$





There is no reason why it should persist at higher s:



The $V + A$ oscillation at $s = 3\ \mathrm{GeV}^2$ is **larger** than at $2.5\ \mathrm{GeV}^2$.

Comments on $V + A$

- DV oscillations are still present in $V + A$ (although of a smaller size than in V and A).

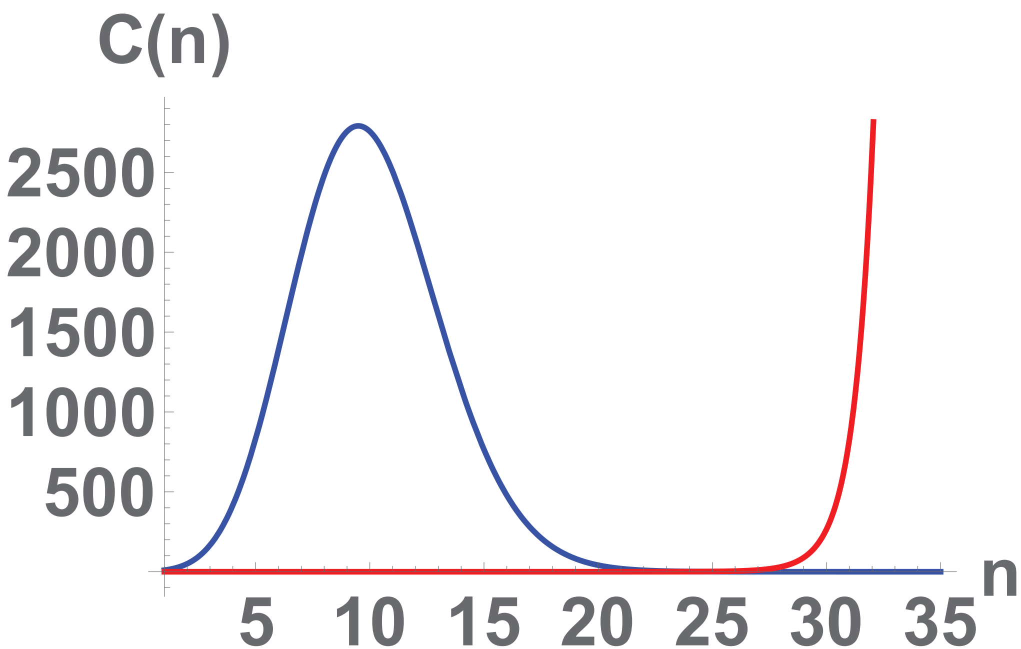
- Since we have a good representation of V and A, we also have it of their sum $V + A$.

- Fit results show that DV's exponent in A is $\ll$ than in V, so the reduction for $s \sim 2 - 3\ GeV^2$ is accidental. At still larger s, the DVs in V will dominate.

- The expectation of strong cancellation of $D = 6$ terms in OPE in $V + A$ is based on the vacuum saturation ansatz. However, the data shows that this ansatz is a very poor approximation.

Asymptotic vs. Convergent

$$f(z) = \sum_{n=1}^{\infty} C(n)\ z^n$$



$C(n) = \left(\frac{10^n}{n!}\right)$, Convergent.

$C(n) = \left(\frac{n!}{10^n}\right)$, Divergent.

Poincaré (circa 1893): discussion Geometers vs. Astronomers ⇔ Convergent vs. Divergent

OPE + DV ($N_c = \infty$)

-Asymptotic Regge behavior:

$$M^2(n) = n + b\log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right)$$

$$F(n) = 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right)$$

produces a Dirichlet series

$$\sigma\mathcal{B}(\sigma) = \sum_n^{\infty} F(n)\, \mathrm{e}^{-\sigma M^2(n)} = \underbrace{1 + \mathcal{O}\left(\log^{k_1}\sigma\right)}_{Pert.\ Theory} + \underbrace{\mathcal{O}\left(\sigma^{k_2}\log^{k_3}\sigma\right)}_{Power\ Corrections} + \underbrace{\mathcal{O}\left(\frac{1}{(\sigma-\hat{\sigma})^{1+\gamma}}\right)}_{Duality\ Violations}$$

and leads to

$$\mathcal{A}(q^2) = \underbrace{1 + \mathcal{O}\left(\frac{1}{\log^{p_1}(-q^2)}\right)}_{Pert.\ Theory} + \underbrace{\mathcal{O}\left(\frac{\log^{p_2}(-q^2)}{(q^2)^{p_3}}\right)}_{Power\ Corrections} + \underbrace{\mathcal{O}\left(\mathrm{e}^{\hat{\sigma}q^2}(-q^2)^{\gamma}\right)}_{Duality\ Violations}$$