The Standard Model Effective Field Theory and Dimension 6 Operators

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Outline

- The SMEFT
- Equations of Motion
- Operators
- Naive Dimensional Analysis
- $\mu \to e\gamma$ and MFV
- Holomorphy

Talk based on:

Rodrigo Alonso, Elizabeth Jenkins, Michael Trott, AM

- C. Grojean, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Scaling of Higgs Operators and $h \to \gamma\gamma$ Decay*, JHEP 1304:016 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, *On Gauge Invariance and Minimal Coupling*, JHEP 1309:063 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and λ Dependence*, JHEP 1310:087 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, *Naive Dimensional Analysis Counting of Gauge Theory Amplitudes and Anomalous Dimensions*, Phys. Lett. B726 (2013) 697-702.
- E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, JHEP 1401:035 (2014).
- R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, JHEP 1404:159 (2014).
- R. Alonso, E.E. Jenkins and A.V. Manohar, *Holomorphy without Supersymmetry in the Standard Model Effective Field Theory*, arXiV:1409.0868 [hep-ph].

SMEFT

A model independent way to include the effects of new physics. The assumption is that there are no new particles at the electroweak scale. This is what the data indicates.

Fields are the SM fields. $\langle H \rangle$ breaks $SU(2) \times U(1)$.

$$L = L_{SM} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

Need to include the $+\ldots$. Cannot just stop at a given dimension.

$$H = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}\left(v + h + i\varphi^0\right) \end{pmatrix}$$

φ^+, φ^0 give mass to W, Z, and h is the Higgs scalar.

$$H^\dagger H \; W^2 \to \left(\frac{1}{2}v^2 + hv + \frac{1}{2}h^2\right) W^2$$

Couplings of h fixed because it is part of H.

An alternate approach is to assume $SU(2) \times U(1) \to U(1)$ broken symmetry, i.e. a chiral field U, and a scalar field h, with arbitrary couplings.

Notation

Fields are three generations of fermions

$$\mathrm{L}: q_r,\ l_r, \qquad \mathrm{R}: u_r,\ d_r,\ e_r \qquad r = 1, \ldots, n_g = 3$$

the scalar doublet H, and $SU(3) \times SU(2) \times U(1)$ gauge fields.

$$L = L_{SM} + \frac{1}{\Lambda} L^{(5)} + \frac{1}{\Lambda^2} L^{(6)} + \ldots \quad \textit{note the dots}$$

Λ is the scale of new physics, and assume $\Lambda > v$

Power Counting

$$L^{(6)} \sim \frac{m_H^2}{\Lambda^2} \qquad L^{(8)} \sim \frac{m_H^4}{\Lambda^4} \qquad L^{(6)} \times L^{(6)} \sim \frac{m_H^4}{\Lambda^4}$$

m_H a typical electroweak scale (v, m_t, m_Z)

SMEFT: dim 5

$$L_5 = c_{rs} \left(H^{\dagger\, i}\, l_{i\alpha r} \right) \left(H^{\dagger\, j}\, l_{j\beta s} \right) \epsilon^{\alpha\beta}$$

$$\begin{aligned} i &: SU(2) \text{ index} \\ \alpha &: \text{Lorentz index} \\ r &: \text{flavor index} \end{aligned}$$

Violates lepton number $\Delta L = 2$. This is the operator that gives Majorana mass to the neutrino.

The scale is the seesaw scale and is high. Not relevant for physics at LHC.

SMEFT: dim 6

Lots of operators at dimension six. 59 operators that preserve baryon number, and 4/5 that violate baryon number.

Buchmuller and Wyler, Nucl.Phys. B268 (1986) 621

Grzadkowski et al. JHEP 1010 (2010) 085

Flavor indices run over $n_g = 3$ values.
2499 baryon number conserving operators, including flavor indices.
1350 CP-even and 1149 CP-odd operators.

Alonso, Jenkins, AM, and Trott, JHEP 1404 (2014) 159.

Notation:

$$\mathrm{L}: q_r,\ l_r, \qquad \mathrm{R}: u_r,\ d_r,\ e_r \qquad r = 1, \ldots, n_g = 3$$

$$H_j,\ \widetilde{H}_j = \epsilon_{ij} H^{\dagger\, j}$$

$$X_{\mu\nu}: G^A_{\mu\nu},\ W^I_{\mu\nu}, B_{\mu\nu}$$

Fierz Identities

Write down all possible gauge invariant operators of dimension 6.

Use Fierz identities:

$$\overline{\psi}_1\gamma^\mu P_L\psi_2\,\overline{\psi}_3\gamma^\mu P_L\psi_4 = \overline{\psi}_3\gamma^\mu P_L\psi_2\,\overline{\psi}_1\gamma^\mu P_L\psi_4$$

So in the four-quark operators:

$$\overline{q}_p^{\alpha i}\gamma^\mu q_{r\beta j}\,\overline{q}_s^{\lambda k}\gamma_\mu q_{t\sigma l} \to \overline{q}_s^{\lambda k}\gamma^\mu q_{r\beta j}\,\overline{q}_p^{\alpha i}\gamma_\mu q_{t\sigma l}$$

$$\overline{q}\,\gamma^\mu\Gamma\,q\,\overline{q}\,\gamma_\mu\Gamma\,q, \qquad \Gamma\otimes\Gamma \propto 1\otimes 1,\; T^A\otimes T^A,\; \tau^I\otimes\tau^I,\; T^A\tau^I\otimes T^A\tau^I$$

Two independent contractions:

$$Q_{qq}^{(1)} = \overline{q}_p\gamma^\mu q_r\,\overline{q}_s\gamma_\mu q_t \qquad 1\otimes 1$$

$$Q_{qq}^{(3)} = \overline{q}_p\gamma^\mu\tau^I q_r\,\overline{q}_s\gamma_\mu\tau^I q_t \qquad \tau^I\otimes\tau^I$$

not needed:

$$Q_{qq}^{(8)} = \overline{q}_p \gamma^\mu T^A q_r \, \overline{q}_s \gamma_\mu T^A q_t \qquad T^A \otimes T^A$$

$$Q_{qq}^{(3,8)} = \overline{q}_p \gamma^\mu \tau^I T^A q_r \, \overline{q}_s \gamma_\mu \tau^I T^A q_t \qquad \tau^I T^A \otimes \tau^I T^A$$

$$\left[T^A\right]^\alpha{}_\beta \left[T^A\right]^\lambda{}_\sigma = \frac{1}{2}\delta^\alpha_\sigma \, \delta^\lambda_\beta - \frac{1}{2N_c}\delta^\alpha_\beta \, \delta^\lambda_\sigma \qquad \propto 1 + \text{swap color}$$

$$\left[\tau^I\right]^i{}_j \left[\tau^I\right]^k{}_l = 2\delta^i_l \, \delta^k_j - \delta^i_j \, \delta^k_l \qquad \propto 1 + \text{swap weak}$$

$1 \leftrightarrow$ swap both   swap weak $\leftrightarrow$ swap color

SM Equations of Motion

$$D^2 H_k - \lambda v^2 H_k + 2\lambda (H^\dagger H) H_k + \overline{q}^j\, Y_u^\dagger\, u\, \epsilon_{jk} + \overline{d}\, Y_d\, q_k + \overline{e}\, Y_e\, l_k = 0\,,$$

for the Higgs field (mix H and fermion operators)

$$i\not{D}\, q_j = Y_u^\dagger\, u\, \widetilde{H}_j + Y_d^\dagger\, d\, H_j\,, \quad i\not{D}\, d = Y_d\, q_j\, H^{\dagger\, j}\,, \quad i\not{D}\, u = Y_u\, q_j\, \widetilde{H}^{\dagger\, j}\,,$$

$$i\not{D}\, l_j = Y_e^\dagger\, e H_j\,, \qquad i\not{D}\, e = Y_e\, l_j H^{\dagger\, j}\,,$$

for the fermion fields, and

$$[D^\alpha, G_{\alpha\beta}]^A = g_3\, j_\beta^A, \qquad [D^\alpha, W_{\alpha\beta}]^I = g_2\, j_\beta^I, \qquad D^\alpha B_{\alpha\beta} = g_1\, j_\beta,$$

for the gauge fields, where $[D^\alpha, F_{\alpha\beta}]$ is the covariant derivative in the adjoint representation.

Equations of Motion

Can use classical equations of motion in a quantum field theory.

No quantum corrections needed.

Green's functions can change, but the S-matrix does not.

The two versions of the operator do not agree diagram by diagram. Only the total contribution to the S-matrix is unchanged.

SMEFT Operators

- Leading higher dimension operators are $d = 6$.
- Assuming B and L conservation, there are 59 independent dimension-six operators which form complete basis of $d = 6$ operators.
- 59 operators divided into eight operator classes.

1 : X^3	2 : H^6	3 : $H^4 D^2$	4 : $X^2 H^2$
5 : $\psi^2 H^3$	6 : $\psi^2 X H$	7 : $\psi^2 H^2 D$	8 : ψ^4

$$X = G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu} \qquad \psi = q, l, u, d, e$$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

Dimension Six Operators

1 : X^3

Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

2 : H^6

Q_H	$(H^\dagger H)^3$

3 : $H^4 D^2$

$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

5 : $\psi^2 H^3 + \text{h.c.}$

Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$

4 : $X^2 H^2$

Q_{HG}	$H^\dagger H\, G^A_{\mu\nu} G^{A\mu\nu}$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W^I_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde W B}$	$H^\dagger \tau^I H\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$

6 : $\psi^2 X H + \text{h.c.}$

Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}$
Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde H\, G^A_{\mu\nu}$
Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde H\, W^I_{\mu\nu}$
Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde H\, B_{\mu\nu}$
Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) H\, G^A_{\mu\nu}$
Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I H\, W^I_{\mu\nu}$
Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

7 : $\psi^2 H^2 D$

$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar l_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar l_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar e_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar q_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar q_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar u_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar d_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p \gamma^\mu d_r)$

Dimension Six Operators

$8:(\bar{L}L)(\bar{L}L)$		$8:(\bar{R}R)(\bar{R}R)$		$8:(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8:(\bar{L}R)(\bar{R}L)+\text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8:(\bar{L}R)(\bar{L}R)+\text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

$$\psi^4 \to JJ,\ (\bar{L}R)(\bar{L}R),\ (\bar{L}R)(\bar{R}L)$$

In the broken phase:

$$H = \begin{bmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}\left(v + h + \varphi^0\right) \end{bmatrix}$$

$$X^2 H^2 : \quad h \to \gamma\gamma, h \to gg$$

$$\psi^2 X H : \quad \bar{l}_r \, \sigma_{\mu\nu} \, e_s \, F^{\mu\nu} \qquad\qquad \mu \to e\gamma$$

$$\psi^2 H^3 : \frac{\text{higgs coupling}}{\text{mass}} = \frac{3}{1}$$

Power Counting for the RGE

Amplitudes and anomalous dimensions obey power counting:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(6)} \propto C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(8)} \propto C^{(8)} + \left[C^{(6)}\right]^2$$

In the SM, because of the dimension two operator $H^\dagger H$, have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto C^{(4)} + m_H^2 C^{(6)} + \ldots$$

equivalently $m_H^2 \to v^2$

$\star$ SM parameter RG evolution affected.

Anomalous Dimension Matrix

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	g^2	0	0	1	0	0	0	0
H^6	2	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	3	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	4	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	5	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	6	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	7	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	8	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

Structure of anomalous dimension matrix. Jenkins, Trott, AM: 1309.0819

Many entries exist because of EOM

Naive Dimensional Analysis

Georgi, AM: NPB234 (1984) 189

Jenkins, Trott, AM: 1309.0819

Related recent work by Buchalla et al. arXiv:1312.5624

$$L = f^2\Lambda^2 \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^a \left(\frac{H}{f}\right)^b \left(\frac{yH}{\Lambda}\right)^c \left(\frac{D}{\Lambda}\right)^d \left(\frac{gX}{\Lambda^2}\right)^e, \qquad \Lambda = 4\pi f$$

NDA weight $w \equiv$ powers of f^2 in denominator.

$$L = f^2\Lambda^2 \left(\frac{H}{f}\right)^6 = \Lambda^2 \frac{(H^\dagger H)^3}{f^4}, \qquad w = 2$$

$$\gamma_{ij} \propto \left(\frac{\lambda}{16\pi^2}\right)^{n_\lambda} \left(\frac{y^2}{16\pi^2}\right)^{n_y} \left(\frac{g^2}{16\pi^2}\right)^{n_g}, \qquad N = n_\lambda + n_y + n_g$$

$$N = 1 + w_i - w_j$$

Familiar Example: $b \to s\gamma$

Use

$$O_q = \overline{b}\gamma^\mu P_L u \, \overline{u}\gamma_\mu P_L s \qquad\qquad w = 1$$

$$O_g = \frac{g}{16\pi^2} m_b \, \overline{b} \, \sigma^{\mu\nu} G_{\mu\nu} P_L s \qquad\qquad w = 0$$

Then

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{bmatrix} C_q \\ C_g \end{bmatrix} = \begin{bmatrix} L & L+1 \\ L-1 & L \end{bmatrix} \begin{bmatrix} C_q \\ C_g \end{bmatrix}$$

where L is the number of loops of the diagram.

Tree-Loop Mixing (incorrect) Claim

- Operators can be classified into “tree” and “loop”
- Based on some notion of “minimal coupling”
- The mixing of “tree” and “loop” operators vanishes at one loop

Bauer et al. for HQET provides a counterexample

γ_{68} as a counterexample

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{eB\\pr}} = \frac{1}{16\pi^2}\left[4g_1 N_c\,(\mathsf{y}_u+\mathsf{y}_q)C^{(3)}_{\substack{lequ\\prst}}\,[Y_u]_{ts}\right] + \ldots$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{eW\\pr}} = \frac{1}{16\pi^2}\left[-2g_2 N_c\,C^{(3)}_{\substack{lequ\\prst}}\,[Y_u]_{ts}\right] + \ldots$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{uB\\pr}} = \frac{1}{16\pi^2}\left[4g_1(\mathsf{y}_e+\mathsf{y}_l)\,C^{(3)}_{\substack{lequ\\stpr}}\,[Y_e]_{ts}\right] + \ldots$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{uW\\pr}} = \frac{1}{16\pi^2}\left[-2g_2 C^{(3)}_{\substack{lequ\\stpr}}\,[Y_e]_{ts}\right] + \ldots,$$

Class 6 are the magnetic dipole operators:

$$Q_{\substack{eB\\pr}} = (\bar{l}_{p,a}\sigma^{\mu\nu}e_r)H^a\,B_{\mu\nu}$$

Class 8 are four-fermion operators

$$\begin{aligned} Q^{(3)}_{lequ} &= (\bar{l}^j_p\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar{q}^k_s\sigma^{\mu\nu}u_t) = -4(\bar{l}^j_p e_r)\epsilon_{jk}(\bar{q}^{k\alpha}_s u_{\alpha t}) - 8(\bar{l}^j_p u_{\alpha t})\epsilon_{jk}(\bar{q}^{k\alpha}_s e_r) \\ &= -4Q^{(1)}_{lequ} - 8(\bar{l}^j_p u_{\alpha t})\epsilon_{jk}(\bar{q}^{k\alpha}_s e_r) \end{aligned}$$

Full 2499×2499 anomalous dimension matrix computed, including all Yukawa couplings, not just y_t.

Alonso, Jenkins, Trott, AM:

JHEP **1310** (2013) 087, JHEP **1401** (2014) 035, arXiv:1312.2014

Group theory can be quite complicated — the penguin graph gives an anomalous dimension 7 pages long.

Concentrate on a small part of the result.

There are some big numbers:

The evolution of the H^6 coefficient is

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_H = \frac{1}{16\pi^2} \left[108\, \lambda\, C_H - 160\, \lambda^2\, C_{H\Box} + 48\, \lambda^2\, C_{HD} \right] + \ldots$$

Independent of normalization of C_H.

For $m_H \sim 126$ GeV, $108\, \lambda / (16\pi^2) \approx 0.1$.

$X^2 H^2$ (Gauge-Higgs) Operators

Grojean, Jenkins, Trott, AM: JHEP 1304 (2013) 016

Look at only 4 operators (since there are 59 operators)

$C_{HW} \to c_W$, etc. rescaled by $g_i g_j/(2\Lambda^2)$

$$\mathcal{O}_G = \frac{g_3^2}{2\,\Lambda^2} H^\dagger\, H\, G^A_{\mu\,\nu} G^{A\,\mu\,\nu}, \qquad \mathcal{O}_B = \frac{g_1^2}{2\,\Lambda^2} H^\dagger\, H\, B_{\mu\,\nu} B^{\mu\,\nu},$$

$$\mathcal{O}_W = \frac{g_2^2}{2\,\Lambda^2} H^\dagger\, H\, W^a_{\mu\,\nu} W^{a\,\mu\,\nu}, \qquad \mathcal{O}_{WB} = \frac{g_1\, g_2}{2\,\Lambda^2} H^\dagger\, \tau^a\, H\, W^a_{\mu\,\nu} B^{\mu\,\nu},$$

Also CP-odd versions, which do not interfere with the SM amplitude.

When you expand them out in the broken phase, $H \to h + v$, get $h \to \gamma\gamma$, $h \to \gamma Z$ and $gg \to h$.

Considered these operators in an earlier work, and an explicit model that produced these operators.

AM, M.B. Wise, PLB636 (206) 107, PRD74 (2006) 035009

An exactly solvable model that produces only the O_W, O_{WB} and O_B operators and the W^3 operator: AM: PLB 726 (2013) 347

$$\mathcal{L} = \mathcal{L}_{\rm SM} + D_\mu S^{\dagger\alpha} D^\mu S_\alpha - V,$$

the usual Standard Model Lagrangian $\mathcal{L}_{\rm SM}$, the S_α, $\alpha = 1, \ldots, N$ kinetic energy term, and the potential

$$V = m_S^2 S^{\dagger\alpha} S_\alpha + \frac{\lambda_1}{N} H^\dagger H \, S^{\dagger\alpha} S_\alpha + \frac{\lambda_2}{N} H^\dagger \tau^a H \, S^{\dagger\alpha} \tau^a S_\alpha$$
$$+ \frac{\lambda_3}{N} S^{\dagger\alpha} S_\alpha \, S^{\dagger\beta} S_\beta + \frac{\lambda_4}{N} S^{\dagger\alpha} \tau^a S_\alpha \, S^{\dagger\beta} \tau^a S_\beta$$

$$c_W = \frac{(\lambda_1/\lambda_3)}{48 \log \frac{\Lambda_3^2}{\langle \Phi \rangle}}, \qquad c_B = \frac{(\lambda_1/\lambda_3)\, Y_S^2}{12 \log \frac{\Lambda_3^2}{\langle \Phi \rangle}}, \qquad c_{WB} = \frac{(\lambda_2/\lambda_4)\, Y_S}{24 \log \frac{\Lambda_4^2}{\langle \Phi \rangle}}$$

$$c_{W^3} = \frac{N\, g_2^3}{2880\pi^2 \langle \Phi \rangle}\,.$$

disproves claims that these are “loop-supressed” operators, and smaller than other contributions such as four-fermion operators.

All terms depend on RG invariant quantities.

Constraint on c_{WB} from the S parameter.

$$c_{WB} = -\frac{1}{8\pi}\frac{\Lambda^2}{v^2}S.$$

The $gg \to h$ amplitude gets contribution from c_G

For $h \to \gamma\gamma$,

$$c_{\gamma\gamma} = c_W + c_B - c_{WB}$$

For $h \to \gamma Z$,

$$c_{\gamma Z} = c_W \cot\theta_W - c_B \tan\theta_W - c_{WB} \cot 2\theta_W,$$

Contribution to the Higgs Decay Rate

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\rm SM}(h \to \gamma\gamma)} \simeq \left| 1 - \frac{4\pi^2 v^2 \ c_{\gamma\gamma}}{\Lambda^2 \ I^\gamma} \right|^2$$

and $I^\gamma \approx -1.64$. Note the $4\pi^2$.

Similar expressions for $gg \to h$ and $h \to \gamma Z$.

Brief Experimental Summary

ATLAS: $\mu_{\gamma\gamma} = 1.17 \pm 0.27$ CMS: $\mu_{\gamma\gamma} = 1.14 \pm 0.25$

Naive combination of these results (not recommended) gives

$$\mu_{\gamma\gamma} \simeq 1.155 \pm 0.18$$

If due to $c_{\gamma\gamma}$:

$$\frac{v^2}{\Lambda^2} c_{\gamma\gamma}(M_h) \simeq -0.086,\ 0.003 \pm 0.003$$

The second solution is preferred. The first solution is when $c_{\gamma\gamma}$ switches the sign of the standard model $h \to \gamma\gamma$ amplitude.

The experiments are sensitive to these effects if the new physics scale Λ is near a few TeV. (4.5 TeV)

Magnetic Dipole Operator

$$\mathscr{C}_{\substack{e\gamma\\rs}} = \frac{1}{g_1} C_{\substack{eB\\rs}} - \frac{1}{g_2} C_{\substack{eW\\rs}} \qquad \mathcal{L} = \frac{ev}{\sqrt{2}} \mathscr{C}_{\substack{e\gamma\\rs}} \, \overline{e}_r \sigma^{\mu\nu} P_R e_s \, F_{\mu\nu} + h.c.$$

where r and s are flavor indices ($\{e_e\,,\ e_\mu\,,\ e_\tau\} \equiv \{e\,,\ \mu\,,\ \tau\}$) and

$$\begin{aligned}
\dot{\mathscr{C}}_{\substack{e\gamma\\rs}} = & \left\{ Y(s) + e^2 \left(12 - \frac{9}{4} \csc^2\theta_W + \frac{1}{4}\sec^2\theta_W \right) \right\} \mathscr{C}_{\substack{e\gamma\\rs}} \\
& + 2 \mathscr{C}_{\substack{e\gamma\\rv}} [Y_e Y_e^\dagger]_{vs} + \left(\frac{1}{2} + 2\cos^2\theta_W \right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\substack{e\gamma\\ws}} + e^2 \left(12 \cot 2\theta_W\right) \mathscr{C}_{\substack{eZ\\rs}} \\
& - \left(2 \sin\theta_W \cos\theta_W\right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\substack{eZ\\ws}} - \cot\theta_W [Y_e^\dagger]_{rs} \left(C_{HWB} + i C_{H\widetilde{W}B} \right) \\
& + \frac{8}{3} e^2 [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma\gamma} + i \widetilde{\mathscr{C}}_{\gamma\gamma} \right) + e^2 \left(\cot\theta_W - \frac{5}{3}\tan\theta_W \right) [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma Z} + i \widetilde{\mathscr{C}}_{\gamma Z} \right) \\
& + 16 [Y_u]_{wv} C^{(3)}_{\substack{lequ\\rsvw}}
\end{aligned}$$

as corrected by Signer and Pruna, arXiv:1408:3565

The current experimental limit $\mathrm{BR}(\mu \to e\gamma) < 5.7 \times 10^{-13}$ from the MEG experiment implies

$$\frac{v}{\sqrt{2}\, m_e} \mathscr{C}_{\substack{e\gamma \\ \mu e}} \lesssim 2.7 \times 10^{-4}\ \mathrm{TeV}^{-2}$$

at the low energy scale $\mu \sim m_\mu$.

This bound implies

$$\frac{m_t}{m_e} C^{(3)}_{\substack{lequ \\ \mu ett}} \lesssim 1.4 \times 10^{-3}\ \mathrm{TeV}^{-2}$$

using the estimate $\ln(\Lambda/m_H)/(16\pi^2) \sim 0.01$ for the renormalization group evolution, and assuming that this term is the only contribution to $\mathscr{C}_{\substack{e\gamma \\ \mu e}}$ at low energies.

The anomalous magnetic moment of the muon is

$$\delta a_\mu = -\frac{4 m_\mu v}{\sqrt{2}} \mathrm{Re}\, \mathscr{C}_{\substack{e\gamma \\ \mu\mu}}$$

which yields the limits

$$|C_{HWB}| \lesssim 0.6\,\mathrm{TeV}^{-2}, \qquad |\mathscr{C}_{\gamma\gamma}| \lesssim 4\,\mathrm{TeV}^{-2}, \qquad \left|\frac{m_t}{m_\mu} \mathrm{Re}\, C^{(3)}_{\substack{lequ \\ \mu\mu tt}}\right| \lesssim 7\ \mathrm{TeV}^{-2},$$

assuming that each of these is the only contribution to $\mathscr{C}_{\substack{e\gamma \\ \mu\mu}}$.

The bound on the electric dipole moment of the electron translates to the limits

$$\left|C_{H\widetilde{W}B}\right| \lesssim 2 \times 10^{-4}\,\mathrm{TeV}^{-2}, \quad \left|\widetilde{\mathscr{C}}_{\gamma\gamma}\right| \lesssim 1 \times 10^{-3}\,\mathrm{TeV}^{-2}, \quad \left|\frac{m_t}{m_e} \mathrm{Im}\, C_{\substack{lequ \\ eett}}\right| \lesssim 2 \times 10^{-5}\ \mathrm{TeV}^{-}$$

using the recently measured upper bound

$$d_e < 8.7 \times 10^{-29}\, e\,\mathrm{cm}$$

from the ACME collaboration, again assuming each of these terms is the only contribution.

Minimal Flavor Violation

MFV based on the structure of flavor violation in the SM.

$$\otimes_i \;\; SU(3)_i \qquad i = Q, U, D, L, E \qquad\qquad Y_D \to U_Q Y_D U_D^\dagger$$

The RGE preserves MFV, since it is a symmetry

But if MFV is violated by dimension-six operators, then MFV feeds into all the sectors via the RGE

Allows one to test MFV in a model-independent way

MFV not compatible with GUTS. Accidental symmetry of the SM. Recent polarization B modes indicate a GUT scale.

Empirical Observation

$$\dot{\mathscr{C}}_{e\gamma} \propto \left(\mathscr{C}_{\gamma\gamma} + i\widetilde{\mathscr{C}}_{\gamma\gamma}\right),\ \mathscr{C}_{e\gamma}$$

no

$$\left(\mathscr{C}_{\gamma\gamma} - i\widetilde{\mathscr{C}}_{\gamma\gamma}\right),\ \mathscr{C}^{*}_{e\gamma}$$

$f(z)$ depends only on z but not z^*

Find this after adding all graphs and using the EOM. Individual contributions are not holomorphic, and only the total respects holomorphy.

- Observation: 1-loop anomalous dimension matrix respects holomorphy to a large extent.
- Using EOM, so equivalent to computing (on-shell) S-matrix elements.

Holomorphy

Recall $d = 6$ operator classes

1 : X^3	2 : H^6	3 : $H^4 D^2$	4 : $X^2 H^2$
5 : $\psi^2 H^3$	6 : $\psi^2 XH$	7 : $\psi^2 H^2 D$	8 : ψ^4

Divide $d = 6$ Operators into Holomorphic, Antiholomorphic and Non-Holomorphic Operators

$$X^{\pm}_{\mu\nu} = \frac{1}{2}\left(X_{\mu\nu} \mp i\widetilde{X}_{\mu\nu}\right), \qquad \widetilde{X}^{\pm}_{\mu\nu} = \pm i X^{\pm}_{\mu\nu},$$

$$\widetilde{X}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} X^{\alpha\beta}/2 \qquad \widetilde{\widetilde{X}}_{\mu\nu} = -X_{\mu\nu}$$

$$X \to X^{\pm}$$

$$\psi \to L, R$$

Complex self-duality condition in Minkowski space.

Holomorphy

Definition

The holomorphic part of the Lagrangian, $\mathcal{L}_{\mathfrak{h}}$, is the Lagrangian constructed from the fields X^+, R, $\overline{L}$, but none of their hermitian conjugates.

These transform as $(0, \frac{1}{2})$ or $(0, 1)$ under the Lorentz group, i.e. only under the $SU(2)_R$ part of $SU(2)_L \times SU(2)_R$.

Holomorphy

$$\mathcal{L}^{d=6} = \mathcal{L}_{\mathfrak{h}} + \mathcal{L}_{\overline{\mathfrak{h}}} + \mathcal{L}_{\mathfrak{n}} = C_{\mathfrak{h}} Q_{\mathfrak{h}} + C_{\overline{\mathfrak{h}}} Q_{\overline{\mathfrak{h}}} + C_{\mathfrak{n}} Q_{\mathfrak{n}}$$

$$Q_{\mathfrak{h}} \subset \left\{ X^{+3},\ X^{+2} H^2,\ (\overline{L} \sigma^{\mu\nu} R)\ X^{+} H,\ (\overline{L} R)(\overline{L} R) \right\}$$

$$Q_{\overline{\mathfrak{h}}} \subset \left\{ X^{-3},\ X^{-2} H^2,\ (\overline{R} \sigma^{\mu\nu} L)\ X^{-} H,\ (\overline{R} L)(\overline{R} L) \right\}$$

$$Q_{\mathfrak{n}} \subset \left\{ H^6,\ H^4 D^2,\ \psi^2 H^3,\ \psi^2 H^2 D,\ (\overline{L} R)(\overline{R} L),\ JJ \right\}$$

Some of the $\mathfrak{n}$ operators are complex.

Holomorphy

$$\dot{C}_i \equiv 16\pi^2 \mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_i = \sum_{j=\mathfrak{h},\bar{\mathfrak{h}},\mathfrak{n}} \gamma_{ij} C_j \; , \qquad\qquad i = \mathfrak{h}, \bar{\mathfrak{h}}, \mathfrak{n}$$

$$\begin{pmatrix} \gamma_{\mathfrak{h}\mathfrak{h}} & \gamma_{\mathfrak{h}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{h}\mathfrak{n}} \\ \gamma_{\bar{\mathfrak{h}}\mathfrak{h}} & \gamma_{\overline{\mathfrak{h}\mathfrak{h}}} & \gamma_{\bar{\mathfrak{h}}\mathfrak{n}} \\ \gamma_{\mathfrak{n}\mathfrak{h}} & \gamma_{\mathfrak{n}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{pmatrix}$$

$$\gamma_{\mathfrak{h}\mathfrak{h}} = \gamma_{\overline{\mathfrak{h}\mathfrak{h}}} \qquad\qquad \gamma^*_{\mathfrak{h}\bar{\mathfrak{h}}} = \gamma_{\bar{\mathfrak{h}}\mathfrak{h}} \qquad\qquad \gamma^*_{\mathfrak{h}\mathfrak{n}} = \gamma_{\bar{\mathfrak{h}}\mathfrak{n}}$$

only need to look at:

$$\left(\begin{array}{cc|c} \gamma_{\mathfrak{h}\mathfrak{h}} & \gamma_{\mathfrak{h}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{h}\mathfrak{n}} \\ \hline \gamma_{\mathfrak{n}\mathfrak{h}} & \gamma_{\mathfrak{n}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{array}\right)$$

- 0: Vanishes by NDA, i.e. NDA gives a negative loop order
- $\nexists$: There is no one-loop diagram (including from EOM)
- $\mathfrak{h}_F$: Holomorphic. Nonholomorphic terms forbidden by NDA and flavor symmetry
- $\to 0$: Vanishes by explicit computation, after adding all contributions. Individual graphs need not vanish.
- $\mathfrak{h}$: Holomorphic, by explicit computation
- $*$: Non-zero

$$Q_{eW} = (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu} \qquad Q^{(1)}_{lequ} = (\bar{l}^j_p e_r)\epsilon_{jk}(\bar{q}^k_s u_t)$$

$$\dot{C}_{eW} \propto Y_e Y_e Y_u^\dagger C^{(1)*}_{lequ}$$

but NDA says only order y^2.

Holomorphy

	$(X^+)^3$	$(X^+)^2H^2$	ψ^2X^+H	$(\bar{L}R)(\bar{L}R)$	$(\bar{L}R)(\bar{R}L)$	JJ	ψ^2H^3	H^6	H^4D^2	ψ^2H^2D
$(X^+)^3$	$\mathfrak{h}$	$\to 0$	0	0	0	0	0	0	0	0
$(X^+)^2H^2$	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}$	0	0	$\nexists$	0	0	$\to 0$	$\to 0$
ψ^2X^+H	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}_F$	$\to 0$	$\to 0$	$\to 0$	0	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{L}R)$	$\to 0$	$\nexists$	$\mathfrak{h}_F$	$\mathfrak{h}_F$	$Y_u^\dagger Y_{e,d}^\dagger$	$Y_u^\dagger Y_{e,d}^\dagger$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{R}L)$	$\to 0$	$\nexists$	$\to 0$	$Y_uY_d,\ Y_u^\dagger Y_e^\dagger$	$\mathfrak{h}_F$	$\ast$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
JJ	$\to 0$	$\nexists$	$\to 0$	$Y_uY_{e,d}$	$\ast$	$\ast$	$\nexists$	$\nexists$	$\nexists$	$\ast$
ψ^2H^3	$\to 0$	$Y_{u,d,e}^\dagger$	$\mathfrak{h}$	$\mathfrak{h}$	$\ast$	$\ast$	$\ast$	$\nexists$	$\ast$	$\ast$
H^6	$\to 0$	$\ast$	$\nexists$	$\nexists$	$\nexists$	$\nexists$	$\ast$	$\ast$	$\ast$	$\ast$
H^4D^2	$\to 0$	$\to 0$	$\to 0$	$\nexists$	$\nexists$	$\nexists$	$\to 0$	$\nexists$	$\ast$	$\ast$
ψ^2H^2D	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$\ast$	$\to 0$	$\nexists$	$\ast$	$\ast$

- The 1 1 block is holomorphic

$$\gamma_{\mathfrak{h}\bar{\mathfrak{h}}} = 0$$

- The 1 2 block vanishes except for the red terms proportional to $Y_u Y_e$ or $Y_u Y_d$.

$$\mathcal{L}_Y = -\overline{q}^j\, Y_d^\dagger\, d\, H_j - \overline{q}^j\, Y_u^\dagger\, u\, \widetilde{H}_j - \overline{l}^j\, Y_e^\dagger\, e\, H_j + \text{h.c.}$$
$$\widetilde{H}_j = \epsilon_{ij} H^{\dagger j}$$

- $\psi^2 H^3$ behaves to some extent like a holomorphic operator.
- one entry $*$ present even if Yukawa couplings set to zero.

RGE of SM parameters

Recall that

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto m_H^2 C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tau = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \left(\frac{4\pi}{g_X^2} - i \frac{\theta_X}{2\pi} \right) = \frac{2 m_H^2}{\pi g_X^2} C_{HX,+}$$

where θ-terms are normalized as $\mathcal{L} \supset (\theta_X g_X^2 / 32\pi^2) X \widetilde{X}$ and $X \in \{SU(3), SU(2), U(1)\}$.

τ is the SUSY holmorphic gauge coupling

* Entry: Some Numerology

$$\dot{C}_H = -3g_2^2\left(g_1^2+3g_2^2-12\lambda\right)\mathrm{Re}(C_{HW,+})$$
$$-3g_1^2\left(g_1^2+g_2^2-4\lambda\right)\mathrm{Re}(C_{HB,+})$$
$$-3g_1g_2\left(g_1^2+g_2^2-4\lambda\right)\mathrm{Re}(C_{HWB,+})+\ldots$$

The $C_{HB,+}$ and $C_{HWB,+}$ terms vanish if $g_1^2+g_2^2=4\lambda$:

$$m_H^2 = 2m_Z^2 = (129\,\mathrm{GeV})^2\,,$$

and the $C_{HW,+}$ term vanishes if $g_1^2+3g_2^2=12\lambda$:

$$m_H^2 = \frac{2}{3}m_Z^2+\frac{4}{3}m_W^2 = (119\,\mathrm{GeV})^2\,,$$

If $g_1^2+g_2^2=4\lambda$:

$$\dot{C}_H = 6g_1^2g_2^2\,\mathrm{Re}(C_{HW,+})+\ldots$$

No explanation for this at present.

Summary

- Complete RGE of dimension-six operators of SM EFT computed for first time.
- Contribution of dimension-six operators to running of SM parameters calculated for first time.
- RG evolution of dimension-six operators important for Higgs boson production $gg \to h$ and decay $h \to \gamma\gamma$ and $h \to Z\gamma$, which occur at one loop in SM.
- Significant constraints on flavor structure of SM EFT. Test of MFV hypothesis.
- Approximate holomorphy of 1-loop anomalous dimension matrix of dimension-six operators.
  - Does it hold in a more general gauge theory?
  - Does any of it extend beyond one loop?