C-Parameter with Massive Quarks

Moritz Preisser

in collaboration with

Vicent Mateu and André Hoang

universität wien

Teilchenphysikseminar Universität Wien
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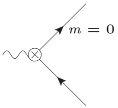
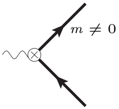
Motivation

- look at $e^+e^- \to$ hadrons
- *event shapes* e.g. thrust, heavy jet mass and **C-parameter** quantify geometric shape of the final state's 3-momentum distribution
  - well suited for many particles in the final state
- event-shape studies in QCD (since the 1980s)
- event-shape studies in Soft-Collinear Effective Theory (SCET) $\alpha_S(m_Z)$ determinations:
  - Thrust including mass effects of primary bottom quarks [Abbate et al., 2011]
  - C-parameter not including mass effects (w.i.p.)
- **goal**: clarify the role of primary quark mass-effects in C-parameter studies



vs.



Outline

Introduction

C-Parameter

- late 1970s: Introduction of the *linearized momentum tensor* $\rightarrow$ IRC-safe
  p_i the i-th final state particle momentum [Parisi, 1978, Donoghue et al., 1979]

$$\theta^{kl} = \frac{1}{\sum_j |\vec{p}_j|} \sum_i \frac{p_i^k p_i^l}{|\vec{p}_i|} \qquad \text{with eigenvalues} \qquad \lambda_1, \lambda_2, \lambda_3$$

- C-parameter definition: look at characteristic polynomial [Ellis et al., 1981]

$$A\lambda^3 - B\lambda^2 + C\tfrac{1}{3}\lambda - D\tfrac{1}{27} = 0 \qquad \rightarrow \qquad A = 1\,,\ B = \mathrm{Tr}[\theta] = 1$$

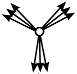
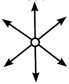
in terms of the eigenvalues

$$C = 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)$$
$$D = 27(\lambda_1\lambda_2\lambda_3)$$

in terms of particle momenta

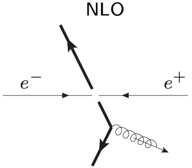
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$

<u>typical C values:</u>

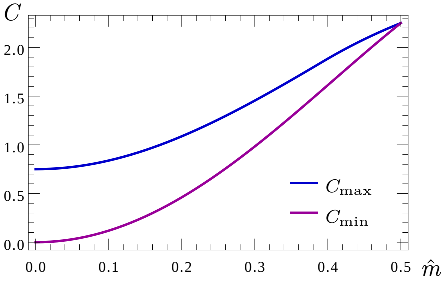
dijet	$C \sim 0$	
three-jet	$C \sim \frac{3}{4}$	
spherical multi-jet	$C \sim 1$	

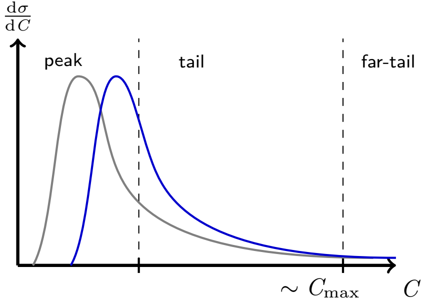
- define massive C-parameter → coincides for the massless case [Gardi and Magnea, 2003] with c.o.m. energy Q

$$C = \frac{3}{2}\left[2 - \frac{1}{Q^2}\sum_{i\neq j}\frac{(p_i\cdot p_j)^2}{p_i^0 p_j^0}\right]$$



- NLO: final state $\overline{Q}Qg$ → dijet and three-jet events
  $C_{\rm min}$ & $C_{\rm max}$ depend on $\hat{m} = m/Q$





C-Parameter in QCD

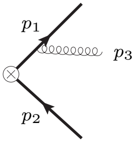
Massless C-Parameter in QCD

- total cross-section given by: $\mathcal{J}_i^\mu = \overline{\psi}\Gamma_i^\mu\psi$

$$\sigma = \sum_X (2\pi)^4 \delta^{(4)}(q - P_X) \sum_{i=a,v} L^i_{\mu\nu} \langle 0| \mathcal{J}_i^{\nu\dagger} |X\rangle \langle X| \mathcal{J}_i^\mu |0\rangle$$

- NLO: double differential cross-section, using $x_i = \frac{2p_i^0}{Q}$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{ml}}_{\mathrm{rad}}}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$



- differential cross-section in integral form [Ellis et al., 1981]
  $\rightarrow$ elliptic integral

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{ml}}_{\mathrm{rad}}}{\mathrm{d}C} = \int_f^g \mathrm{d}x \, \frac{r(C,x)}{\sqrt{(x-e)(x-f)(x-g)(x-h)}}$$

- use representation in Legendre normal form [Gardi and Magnea, 2003]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{ml}}_{\mathrm{rad}}}{\mathrm{d}C} = R(C) + k(C)\mathbf{K} + m(C)\mathbf{E} + n(C)\mathbf{\Pi}$$

  $\rightarrow$ "easy" to expand for $C \sim 0$ and numerically well behaved

- massless NLO result → singular terms [Catani and Webber, 1998]

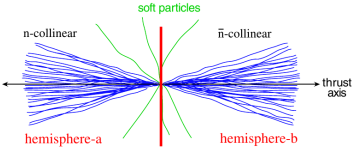
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{rad}}}{\mathrm{d}C} = \frac{\alpha_s C_F}{2\pi}\left[-\frac{3+4\ln(\frac{C}{6})}{C}+\ldots\right]+\mathcal{O}(\alpha_s^2)$$

look at *cumulant distribution*: $\Sigma(C) = \int_0^C \mathrm{d}C' \,\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}C'}$ → Sudakov double log

$$\Sigma_{\mathrm{rad}}(C) = \frac{\alpha_s C_F}{2\pi}\left[-2\ln^2(\tfrac{C}{6})-3\ln(\tfrac{C}{6})+\ldots\right]+\mathcal{O}(\alpha_s^2)$$

→ resummation of large logs needed

- logs of ratios of characteristic scales
  - hard scale $\sim Q$
  - jet scale $\sim Q\sqrt{\frac{C}{6}}$
  - soft scale $\sim Q\frac{C}{6}$



Ref.: [Fleming et al., 2008]

- log-counting: $\ln \sim \alpha_s^{-1}$

$$\ln(\Sigma) \sim \ln\sum_{i=0}(\alpha_s\ln)^{i+1} + \sum_{i=0}(\alpha_s\ln)^{i+1} + \alpha_s\sum_{i=0}(\alpha_s\ln)^{i} + \ldots + \text{non-sing}$$

LL NLL N^2LL

C-Parameter in SCET

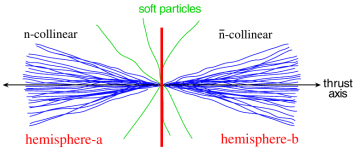
Soft-Collinear Effective Theory (SCET)

- Effective Field Theory (EFT) of QCD which is constructed for jet situation
  [Bauer et al., 2000, Bauer et al., 2001, Bauer and Stewart, 2001, Bauer et al., 2002a, Bauer et al., 2002b]
  consider hierarchy: $Q^2 \gg m_{jet}^2 \gg \mu_S^2 \gg \Lambda_{\rm QCD}^2$
  $\rightarrow$ power counting parameter: $\lambda \sim \frac{m_{jet}}{Q} \sim \sqrt{\frac{C}{6}}$
- use light cone coordinates $\rightarrow n^\mu = (1,0,0,-1)$ & $\bar{n}^\mu = (1,0,0,1)$

$$p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu \qquad \rightarrow \qquad (p^+, p^-, p_\perp) = (\bar{n}\cdot p, n\cdot p, |\vec{p}_\perp|)$$

- look at the *dijet limit* and identify the relevant modes

mode	$p^\mu = (+,-,\perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	-
n-coll.	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
$\bar{n}$-coll.	$Q(1,\lambda^2,\lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$
soft	$Q(\lambda^2,\lambda^2,\lambda^2)$	$Q^2\lambda^4$	A_{us}



Ref.: [Fleming et al., 2008]

- integrate out modes which are far off-shell

- associate effective field operators with relevant modes
- expand Lagrangian in terms of effective fields to leading order in λ
  $\rightarrow$ SCET Feynman rules

- restrict sum to dijet final states $\rightarrow$ SCET applicable
  replace full theory with effective field theory current $\rightarrow$ singular contributions

$$\sigma = \sum_X^{\text{res}} (2\pi)^4 \delta^{(4)}(q - P_X) \sum_{i=a,v} L^i_{\mu\nu} \langle 0 | J_i^{\nu\dagger} | X \rangle \langle X | J_i^\mu | 0 \rangle + \text{non-singular}$$

- need matching between full and effective field theory operators
  would naively expect $\rightarrow$ but not correct!

$$\mathcal{J}_i^\mu = \overline{\psi} \Gamma_i^\mu \psi \quad \rightarrow \quad J_i^\mu = \int \mathrm{d}\omega \, \mathrm{d}\overline{\omega} \; C(\omega, \overline{\omega}) \, \overline{\xi}_{n,\omega} \Gamma_i^\mu \xi_{\bar{n},\overline{\omega}}$$

with $\Gamma_v^\mu = \gamma^\mu, \; \Gamma_a^\mu = \gamma^\mu \gamma_5$

- correct replacement for the current

$$\mathcal{J}_i^\mu = \overline{\psi}\Gamma_i^\mu \psi \quad \to \quad J_i^\mu = \int \mathrm{d}\omega \, \mathrm{d}\overline{\omega} \; C(\omega, \overline{\omega}) \, \overline{\xi}_{n,\omega}^{(0)} \, W_n \, Y_n^\dagger \Gamma_i^\mu \, Y_{\bar{n}} \, W_{\bar{n}}^\dagger \xi_{\bar{n},\overline{\omega}}^{(0)}$$

- need collinear Wilson lines $W_{n(\bar{n})}$ to preserve collinear gauge invariance
- decouple soft gluons from collinear particles $\to$ usoft Wilson lines $Y_{n(\bar{n})}$

- can factorize SCET cross-section into matrix elements containing only collinear or soft dynamics
- additionally one can show that

$$C^{\text{dijet}} = C_n + C_{\bar{n}} + C_s + \mathcal{O}(\lambda^4)$$

and with this derive a factorization theorem for the differential C-parameter cross-section

Factorization

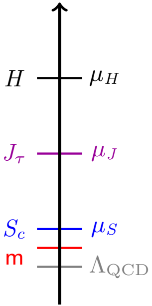
- factorization theorem for the differential C-parameter cross-section

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}C} = 6Q\, H(Q,\mu) \int \mathrm{d}s\; J_\tau(s,m,\mu)\; S_C\big(\tfrac{Q}{6}(C - C_{\min}) - \frac{s}{Q},\mu\big)$$

$$+ \text{ non-singular} + \text{power corrections}$$

- each factor accounts for dynamics at a different scale
  - hard scale $\mu_H \sim Q$
  - jet scale $\mu_J \sim Q\sqrt{\frac{1}{6}(C - C_{\min})}$
  - soft scale $\mu_S \sim \frac{Q}{6}(C - C_{\min})$
- consider hierarchy with mass as the lowest perturbative scale

$\mu_H > \mu_J > \mu_S > m \gg \Lambda_{\mathrm{QCD}}$

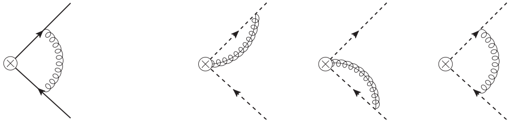


Hard Function

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}C} = 6Q\; H(Q,\mu) \int \mathrm{d}s\; J_\tau(s,m,\mu)\; S_C\left(\tfrac{Q}{6}(C - C_{\min}) - \frac{s}{Q}, \mu\right)$$

+ non-singular + power corrections

- hard function contains dynamics at the hard scale → physics of the hard process given by the square of the SCET matching coefficient
- universal for all e^+e^- event-shapes
- no primary quark mass-effect
- hard-function contains the log: $\ln\left(\frac{Q^2}{\mu^2}\right)$



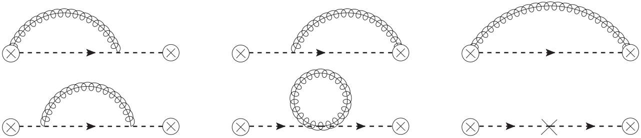
Jet Function

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}C} = 6Q\, H(Q,\mu) \int \mathrm{d}s\; J_\tau(s,m,\mu)\; S_C\big(\tfrac{Q}{6}(C - C_{\min}) - \frac{s}{Q},\mu\big)$$

$+$ non-singular $+$ power corrections

- jet function accounts for dynamics at the jet scale $\rightarrow$ dynamics of collinear quarks and gluons within the jet(s)
- corrections due to primary quark mass-effects already at one-loop
- the same as for thrust
- jet-function contains two types of logs: roughly the same for small $\hat{m}$

$$\ln\left(\frac{Q^2(C - C_{\min})}{\mu^2}\right) \qquad \& \qquad \ln\left(\frac{Q^2(C - C_{\min} + \hat{m}^2)}{\mu^2}\right)$$

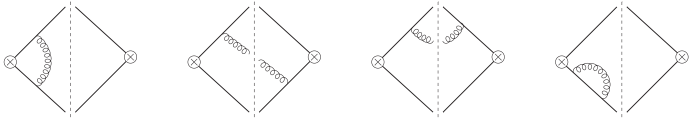


Soft Function

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}C} = 6Q\ H(Q,\mu)\int \mathrm{d}s\ J_\tau(s,m,\mu)\ S_C\big(\tfrac{Q}{6}(C-C_{\min}) - \frac{s}{Q},\mu\big)$$

$+$ non-singular $+$ power corrections

- soft function accounts for dynamics at the soft scale $\rightarrow$ dynamics of soft radiation and soft cross-talk between the jets
- no primary quark mass-effects $\rightarrow$ secondary quark mass-effects at $\mathcal{O}(\alpha_s^2)$
- soft-function contains the log: $\ln\left(\frac{Q(C-C_{\min})}{\mu}\right)$



...

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}C} = 6Q\; H(Q,\mu)\int \mathrm{d}s\; J_\tau(s,m,\mu)\; S_C\big(\tfrac{Q}{6}(C - C_{\min}) - \frac{s}{Q},\mu\big)$$

$+$ non-singular $+$ power corrections

- SCET reproduces singular part of the cross-section
- include non-singular parts → different for vector and axial-vector

$$\frac{\mathrm{d}\sigma^i_{\mathrm{ns}}}{\mathrm{d}C} = \frac{\mathrm{d}\sigma^i_{\mathrm{QCD}}}{\mathrm{d}C} - \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}C} \qquad\qquad i = a, v$$

- include non-perturbative effects (hadronization effects) by convoluting with a shape function

$$\frac{\mathrm{d}\sigma}{\mathrm{d}C} = \left(\frac{\mathrm{d}\sigma_{\mathrm{s}}}{\mathrm{d}C} + \frac{\mathrm{d}\sigma_{\mathrm{ns}}}{\mathrm{d}C}\right) \otimes S_{\mathrm{mod}}$$

Large Log Resummation in SCET

- **how** is this useful for dealing with large logarithms?
- evolve each cross-section factor to common scale μ by using its RG evolution
- each cross-section factor has non-trivial anomalous dimension
  - $\gamma_H(Q,\mu) = \Gamma_H^{\rm cusp}[\alpha_s]\ln(\frac{Q^2}{\mu^2}) + \gamma_H[\alpha_s]$
  - $\tilde{\gamma_F}(y,\mu) = \Gamma_F^{\rm cusp}[\alpha_s]\ln(iy'\mu) + \gamma_F[\alpha_s]$ with $F = J, S$
- introduce additional scales μ_H, μ_J, μ_S
- evolution kernel for corresponding cross-section factor
  - $H(Q,\mu) = H(Q,\mu_H)\,U_H(Q,\mu_H,\mu)$
  - $\tilde{F}(y,\mu) \;=\tilde{U}_F(y,\mu,\mu_F)\tilde{F}(y,\mu_F)$ with $F = J, S$

Large Log Resummation in SCET

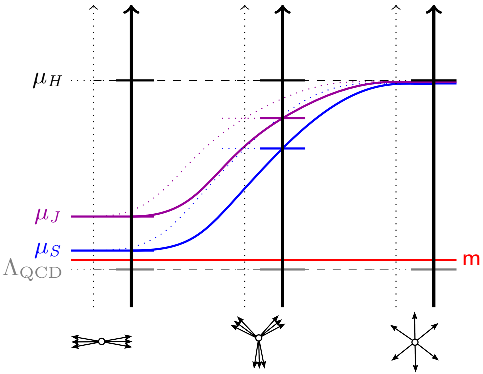
- this method allows to resum logs of order N^{n}LL' by calculating
  - cusp anomalous dimension to order $(n+1) \rightarrow$ resum Sudakov double logs
  - non-cusp anomalous dimension to order $n \rightarrow$ resum single logs
  - matrix elements (ME) to order n

- resum large logs $\sim \ln(\frac{\mu}{\mu_F})$ but ME still involve logs $\sim \ln(\frac{\mu_F}{\text{char. scale}})$
- Choice of μ_F should minimize logs in ME

  $\rightarrow \mu_F \sim$ characteristic scale of F which is in general C-dependent

Profile Functions

- introduce *profile functions* $\mu_F(C) \to$ use the massless ones

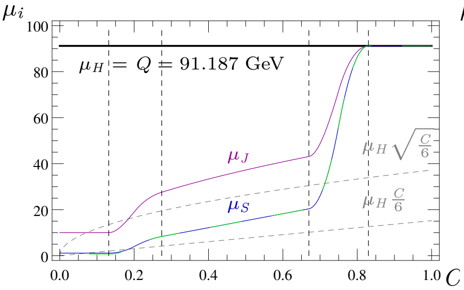


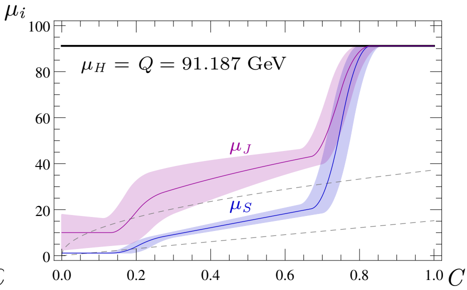
- generalize massless profile functions $\to$ recall shift of $C_{\min}$ & $C_{\max}$
- rescale massless profile functions to fit the massive case

Profile Functions

- take massless profile functions → generalization to massive case ✓
- theoretical error can be estimated through profile variations

  → convergence of the cross-section: N^{k}LL vs. N^{k+1}LL



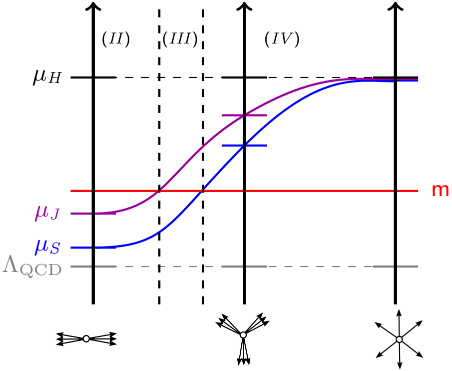


- where is current description **applicable**?

Applicability

- the considered scale hierarchy is not always valid

  for big m or small Q other hierarchies have to be considered



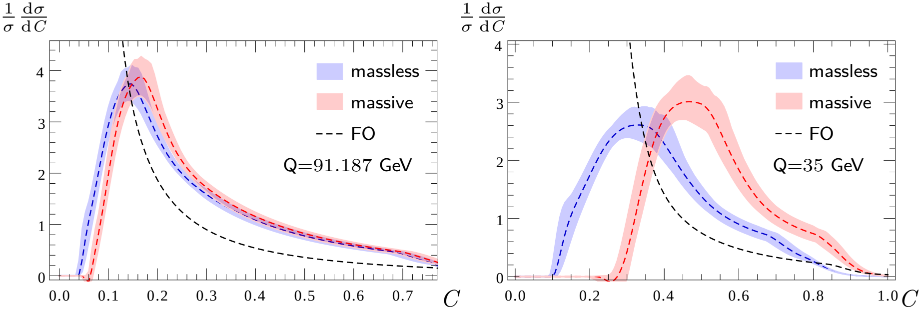
- implement general setup to treat different scale hierarchies [Pietrulewicz et al., 2014]
  - (III) modifies evolution of cross-section factors
  - $(II)/(III)$ bHQET region: unresummed large log in the jet-function

Applicability

- massive case: current description valid for scenario (IV)
- hadronization effects are implemented by convoluting with a shape function
  $\rightarrow$ strictly valid only in (far-)tail region
- **overall**: cross-section shows correct qualitative behaviour
  but strictly only valid in (far-)tail region

Results & Outlook

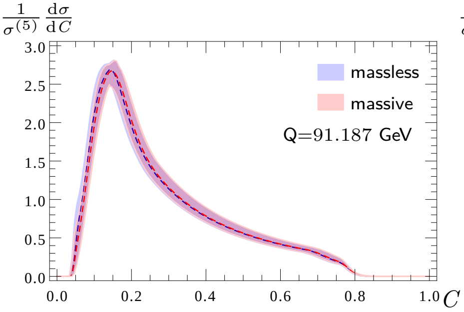
- NLL' resummation with $\overline{m}_b(\overline{m}_b) = 4.2$ GeV

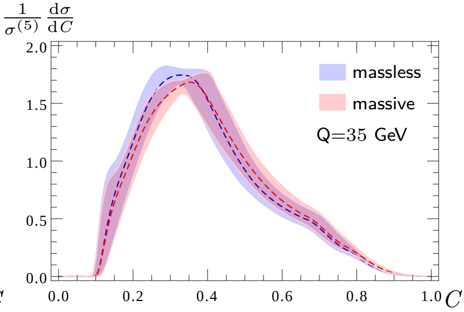


- big effect for large $\hat{m} = m/Q \rightarrow$ shift + shape change
- can be used to extract m_b from LEP and JADE data $\rightarrow$ data should be reanalyzed
- theoretically clean description of $\hat{m}$-sensitive observable
  $\rightarrow$ can be used to study m_t^{MC}

Results 2: *all flavors*

- NLL' resummation with $\overline{m}_b(\overline{m}_b) = 4.2$ GeV





- expect small contribution of $b\bar{b}$ to *all flavor production* cross-section
  $\rightarrow$ EW factors
- effect on extracted $\alpha_S(m_Z)$ value
  $\rightarrow$ implement as a correction

Summary & Outlook

achievements:

- calculated C-parameter cross-section in SCET and QCD with primary massive quarks
- provided a starting point for a more general treatment of mass effects in the C-parameter distribution

next steps:

- α_s fits including corrections from primary bottom quark mass effects
- generalize the used setup for top quark production
- study the MC mass parameter m_t^{MC} and clarify its relation to a theoretically well defined mass scheme

Thank you for your attention!