Renormalization of a toy model with spontaneously broken symmetry

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Contents

Toy model

Lagrangian density

$$\mathcal{L} = i\overline{\psi}_L \partial_\mu \gamma^\mu \psi_L + \frac{Y}{2}(\psi_L^{\mathrm{T}} C^{-1} \psi_L \varphi + \mathrm{H.c.}) + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2}\mu^2 \varphi^2 - \frac{1}{4}\lambda \varphi^4.$$

- Lagrangian density. Most general one under the requirement of renormabizability and global symmetry:
  $\varphi \longrightarrow -\varphi$,
  $\psi_L \longrightarrow i\psi_L$.
- Two interacting fields, a massless fermion field ψ_L and a real massive scalar field φ.
- Mass term for scalar field. No mass term allowed for the fermion.
- Two kinds of couplings:
  1. Yukawa coupling with constant Y between both fields.
  2. Scalar φ^4 with coupling with constant λ.
- C charge-conjugation matrix.
- Lorentz invariance
- Interesting renormalization scheme allowed in the model.

Spontaneous symmetry breaking

Potential given by $V = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4$. With the conditions:

- $\lambda > 0$. Always true for consistency of the theory.
- $\mu^2 < 0$. The potential shows two symmetrical minima. To reach the minima $\longrightarrow \varphi \neq 0$.
- $v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$ values of field for minimal potential.

Definition of new field:

1. Take the system close to one of the minima.
2. New shifted field $\varphi(x) = h(x) + v$.
3. Lagrangian in terms of h:
$$\mathcal{L} = i\overline{\psi}_L\partial_\mu\gamma^\mu\psi_L + \frac{Yv}{2}(\psi_L^{\mathrm{T}}C^{-1}\psi_L + \mathrm{H.c.}) + \frac{Y}{2}(\psi_L^{\mathrm{T}}C^{-1}\psi_L h + \mathrm{H.c.}) + \frac{1}{2}\partial_\mu h\partial^\mu h - \lambda v^2h^2 - \lambda vh^3 - \frac{1}{4}\lambda h^4 + \frac{1}{4}\lambda v^4.$$
4. Linear terms vanish $\longrightarrow$ $h = 0$ for minimal potential.
5. Additional interaction term h^3.

Remark: From massless Weyl fields to massive Majorana field

- Massless particles: helicity and chirality coincide and are well defined.
- Dirac equation solutions as reducible representations of Lorentz group.
- Chirality(Helicity) invariant under Lorentz tranformations $\longrightarrow$ irreducible representation.
- Fermion field divided in two left/right Weyl fields $\psi = \psi_L + \psi_R$.

Majorana fields are massive. We need left and right Weyl fields to build them and the result should be real.

- Projectors
  1. Left handed $L = \frac{1}{2}(1 - \gamma_5)$.
  2. Right handed $R = \frac{1}{2}(1 + \gamma_5)$.
- If ψ_L is a left Weyl field, then $(1 + \gamma_5)\psi_L = 0$.
- Complex conjugate field $(\psi_L)^c = C\gamma_0^T \psi_L^* \longrightarrow (1 - \gamma_5)(\psi_L)^c = 0$. Therefore $(\psi_L)^c$ is right-handed.
- The complex conjugation operator acts as $((\psi_L)^c)^c = \psi_L$.

Majorana massive field $\chi = \psi_L + (\psi_L)^c$ from Weyl massless field ψ_L.

Majorana field χ

Massive Majorana field

Majorana field$\longrightarrow$ $\chi := \psi_L + (\psi_L)^c$.

1. Hermitian conjugate integral action: terms and its complex conjugates ones.
2. Mathematical remarks:
   - $\psi_L^T C^{-1} = -\overline{(\psi_L)^c}$.
   - $(\overline{(\psi_L)^c}\psi_L)^\dagger = \overline{\psi}_L(\psi_L)^c$.
   - $\overline{\chi}\chi = \overline{\psi}_L(\psi_L)^c + \overline{(\psi_L)^c}\psi_L$.
3. New form for the Lagrangian:

$$\mathcal{L} = i\overline{\psi}_L\partial_\mu\gamma^\mu\psi_L - \frac{1}{2}Yv\overline{\chi}\chi + \frac{1}{2}\partial_\mu h\partial^\mu h - \frac{1}{2}2\lambda v^2h^2 - \frac{Y}{2}\overline{\chi}\chi h - \lambda v h^3 - \frac{1}{4}\lambda h^4.$$

4. As expected, two mass terms:
   - Massive Majorana fermion field $m_0 = Yv$.
   - Massive real scalar field $M_0^2 = 2\lambda v^2$.

Propagator in interaction picture

- With no interaction, two-point correlation function equal to Feynman propagator with pole in the "bare mass"(M_0/m_0) value.
- With interaction, the pole is shifted to a different value$\longrightarrow$ physical mass (M/m).
- Conclusion: Interaction determines the physical mass of the particles.
- Mass related with coupling constants$\longrightarrow$ physical coupling constants.

Exact propagator in the interaction scheme and expansion around pole:

Fermion case

$$\frac{i}{\not{p} - m_0 - \Sigma(p)} \underset{p^0 \to E_{\vec{p}}}{\sim} \frac{iZ_2}{\not{p} - m}.$$

Scalar case

$$\frac{i}{p^2 - M_0^2 - \Pi(p^2)} \underset{p^0 \to E_{\vec{p}}}{\sim} \frac{iZ_1}{p^2 - M^2}.$$

Constants Z_1 and Z_2 field renormalization factors.

Aim

- $\Pi(p^2)/\Sigma(p)$ shift in the pole $\longrightarrow$ contribution to self energy.
- $-i\Pi(p^2)/-i\Sigma(p)$ sum of one-particle irreducible insertions in the scalar/fermion propagator.
- Bare terms give divergent contributions.
- Contributions of the interaction shifts?
- Coupling constants dimensionless $\longleftrightarrow$ Renormalizable system.
- In d dimensions, renormalized contants to keep them dimensionless.$\epsilon = 4 - d$.
  - $Y \longrightarrow Y\mathcal{M}^{\frac{\epsilon}{2}}$.
  - $\lambda \longrightarrow \lambda\mathcal{M}^{\epsilon}$.
  - $v \longrightarrow v\mathcal{M}^{-\frac{\epsilon}{2}}$.

Aim

1. Renormalize the model.
2. Compute all the one particle irreducible diagrams (1PI).
3. Add them up and obtain $-i\Pi(p^2)$ or $-i\Sigma(p)$.
4. Obtain the shift in the propagator. Take mass shell limit.
5. Compute the physical masses of the fields.

Renormalization

Initial Lagrangian density before S.S.B.

Renormalization $\longrightarrow$ reach finite results absorbing the divergent constants into unobservable bare parameters.

1. Eliminate residue Z_1 and Z_2 by rescaling the bare fields ψ_{L0}, φ_0:
   - $\varphi_0 = \sqrt{Z_1}\varphi$.
   - $\psi_{L0} = \sqrt{Z_2}\psi_L$.
   - $\mathcal{L} =$
     $iZ_2\overline{\psi}_L\partial_\mu\gamma^\mu\psi_L + \frac{Y_0}{2}Z_2\sqrt{Z_1}(\psi_L^{\mathrm{T}}C^{-1}\psi_L\varphi + \mathrm{H.c.}) + \frac{1}{2}Z_1\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}Z_1\mu_0^2\varphi^2 - \frac{1}{4}Z_1^2\lambda_0\varphi^4.$
2. Eliminate bare quantities $Y_0,\ \lambda_0,\ \mu_0^2$:
   - $\delta_\varphi := Z_1 - 1,$
   - $\delta_\psi := Z_2 - 1,$
   - $\delta_y\mathcal{M}^{\frac{\epsilon}{2}} := Y_0Z_2\sqrt{Z_1} - Y\mathcal{M}^{\frac{\epsilon}{2}},$
   - $\delta_\mu := Z_1\mu_0^2 - \mu^2,$
   - $\delta_\lambda\mathcal{M}^\epsilon := Z_1^2\lambda_0 - \lambda\mathcal{M}^\epsilon.$

Renormalized Lagrangian density

Lagrangian $\longrightarrow$ Two parts

$$\begin{aligned}\mathcal{L} &= i\overline{\psi}_L\partial_\mu\gamma^\mu\psi_L + \frac{Y\mathcal{M}^{\frac{\epsilon}{2}}}{2}(\psi_L^{\mathrm{T}}C^{-1}\psi_L\varphi + \mathrm{H.c.}) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\mu^2\varphi^2 - \frac{1}{4}\lambda\mathcal{M}^\epsilon\varphi^4\\ &+ i\delta_\psi\overline{\psi}_L\partial_\mu\gamma^\mu\psi_L + \frac{\delta_y\mathcal{M}^{\frac{\epsilon}{2}}}{2}(\psi_L^{\mathrm{T}}C^{-1}\psi_L\varphi + \mathrm{H.c.}) + \frac{1}{2}\delta_\varphi\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\delta_\mu\varphi^2 - \frac{1}{4}\delta_\lambda\mathcal{M}^\epsilon\varphi^4\\ &= \mathcal{L}_0 + \mathcal{L}'.\end{aligned}$$

- $\mathcal{L}_0$: Original Lagrangian with physical parameters.
- $\mathcal{L}'$: Counterterms $\longrightarrow$ difference between bare and physical parameters.

Spontaneously symmetry breaking II

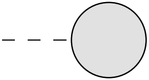
- Shifted field $\varphi = h + v\mathcal{M}^{-\frac{\epsilon}{2}}$.
- Lagrangian:

$$\begin{aligned}\mathcal{L}_0 =& i\overline{\psi}_L \partial_\mu \gamma^\mu \psi_L - \frac{1}{2} m_0 \overline{\chi}\chi + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2} M_0 h^2 \\ &- \frac{Y\mathcal{M}^{\frac{\epsilon}{2}}}{2}\overline{\chi}\chi h - \lambda \mathcal{M}^{\frac{\epsilon}{2}} v h^3 - \frac{1}{4}\lambda\mathcal{M}^{\epsilon} h^4 + \frac{1}{4}\lambda\mathcal{M}^{-\epsilon} v^4. \\ \mathcal{L}' =& i\delta_\psi \overline{\psi}_L \partial_\mu \gamma^\mu \psi_L - \frac{1}{2}\delta_y v \overline{\chi}\chi - \frac{1}{2}\delta_y \mathcal{M}^{\frac{\epsilon}{2}}\overline{\chi}\chi h + \frac{1}{2}\delta_\varphi \partial_\mu h \partial^\mu h \\ &- \frac{1}{2}\delta_\mu \mathcal{M}^{-\epsilon} v^2 - (\delta_\mu v + \delta_\lambda v^3)\mathcal{M}^{-\frac{\epsilon}{2}} h - \frac{1}{2}\left(\delta_\mu + 3\delta_\lambda v^2\right) h^2 \\ &- \frac{1}{4}\delta_\lambda \mathcal{M}^{-\epsilon} v^4 - \frac{1}{4}\delta_\lambda \mathcal{M}^{\epsilon} h^4 - \delta_\lambda \mathcal{M}^{\frac{\epsilon}{2}} v h^3.\end{aligned}$$

- Linear counterterms $\longrightarrow$ Difference $v - v_0$.
- Masses $m_0 = Yv, \; M_0^2 = 2\lambda v^2$.
- χ field: $\chi = \psi_L + (\psi_L)^c$.
- Five parameters $\delta_\psi, \delta_y, \delta_\varphi, \delta_\mu, \delta_\lambda$ to absorb infinities.
- Five renormalization conditions.

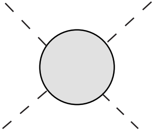
Renormalization conditions

δ_μ :

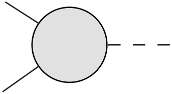


$= 0$

δ_λ : $\overline{\text{MS}}$ *in*



δ_y : $\overline{\text{MS}}$ *in*



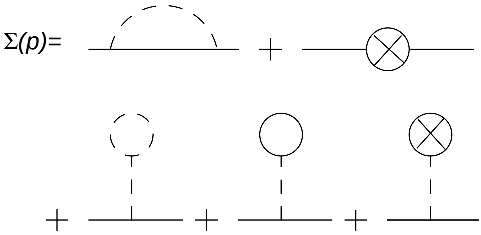
δ_ψ : *Residue of fermion propagator* $= 1$

δ_φ : *Residue of scalar propagator* $= 1$

Majorana fermion field χ

One-particle irreducible diagrams: insertions to the fermion propagator contributing to the self energy of the particle.
Three allowed contributions from $\mathcal{L}_0$ and two associated counterterms from $\mathcal{L}'$.

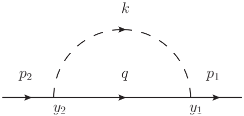
$$\langle\Omega|\,\chi(x_1)\overline{\chi}(x_2)\,|\Omega\rangle = \langle 0|\,T\{\chi_{ip}(x_1)\overline{\chi}_{ip}(x_2)e^{-i\int \mathrm{d}^4y\mathcal{H}_{int}(y)}\}\,|0\rangle\,.$$



Counterterms: from $i\delta_\psi\overline{\psi}_L\partial_\mu\gamma^\mu\psi_L - \frac{1}{2}\delta_y v\overline{\chi}\chi$ and $(\delta_\mu v + \delta_\lambda v^3)\mathcal{M}^{-\frac{\epsilon}{2}}h$ in $\mathcal{L}'$.

Field χ : Diagrams computation I

Taking the first diagram:



Computation methods and steps:

1. Majorana treatment: $(1/2)$ factors cancel in every line $\longrightarrow$ In loops $(-1/2)$ remains.
2. Symmetry factor and Feynman rules in momentum space:
$$8\,\frac{(-i)^2}{2!}\,\frac{Y^2}{4}\,S_F(p_1)\,S_F(p_2)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,S_F(q)\,D_F(k).$$
3. Momentum conservation in the vertices:
$k+q=p_2 \quad k+q=p_1 \;\longrightarrow\; p_1=p_2=p \quad p=k+q.$
4. Simplify the expression and remove the external propagators: Several factors in the integral denominator.

Field χ: Diagrams computation II

4. Use of Feynman parameters: additional integrating parameters.

$$\frac{1}{A_1^{m_1}A_2^{m_2}...A_n^{m_n}} = \int_0^1 \mathrm{d}x_1...\mathrm{d}x_n \;\delta(\sum x_i - 1)\frac{\prod x_i^{mi-1}}{[\sum x_i A_i]^{\sum m_i}}\frac{\Gamma(m_1+...+m_n)}{\Gamma(m_1)...\Gamma(m_n)}.$$

5. Symplified expression with $\Delta := (1-y)m_0^2 + y(M_0^2 - (1-y)p^2)$.

$$Y^2\mathcal{M}^\epsilon\left[\int_0^1 \mathrm{d}y\;\not{p}y\int\frac{\mathrm{d}^d l}{(2\pi)^d}\frac{1}{(l^2-\Delta+i\epsilon)^2} + \int_0^1 \mathrm{d}y\;\; m_0\int\frac{\mathrm{d}^d l}{(2\pi)^d}\frac{1}{(l^2-\Delta+i\epsilon)^2}\right].$$

6. Dimensional regularization:$\epsilon = 4-d,\; C_\infty = 2/\epsilon - \gamma + ln(4\pi)$.
   - $\mathcal{M}^\epsilon\int\frac{\mathrm{d}^d l}{(2\pi)^d}\frac{1}{(l^2-\Delta+i\epsilon)^2} = \frac{i}{16\pi^2}\left(C_\infty - \ln\left(\frac{\Delta}{\mathcal{M}^2}\right)\right).$
   - $\mathcal{M}^\epsilon\int\frac{\mathrm{d}^d l}{(2\pi)^d}\frac{1}{l^2-\Delta+i\epsilon} = \frac{i\Delta}{16\pi^2}\left(C_\infty + 1 - \ln\left(\frac{\Delta}{\mathcal{M}^2}\right)\right).$

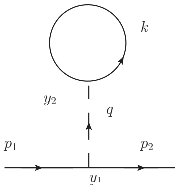
7. We obtain our first contribution multiplying by (-i):

$$\Sigma(p)_1 = -\frac{Y^2}{16\pi^2}\left[C_\infty\left(m_0+\frac{\not{p}}{2}\right) - \int_0^1 \mathrm{d}y\;\ln\left(\frac{\Delta}{\mathcal{M}^2}\right)(\not{p}y+m_0)\right].$$

Field χ: Tadpole diagrams

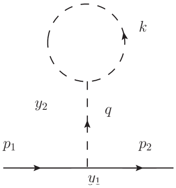
The remaining diagrams, computed in an analogous way, give the following contributions:

Fermion loop



$$\Sigma_2 = -\frac{2Y^2}{16\pi^2}\frac{m_0^3}{M_0^2}\left(C_\infty + 1 - \ln\left(\frac{m_0^2}{\mathcal{M}^2}\right)\right).$$

Scalar loop



$$\Sigma_3 = \frac{3Y\lambda v}{16\pi^2}\left(C_\infty + 1 - \ln\left(\frac{M_0^2}{\mathcal{M}^2}\right)\right).$$

Tadpole condition

The sum of the contributions from one-point amplitudes vanishes

1. From $\mathcal{L}_0$: Tadpole diagrams $\longrightarrow$ No Yukawa coupling $\rightarrow$ T_χ, T_h.
2. From $\mathcal{L}'$: Term $h(\delta_\mu\, v + \delta_\lambda\, v^3)\mathcal{M}^{-\frac{\epsilon}{2}}$.
   - $v_0 = \sqrt{\frac{-\mu_0}{\lambda_0}}$.
   - $v_0 = (\delta_v + v)\mathcal{M}^{-\frac{\epsilon}{2}}\sqrt{Z_1}$.
   - $h\mathcal{M}^{-\frac{\epsilon}{2}}(\delta_\mu\, v + \delta_\lambda\, v^3) = -\delta_v M_0^2 h$
   - Counterterm Contribution:
     $i\int d^4y\, M_0^2\, \delta_v \langle 0|\, h(x)h(y)\,|0\rangle = -i\delta_v$.
3. condition $\delta_v + T_\chi + T_h = 0$.

$$\delta_v = +\frac{2Y}{16\pi^2}\frac{m_0^3}{M_0^2}\left(C_\infty + 1 - \ln\left(\frac{m_0^2}{\mathcal{M}^2}\right)\right) - \frac{3\lambda v}{16\pi^2}\left(C_\infty + 1 - \ln\left(\frac{M_0^2}{\mathcal{M}^2}\right)\right)$$

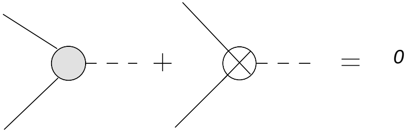
Yukawa vertex renormalization

Vertex contribution: three-point correlation function

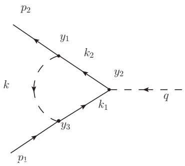
Fermion case

$$6 \cdot 4 \cdot 2 \frac{(-i)^3}{3!} \frac{Y^3}{2^3} S_F(p_1)\, S_F(p_2)\, D_F(q) \times$$
$$\times \int \frac{\mathrm{d}^4 k}{(2\pi)^4} S_F(k_1)\, S_F(k_2)\, D_F(k),$$
$$\Delta_Y := -xq^2(1-x) + m_0^2$$
$$- z(m_0^2 - M_0^2) - zp^2(1-z) + 2xzpq.$$

Divergent part $\frac{iY^3}{16\pi^2} C_\infty$.



Vertex graphic



- Counterterm $\frac{1}{2}\delta_y \chi\overline{\chi} h.$
- Contribution $-i\delta_y.$

$$\boxed{\delta_y = \frac{Y^3}{16\pi^2} C_\infty}$$

Fermion self energy renormalization

1. From $\mathcal{L}_0 \longrightarrow \quad \Sigma_1(p), \Sigma_2, \Sigma_3.$
2. From $\mathcal{L}'$:
   - δ_v renomalization condition $\longrightarrow \delta_v + T_\chi + T_h = 0.$
   - Counterterm $i\delta_\psi \overline{\psi}_L \not{p} - \frac{1}{2}\delta_y v \chi\overline{\chi} h^2.$
   - Counterterm contribution:
     $-\delta_\psi \not{p} + \delta_y v = -\delta_\psi \not{p} + v \frac{Y^3}{16\pi^2} C_\infty = -\delta_\psi \not{p} + \frac{m_0 Y^2}{16\pi^2} C_\infty.$

Self energy:

$$\Sigma(p) = \frac{Y^2}{16\pi^2}\left[-\frac{m_0}{2} C_\infty + \int_0^1 \mathrm{d}y \;\; \ln\left(\frac{\Delta}{\mathcal{M}^2}\right) y + \delta_\psi\right]\not{p}$$
$$+ m_0 \frac{Y^2}{16\pi^2}\left[-C_\infty + C_\infty + \int_0^1 \mathrm{d}y \;\; \ln\left(\frac{\Delta}{\mathcal{M}^2}\right)\right].$$

Self energy approximation

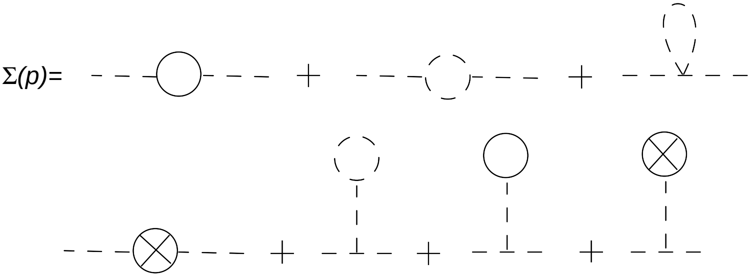
- Form of the self energy $\Sigma(p) = A(p^2)\not{p} + m_0 B(p^2)$.
- Propagator $\dfrac{1}{(1 - A(p^2))\not{p} - (m_0 + m_0 B(p^2))}$.
- Progragator mass shell limit $\dfrac{1}{\not{p} - m}$.
- Close to the pole $p^2 = m^2 \longrightarrow \quad A(p^2) = A(m^2) + A'(m^2)(p^2 - m^2) + \ldots$
  1. General case $A(m^2) = 0 \longrightarrow m = m_0 + m_0 B(m^2)$.
  2. 1-loop case $A(m_0^2) = 0 \longrightarrow m = m_0 + m_0 B(m_0^2)$.
- For our case $A(m_0^2) = 0$.
- Third parameter:

$$\delta_\psi = \frac{Y^2}{16\pi^2}\left[\int_0^1 \mathrm{d}y\, y \ln\left(\frac{\Delta(m_0^2)}{\mathcal{M}^2}\right) - \frac{C_\infty}{2}\right]$$

Real scalar field h

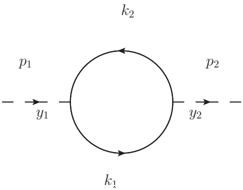
Higher number of diagrams contributing to the self energy. Tadpoles already computed.

$$\langle\Omega|\, h(x_1)h(x_2)\,|\Omega\rangle = \langle 0|\, T\{h_{ip}(x_1)h_{ip}(x_2)e^{-i\int d_y^4 \mathcal{H}_{int}(y)}\}\,|0\rangle\,.$$



Counterterm: from $\frac{1}{2}\delta_\varphi \partial_\mu h \partial^\mu h - \frac{1}{2}\left(\delta_\mu + 3\delta_\lambda v^2\right)h^2$ in $\mathcal{L}'$.

Real scalar field h: Fermion loop



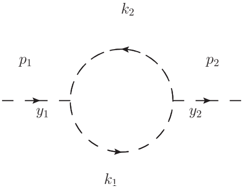
Properties of the first diagram:

1. One fermion loop:
   - Extra (-1/2) factor
   - Trace of the propagator. γ^μ traceless.
2. After Feynman parameters and Dimensional regularization, with:
   - $l := k - yp$.
   - $\Delta_\chi := m_0^2 - yp^2(1-y)$.
3. Contribution:

$$\Pi(p^2)_1 = 2Y^2 \int_0^1 dy\, \frac{\Delta_\chi}{16\pi^2}\left(3C_\infty + 1 - 3\ln\left(\frac{\Delta_\chi}{\mathcal{M}^2}\right)\right).$$

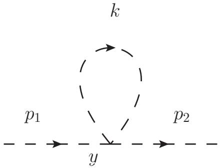
Scalar field loop interaction diagrams. $\Delta_h = M_0^2 - yp^2 + y^2p^2$.

Scalar loop 1



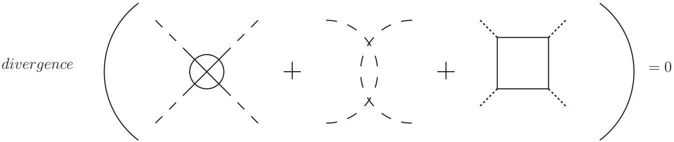
$$\Pi(p^2)_2 = \frac{18\lambda^2 v^2}{16\pi^2}\left[-C_\infty + \int_0^1 \mathrm{d}y \ln\left(\frac{\Delta_h}{\mathcal{M}^2}\right)\right].$$

Scalar loop 2

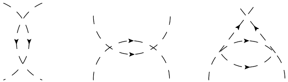


$$\Pi_3 = \frac{-3\lambda M_0^2}{16\pi^2}\left[C_\infty + 1 - \ln\left(\frac{M_0^2}{\mathcal{M}^2}\right)\right].$$

Scalar vertex renormalization



- Counterterm $\frac{1}{4}\delta_\lambda \mathcal{M}^\epsilon h^4 \longrightarrow$ Contribution $-6i\delta_\lambda$.
- 3 equivalent diagrams. Same divergent contribution $3 \times \frac{i6\lambda^2}{16\pi^2} C_\infty$.



- Divergent contribution from Fermion loop diagram: $\frac{-i12Y^4}{16\pi^2} C_\infty$.
- Applying the condition: $\delta_\lambda = \frac{C_\infty}{16\pi^2}(-2Y^4 + 9\lambda^2)$.

Scalar Self energy renormalization

1. From $\mathcal{L}_0 \longrightarrow \quad \Pi_1(p^2), \Pi_2(p^2), \Pi_3$.
2. From $\mathcal{L}'$:
   - Tadpole condition applied.
   - Counterterm:
     $-\frac{1}{2}\delta_\varphi(\partial_\mu h)^2 - \frac{\delta_\mu}{2}h^2 - \frac{3}{2}\delta_\lambda v^2 h^2 = -\frac{1}{2}\delta_\varphi(\partial_\mu h)^2 - \frac{1}{2}(-2v\lambda\delta_v + 2v^2\delta_\lambda)h^2 =$
     $-\frac{1}{2}\delta_\varphi(\partial_\mu h)^2 - \frac{1}{2}\delta_m h^2$.
   - Counterterm contribution:$-p^2\delta_\varphi + \delta_m$.
   - Self energy:
     $\Pi(p^2) = -p^2\delta_\varphi - \frac{2Y^2 m_0^2}{16\pi^2}\left(C_\infty + 1 - \ln\left(\frac{m_0^2}{\mathcal{M}^2}\right)\right) + \frac{3\lambda M_0^2}{16\pi^2}\left(C_\infty + 1 - \ln\left(\frac{M_0^2}{\mathcal{M}^2}\right)\right) -$
     $\frac{4Y^2 m_0^2}{16\pi^2}C_\infty + \frac{9\lambda M_0^2}{16\pi^2}C_\infty + 2Y^2\int_0^1 dy\, \frac{\Delta_\chi}{16\pi^2}\left(3C_\infty + 1 - 3\ln\left(\frac{\Delta_\chi}{\mathcal{M}^2}\right)\right) -$
     $\frac{9\lambda M_0^2}{16\pi^2}C_\infty + \frac{9\lambda M_0^2}{16\pi^2}\int_0^1 \mathrm{d}y\, \ln\left(\frac{\Delta_h}{\mathcal{M}^2}\right) - \frac{3\lambda M_0^2}{16\pi^2}\left(C_\infty + 1 - \ln\left(\frac{M_0^2}{\mathcal{M}^2}\right)\right)$.

Self enegy approximatrion

- Propagator $\dfrac{1}{p^2 - M_0^2 - \Pi(p^2)}$.
- Progragator mass shell limit $\dfrac{1}{p^2 - M^2}$.
- Close to the pole $(p^2 = M^2)$
  $\longrightarrow$ series expansion $\Pi(p^2) = \Pi(M^2) + \Pi'(M^2)(p^2 - M^2) + \ldots$
  - General case $\Pi'(M^2) = 0 \longrightarrow \; M^2 = M_0^2 + \Pi(M^2)$.
  - 1-loop case $\Pi'(M_0^2) = 0 \longrightarrow \; M^2 = M_0^2 + \Pi(M_0^2)$.

$$\Pi'(p^2) = -\delta_\varphi + \frac{2Y^2}{16\pi^2}\int_0^1 \mathrm{d}y(y(1-y))\left(-3C_\infty + 2 + 3\ln\left(\frac{\Delta_\chi}{\mathcal{M}^2}\right)\right) - \frac{9M_0^2\lambda}{16\pi^2}\int_0^2 \mathrm{d}y\,\frac{y(1-y)}{\Delta_h}.$$

Fifth parameter:

$$\delta_\varphi = -\frac{2Y^2}{16\pi^2}\int_0^1 \mathrm{d}y(y(1-y))\left(3C_\infty - 2 - 3\ln\left(\frac{\Delta_\chi(M_0)}{\mathcal{M}^2}\right)\right) - \frac{9\lambda}{16\pi^2}\int_0^2 \mathrm{d}y\,\frac{y(1-y)}{1-y(1-y)}.$$

Physical masses

1. Fermion mass
   - Propagator $\dfrac{1}{\not{p} - m_0 - m_0 B(m_0^2) + i\epsilon}$.
   - $m_0 = Yv$ and $\Delta(m_0^2) = M_0^2 y + (1-y)^2 m_0^2$.
   - Physical mass

     $$m = m_0(1 + B(m_0^2)) = m_0 \left(1 + \frac{Y^2}{16\pi^2} \int_0^1 \mathrm{d}y \;\; \ln\left(\frac{\Delta_\chi(m_0^2)}{\mathcal{M}^2}\right)\right)$$

2. Scalar mass
   - Propagator $\dfrac{1}{p^2 - M_0^2 - \Pi(M_0^2)}$.
   - $M_0^2 = 2\lambda v^2$ and $\Delta_\chi(M_0^2) = m_0^2 - yM_0^2(1-y)$, $\Delta_h(M_0^2) = M_0^2(1 + y(y-1))$.
   - Physical mass

     $$M^2 = M_0^2 + \Pi(M_0^2) =$$
     $$M_0^2\left(1 + \frac{Y^2}{16\pi^2}\left[-1 + \frac{2m_0^2}{M_0^2}\left(-\int_0^1 dy\, 3\ln\left(\frac{\Delta_\chi(M_0^2)}{\mathcal{M}^2}\right) + \ln\left(\frac{m_0^2}{\mathcal{M}^2}\right)\right)\right] + \right.$$
     $$\left.\frac{9\lambda}{16\pi^2}\int_0^1 dy\left[\frac{y(1-y)}{1-y(1-y)} + \ln\left(\frac{\Delta_h(M_0^2)}{\mathcal{M}^2}\right)\right]\right).$$

Conclusions

- Renormalization scheme:
  - Renormalizing coupling constants and v.e.v. $\longrightarrow$ Finite masses.
- Dependence of the masses on the coupling constants: hierarchy problem.
- Generalizable to arbitrary number of fermions and scalars.
- Dependence of the masses on $\mathcal{M}$ is of higher order.

Thank you for your attention