Heavy Quark Mass Determinations From Jets

Bahman Dehnadi

In collaboration with M. Butenschoen, A. H. Hoang, V. Mateu and Iain W. Stewart

University of Vienna

26 - 06 - 2014

Motivations and Aims

- **Full quark mass dependence for two loops corrections in QCD is only known for total cross section!!** **"Total Top-Quark Pair-Production Cross Section at Hadron Collider" M. Czakon, A. Mitov (2013)**

Aim : Systematic treatment of mass dependence in thrust distribution (jet observable).
Analysis of the low $E_{c.m.}$ data on thrust distribution. ⟶ Bottom mass

TASSO and JADE experiments at the PETRA electron positron collider at DESY

- **Top mass is highly correlated to the higgs mass and electroweak observables**

Problem not addressed: What is m_t^{Pythia} ?

→ Additional conceptual uncertainty in m_t^{Pythia}: O(1GeV)

But with respect to what? $m_t^{Pythia} = m_t^{short\text{-}distance} + O(1GeV)$

$$\boxed{m_t \; = \; 173.07 \pm 0.89 \text{ GeV}}$$

Aim: Systematics of heavy quark mass parameter in Monte Carlo generators. (Pythia)

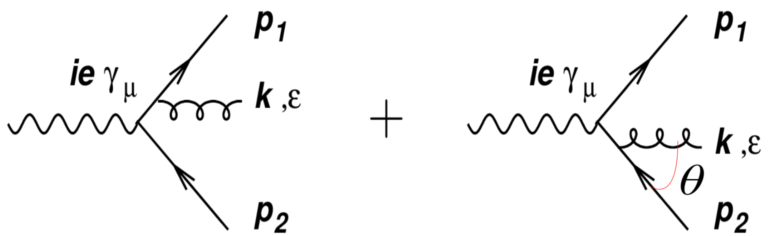
Outlines

- QCD at the electron positron colliders
- Event shapes & thrust distribution
- Soft collinear effective field theory
- Factorization theorem for massless quarks
- Full Massive thrust distribution
- Short distance mass
- Preliminary results
- Conclusion & outlooks

Soft and Collinear limits

- **Hadronic** matrix element

$$e^+e^- \to \gamma^* \to q\bar{q}$$



$k_\mu \ll p_\mu$ ⟶ **Soft gluon**

- **Factorization** of differential cross section

$$|\mathcal{M}_{q\bar{q}g}|^2 d\Phi_{q\bar{q}g} \simeq |\mathcal{M}_{q\bar{q}}|^2 d\Phi_{q\bar{q}} d\mathcal{S}\,,$$

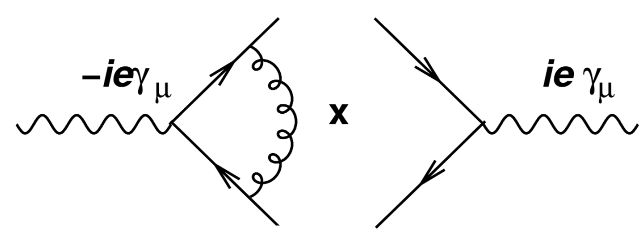
- **Soft gluon** emission probability

$$d\mathcal{S} = \frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\sin\theta}\frac{d\phi}{2\pi}$$

IR soft and collinear divergences

- **Inclusive** total cross section

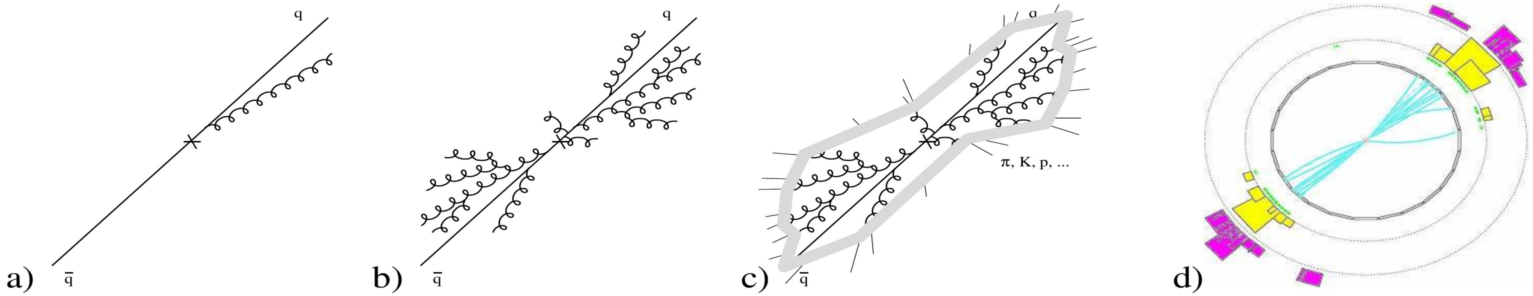
IR soft and collinear divergences cancellation



$$\sigma_{tot} = \sigma_{q\bar{q}}\left(1 + 1.045\frac{\alpha_s(Q)}{\pi} + 0.94\left(\frac{\alpha_s(Q)}{\pi}\right)^2 - 15\left(\frac{\alpha_s(Q)}{\pi}\right)^3 + \cdots\right),$$

⟶ Reasonable perturbative expansion

IRC Safe Observables



Infrared and collinear safe observables

"For an observable distribution to be calculable in [Fixed-order] perturbation theory, the observable should be infrared safe, i.e. insensitive to the emission of soft or collinear gluon. " **R. K, Ellis, W. J. Stirling and B. R. Webber (1996)**

In particular if any momentum occurring in observable definition, it must be invariant under the branching

$$\vec{p}_i \rightarrow \vec{p}_j + \vec{p}_k \longrightarrow \begin{cases} \text{Collinear} \\ \text{soft} \end{cases}$$

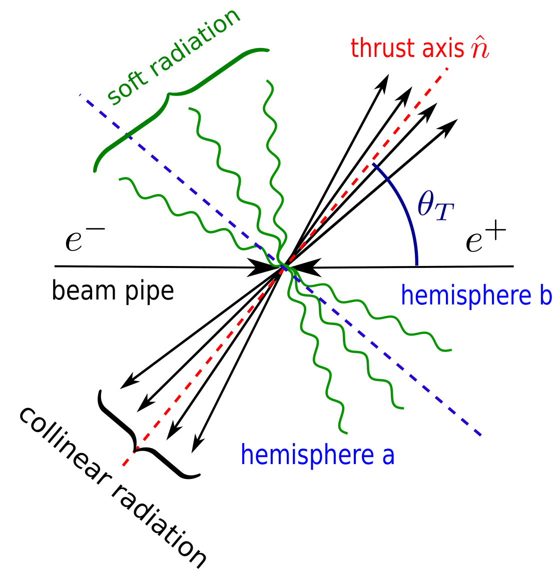
Event shapes: characterizes the final sates in a geometric ways.

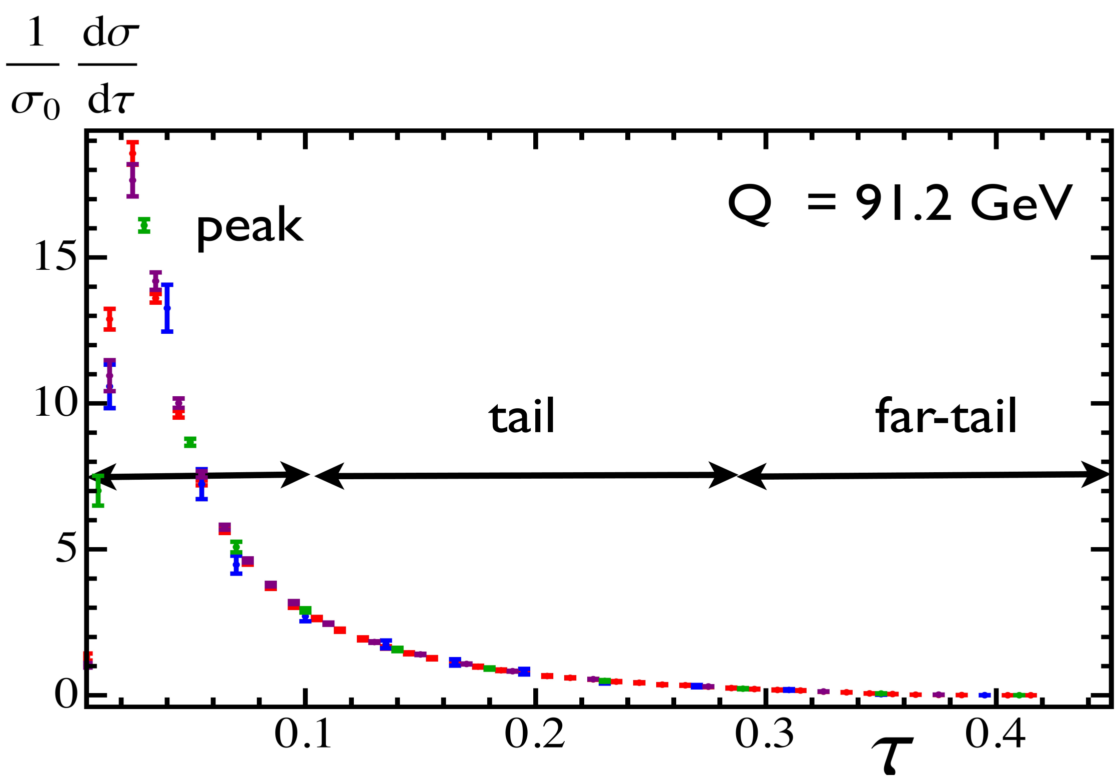
- IRC safe.
- Large log enhancement in small event shape values.

Thrust Distribution

- **Thrust**: Measure for "**Jettiness**" of the final state.

$$\tau = 1 - \max_{\hat{n}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{Q}$$

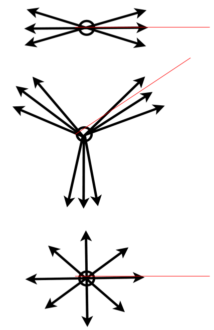




dijet $\tau \sim 0$

three jets $\tau \sim 0.3$

spherical Multijet $\tau \sim 0.5$



Log Resummation

- Thrust distribution

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s}{3\pi}\frac{1}{\tau}\left(3 + 4\lg\tau + \dots\right)$$

Large Logs at small τ

Log counting in the resummation of singular terms

$\alpha_s \lg(\tau) \sim \mathcal{O}(1)$

- Counting more clear in the exponent of cumulant distribution

$$\Sigma(e_c) \equiv \int_0^{e_c} \mathrm{d}e \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e}$$

$$\log\Sigma(e_c) = \lg e \underbrace{\sum_{i=0}\left(\alpha_s \log e\right)^{i+1}}_{\textbf{LL}} + \underbrace{\sum_{i=0}\left(\alpha_s \log e\right)^{i+1}}_{\textbf{NLL}} + \underbrace{\alpha_s \sum_{i=0}\left(\alpha_s \log e\right)^{i}}_{\textbf{NNLL}} + \alpha_s^2 \sum_{i=0}\left(\alpha_s \log e\right)^{i} + \dots$$

+ non singular

(a) Perturbative corrections

	Cusp	Noncusp	Matching	$\beta[\alpha_s]$	Nonsingular
LL	1	None	Tree	1	None
NLL	2	1	Tree	2	None
NNLL	3	2	1	3	1
N^3LL	4^{pade}	3	2	4	2

Effective Field Theories

Effective field theory approach:

- ✓ Transparent Factorization
- ✓ Resummation of large logs via RGE
- ✓ Systematic power correction

SCET: Soft Collinear Effective Theory
EFT of QCD for jet observables.

Bauer, Fleming, Pirjol, Stewart (2001)

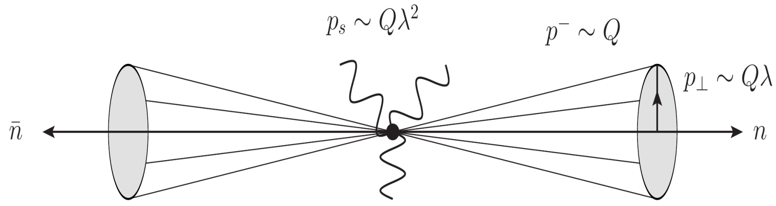
SCET

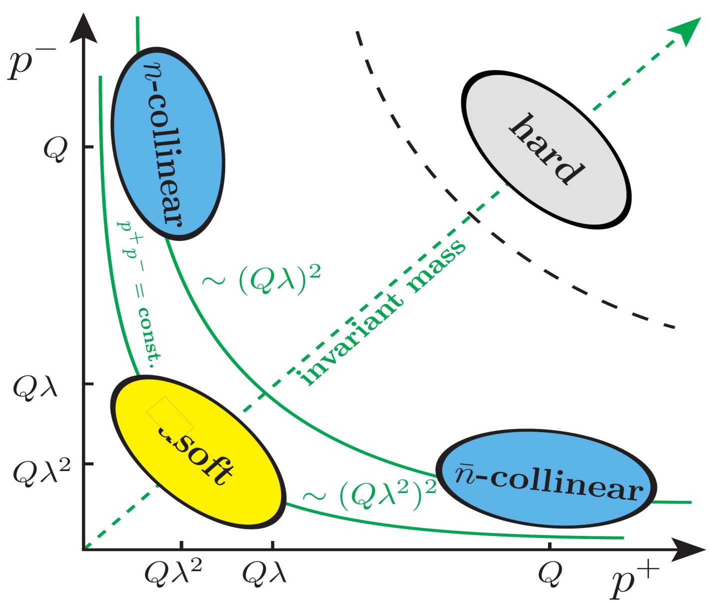
- **Relevant scales for high energy jets** $Q^2 \gg m_{jet}^2 \gg \Lambda_{QCD}^2$ → **Invariant mass of jets** $\sim Q^2\tau$

Relevant modes
soft and collinear modes to preserve the physical IR degrees of freedom of QCD

- **Lightcone coordinate** $n^\mu = (1, \vec{n}), \qquad \bar{n}^\mu = (1, -\vec{n})$

$$\boxed{\lambda = m_{jet}/Q}$$





$$p^\mu = n\cdot p\,\frac{\bar{n}^\mu}{2} + \bar{n}\cdot p\,\frac{n^\mu}{2} + p_\perp^\mu$$

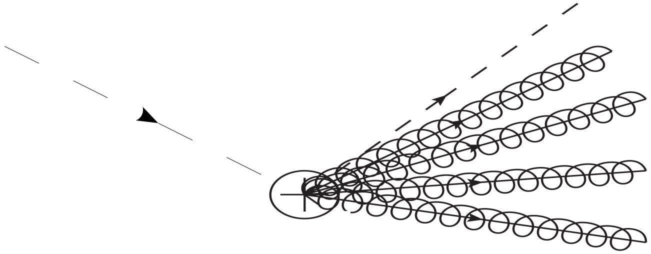
$$(p^+, p^-, p_\perp) = (n\cdot p, \bar{n}\cdot p, p_\perp)$$

mode	$p^\mu = (+,-,\perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	—
n-collinear	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1,\lambda^2,\lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}$, $A_{\bar{n}}^\mu$
soft	$Q(\lambda^2,\lambda^2,\lambda^2)$	$Q^2\lambda^4$	q_s, A_s^μ

SCET

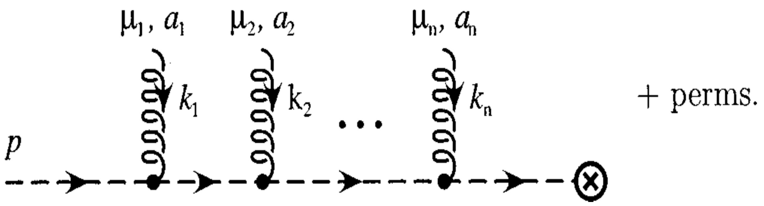
- Apply the **power counting** at the level of Lagrangian (label formalism)
- Operators **matching** (current matching)
- Emergence of **collinear gauge symmetries at high energy** restricts the invariant from of operator and physical fields (Wilson lines)
- Emergence of **soft gauge symmetries at high energy** restricts the invariant from soft interactions (Wilson lines)

Collinear Wilson lines
Multi collinear gluon interactions



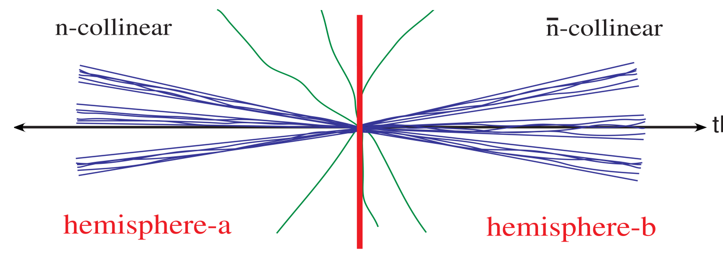
$$W_n^\dagger(x) = \mathrm{P}\exp\left(ig\int_0^\infty ds\bar{n}\cdot A_+(\bar{n}s+x)\right)$$

Soft Wilson lines
Multi soft gluon interactions



$$Y(x) = P\exp\left(ig\int_{-\infty}^{x} n\cdot A_s(n\,s)\right)$$

SCET



- Decoupling of the various modes dynamics

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_{c,n} + \mathcal{L}_{c,\bar{n}}$$

$$\mathcal{L}^q_{c,n} = \bar{\xi}_n \Big[in \cdot \partial + gn \cdot A_n + i\,\not{D}_c^\perp W_n \frac{1}{\overline{\mathcal{P}}} W_n^\dagger \not{D}_c^\perp \Big) \frac{\not{\bar{n}}}{2} \xi_n$$

Small component dynamic

Leading large component dynamic (label dynamic)

Collinear covariant derivative: $iD_c^\mu = \mathcal{P}^\mu + gA_n^\mu$

$$\mathcal{L}_s = \overline{q}_s i\not{D} q_s - \frac{1}{2} tr\{G^{\mu\nu}G_{\mu\nu}\}$$

Soft covariant derivative: $iD^\mu = i\partial^\mu + gA_s^\mu$

SCET production current at leading order

$$J_i^{(0)\mu}(\omega,\overline{\omega},\mu) = \left[\overline{\xi}_n(0)W_n\delta(\omega - n\cdot\mathcal{P})\right]Y_n^\dagger\Gamma_i^\mu Y_{\bar{n}}\left[\delta(\overline{\omega} - \bar{n}\cdot\mathcal{P})W_{\bar{n}}^\dagger\xi_{\bar{n}}(0)\right]$$

Matching relation of QCD and SCET current

$$\mathcal{J}_i^{\mu}(0) = \int d\omega d\bar{\omega} C(\omega,\bar{\omega},\mu) J_i^{(0)\mu}(\omega,\bar{\omega},\mu)$$

Factorization

Total hadronic in QCD $$e^- e^+ \to \gamma^*,\, Z^* \to t\bar{t} + X$$

$$\sigma = \sum_X^{\text{res.}} (2\pi)^4 \delta^4(q - p_X) \sum_{i=a,v} L^i_{\mu\nu} \langle 0|\mathcal{J}_i^{\nu\dagger}(0)|X\rangle\langle X|\mathcal{J}_i^{\mu}(0)|0\rangle$$

Observable definition

Total hadronic in SCET

$$\sigma = \sum_{\vec{n}} \sum_{X_n X_{\bar{n}} X_s}^{\text{res.}} (2\pi)^4 \delta^4(q - P_{X_n} - P_{X_{\bar{n}}} - P_{X_s}) \sum_i L^i_{\mu\nu} \int \mathrm{d}\omega\, \mathrm{d}\overline{\omega}\, \mathrm{d}\omega'\, \mathrm{d}\overline{\omega}' C(\omega, \overline{\omega}) C^*(\omega', \overline{\omega}')$$
$$\langle 0|\mathrm{T}\{\bar{\xi}_{\bar{n}} W_{\bar{n}} Y^\dagger_{\bar{n}} \overline{\Gamma}^\nu_i Y_n W^\dagger_n \xi_n\}|X_n X_{\bar{n}} X_s\rangle\langle X_n X_{\bar{n}} X_s|\mathrm{T}\{\bar{\xi}_n W_n Y^\dagger_n \Gamma^\mu_i Y_{\bar{n}} W^\dagger_{\bar{n}} \xi_{\bar{n}}\}|0\rangle$$

Hard sector

- For event shape distributions the matrix elements can be rearranged (**Factorized**) into two color and spin singlet collinear and soft sectors which are convoluted.

Massless SCET Factorization

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau} = Q^2 \boxed{H_Q^{(n_l)}(Q,\mu)} \int \mathrm{d}l\, J^{(n_l)}(Ql,\mu)\, S^{(n_l)}(Q\tau - l,\mu)$$

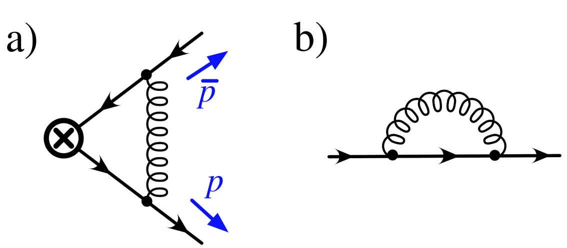
+ non-singular & power correction

Hard function

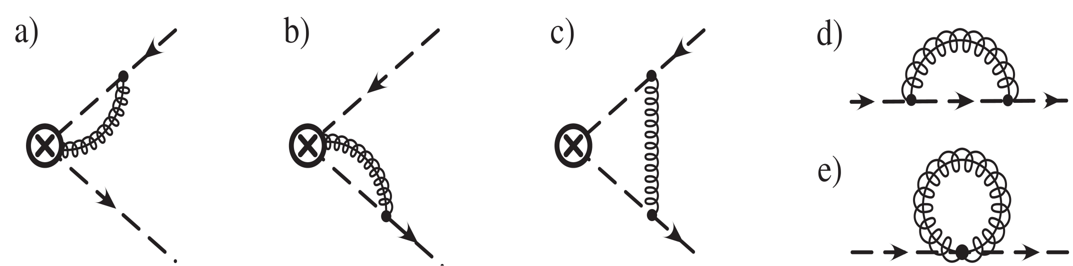
- Purely **Perturbative** hard interaction
- **Matching** SCET to QCD
- Matrix element and anomalous dimension are known at **3 loops**

Baikov et al, Moch et al, Lee et al

QCD



SCET



$$H_Q(Q,\mu) = U_{H_Q}(Q,\mu_Q,\mu) H_Q(Q,\mu_Q)$$

$$\boxed{\mu_Q \sim Q}$$

$$H_Q(Q,\mu_Q) = 1 + \frac{C_F \alpha_s(\mu_Q)}{4\pi}\left\{ -2\ln^2\left(\frac{Q^2}{\mu_Q^2}\right) + 6\ln\left(\frac{Q^2}{\mu_Q^2}\right) - 16 + \frac{7\pi^2}{3}\right\}$$

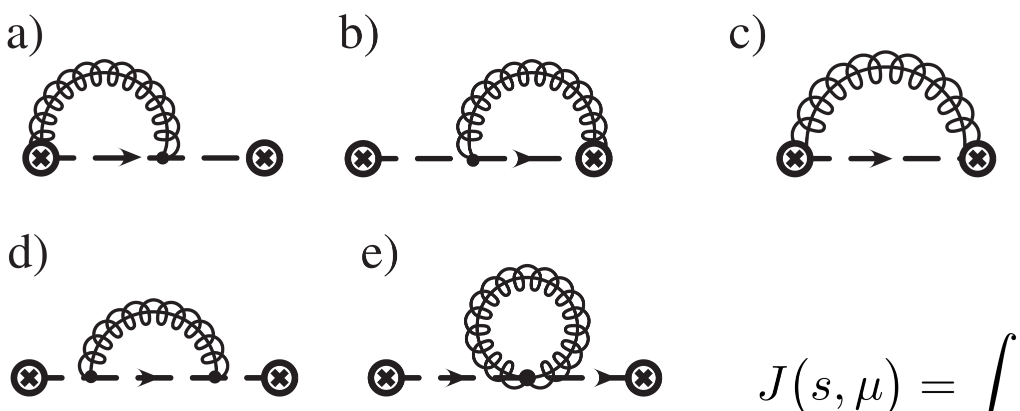
Massless SCET Factorization

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau} = Q^2\, H_Q^{(n_l)}(Q,\mu) \int \mathrm{d}l\, J^{(n_l)}(Ql,\mu)\, S^{(n_l)}(Q\tau - l,\mu)$$

Jet function

- Purely **Perturbative** evolution of produced jets
- Matrix element known at **2 loops** Bacher and Neubert
- Anomalous dimension are known at **3 loops** Moch, Vermaressen and Vogt

Jet function: Cut through the following forward scattering amplitudes



$$\mu_J^2 \sim S \sim M_{\text{Jet}}^2$$

$$J(s,\mu) = \int \mathrm{d}s'\, U_J(s',\mu,\mu_J)\, J(s-s',\mu_J)$$

$$J(s,\mu) = \delta(s) + \frac{\alpha_s(\mu)C_F}{4\pi}\left\{\delta(s)(14-2\pi^2) - \frac{6}{\mu^2}\left[\frac{\theta(s)}{s/\mu^2}\right]_+ + \frac{8}{\mu^2}\left[\frac{\theta(s)\log[s/\mu^2]}{s/\mu^2}\right]_+\right\}$$

Massless SCET Factorization

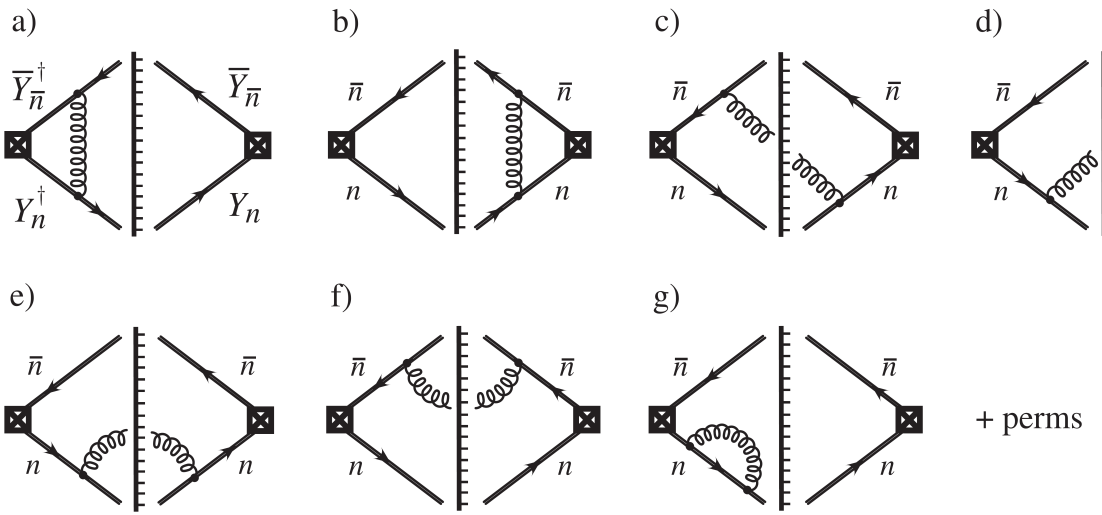
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau} = Q^2\, H_Q^{(n_l)}(Q,\mu)\int \mathrm{d}l\, J^{(n_l)}(Ql,\mu)\, \boxed{S^{(n_l)}(Q\tau - l,\mu)}$$

Soft function

- Convolution of **perturbative and non-perturbative** hadronization effects.
- Matrix elements known at **2 loops**
- Anomalous dimension are known at **3 loops**

$$S = S_{\rm part} \otimes F_{\rm shape}$$

Convolution with model and power corrections can be incorporated at the end.



Final state cut

$$S(l,\mu) = \int \mathrm{d}k\, U_S(k,\mu,\mu_s)\, S(l-k,\mu_s)$$

$$\boxed{\mu_s \sim l \sim k_{\rm soft}}$$

$$S_{\rm part}(l,\mu) = \delta(l) + \frac{C_F\alpha_s(\mu)}{4\pi}\left\{\frac{\pi^2}{3}\,\delta(l) - \frac{16}{\mu}\left[\frac{\theta(l)\ln(l/\mu)}{l/\mu}\right]_+\right\}$$

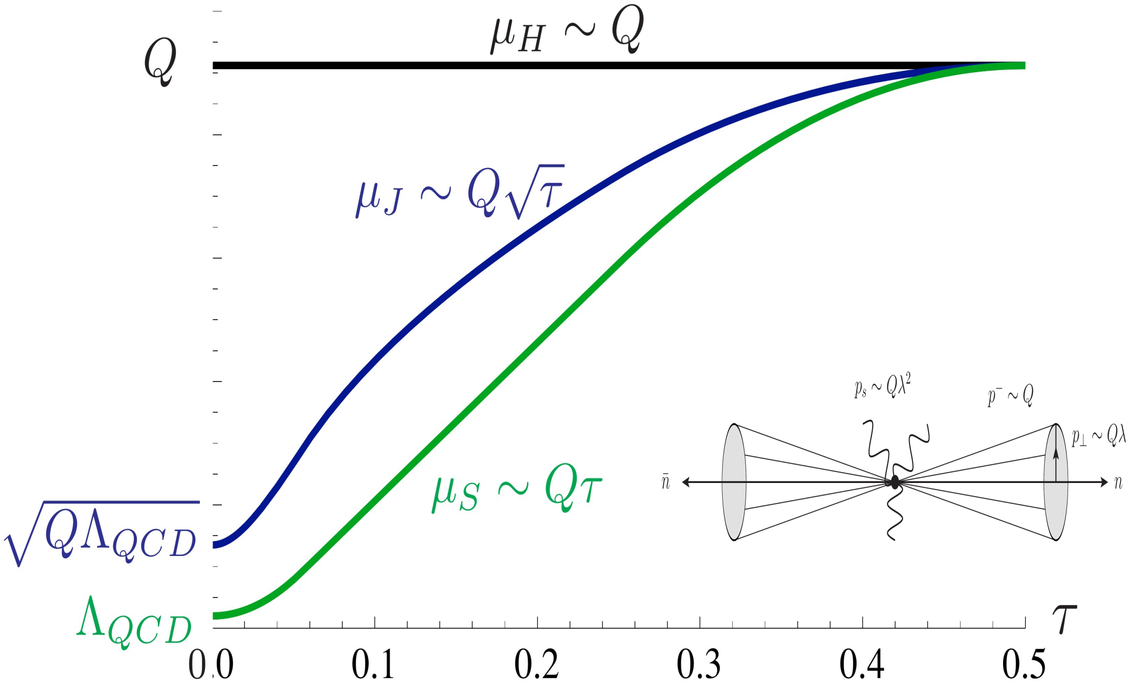
Massless SCET Factorization

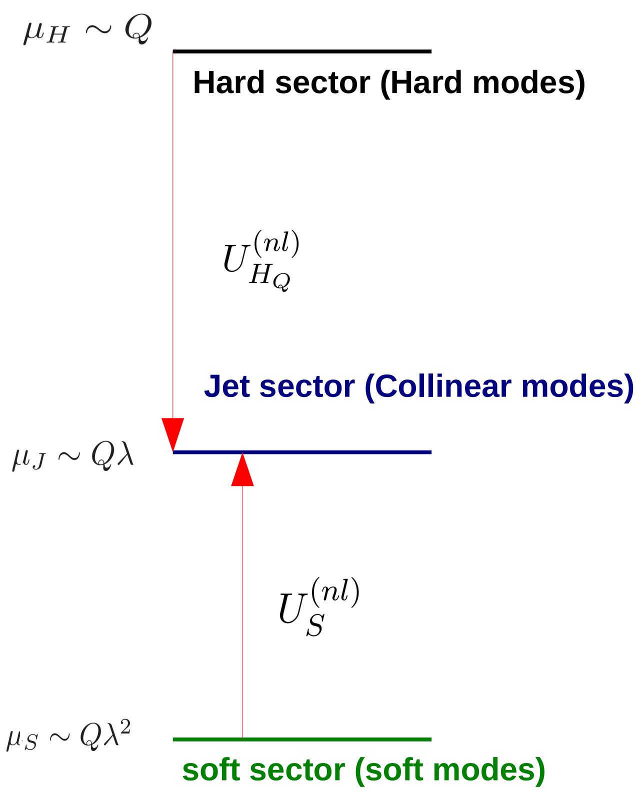
- Resummation of large logarithms via RGE (Evolution kernels).

Fleming, Hoang, Mantry, Stewart (2007) & Bauer, Fleming, Lee, Sterman (2008)

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau} = Q\, H_Q^{(n_l)}(Q,\mu_Q)\, U_{H_Q}^{(n_l)}(Q,\mu_Q,\mu_J) \int \mathrm{d}s \int \mathrm{d}k\, J^{(n_l)}(s,\mu_J) U_S^{(n_l)}(k,\mu_J,\mu_S)\, S_{\text{part}}^{(n_l)}\left(Q\tau - \frac{s}{Q} - k, \mu_S\right)$$

Profile function:
Parametrization of relevant scales in terms of thrust

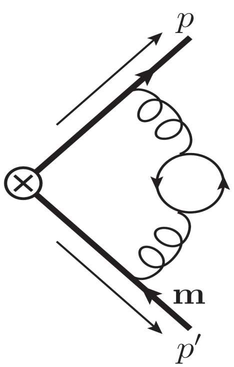


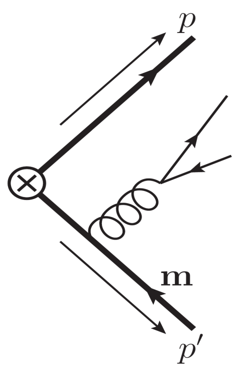


Jets with massive quarks

- **Massive primary quarks**
- Simple modification of the **collinear sector** at the level of Lagrangian
  (**Jet function wil be explicitly modified**)

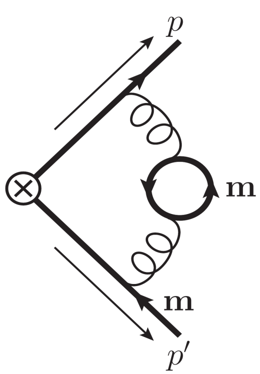
Fleming, Hoang, Mantry, Stewart (2008)

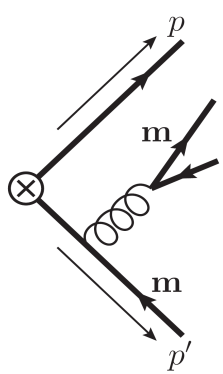




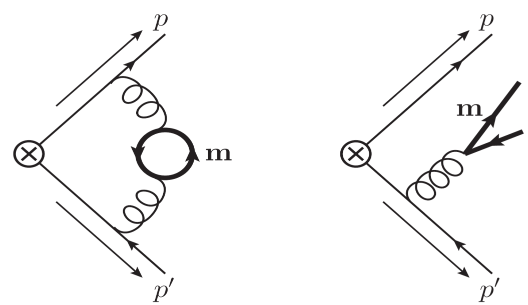
$$\mathcal{L}^q_{c,n} = \bar{\xi}_n\Big[in\cdot\partial + gn\cdot A_n + (i\not{D}^\perp_c - m)W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger(i\not{D}^\perp_c + m)\Big]\frac{\bar{\not{n}}}{2}\xi_n$$

- **Secondary massive quarks**
- New degrees of freedom: **Mass modes**
- Mass modes are not relevant at the order of NNLL (**except large rapidity logs**)





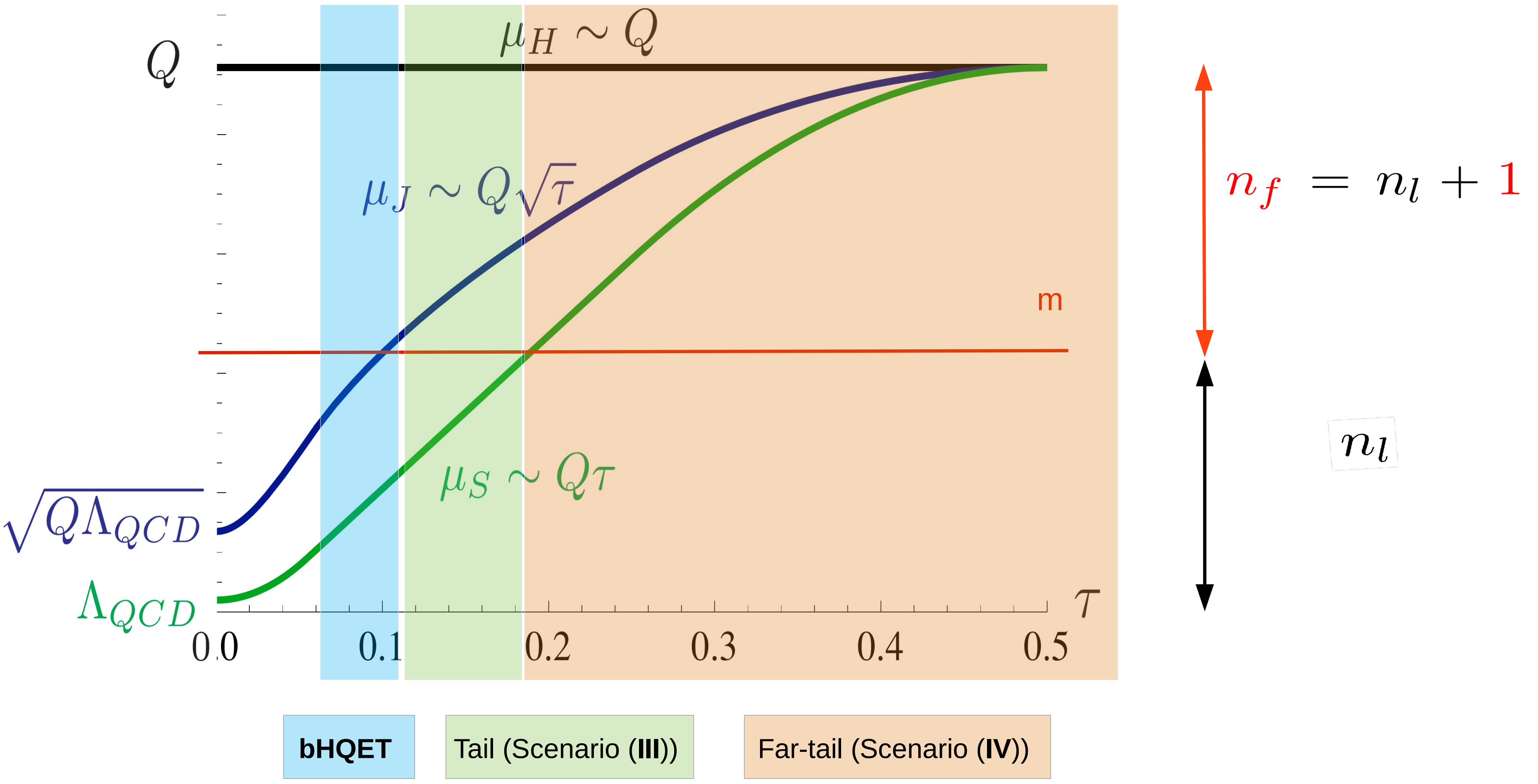
Gritschacher, Hoang, Jemos, Pietrulewicz (2013)



Jets with massive quarks

- Different type of physical situations (hierarchy) along the thrust spectrum (**VFS**).
- **Various scenarios** (factorization theorem) at different regions.
- In all scenarios the **Hard** function remains the same as massless but with nl+1 active flavors.
- **Jet and soft function** should be study separately in each scenario.

Pietrulewicz, Gritschacher, Hoang, Jemos, Mateu (2014)

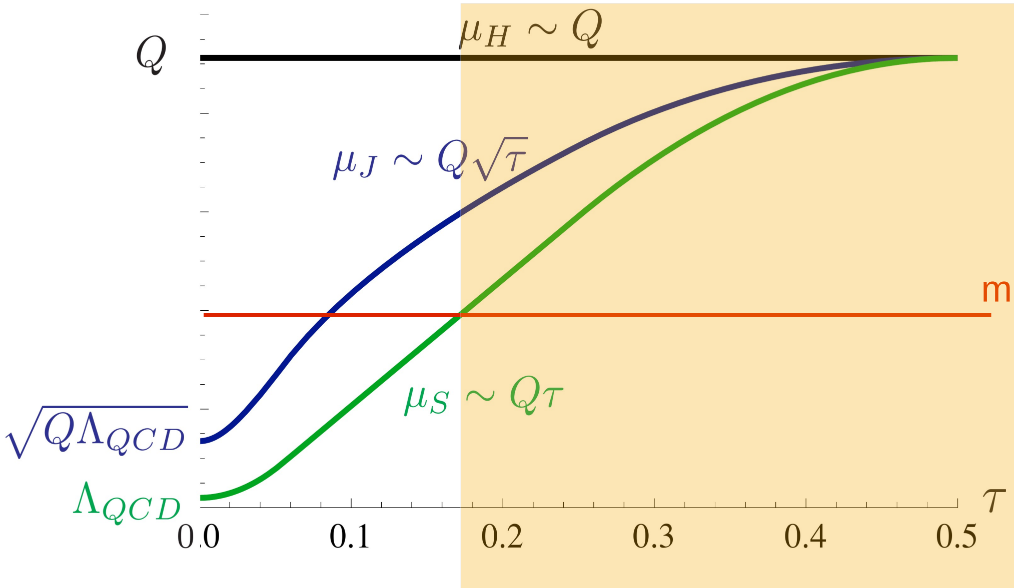


Scenario (IV) - Far Tail

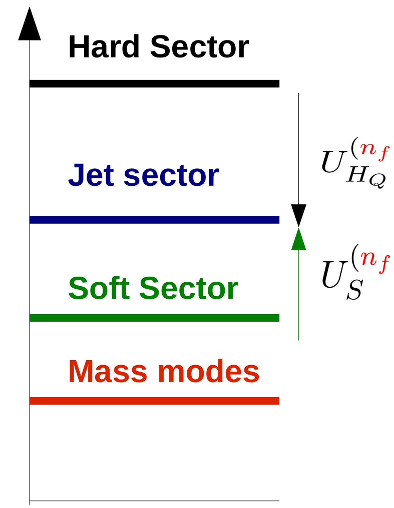
At NNLL

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-IV}} = Q\,\boxed{H_Q^{(n_f)}(Q,\mu_Q)}\,U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int \mathrm{d}s\int \mathrm{d}k\,J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))$$
$$U_S^{(n_f)}(k,\mu_J,\mu_S)\,S_{\mathrm{part}}^{(n_f)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k,\mu_S\right)$$

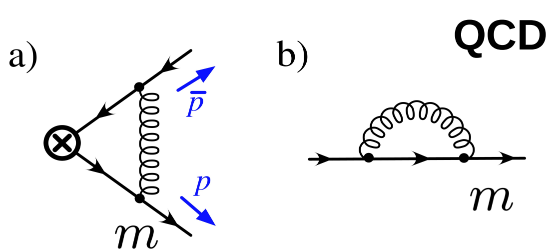
$$n_f = n_l + 1$$

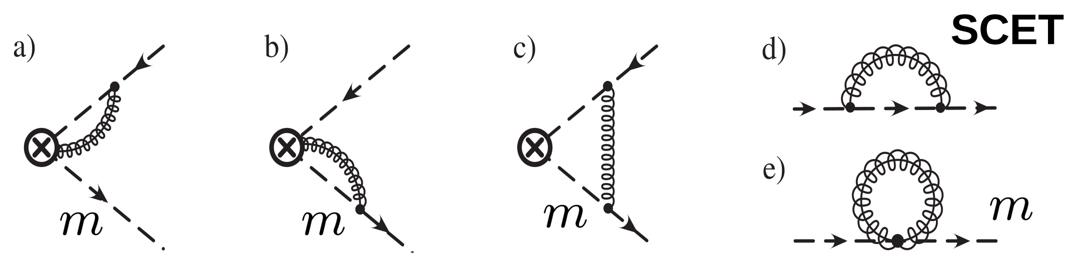


The mass modes enter in all the higher scale sectors.



- **Hard** sector remains the same as mass-less factorization but with **nl+1** number of active flavors.



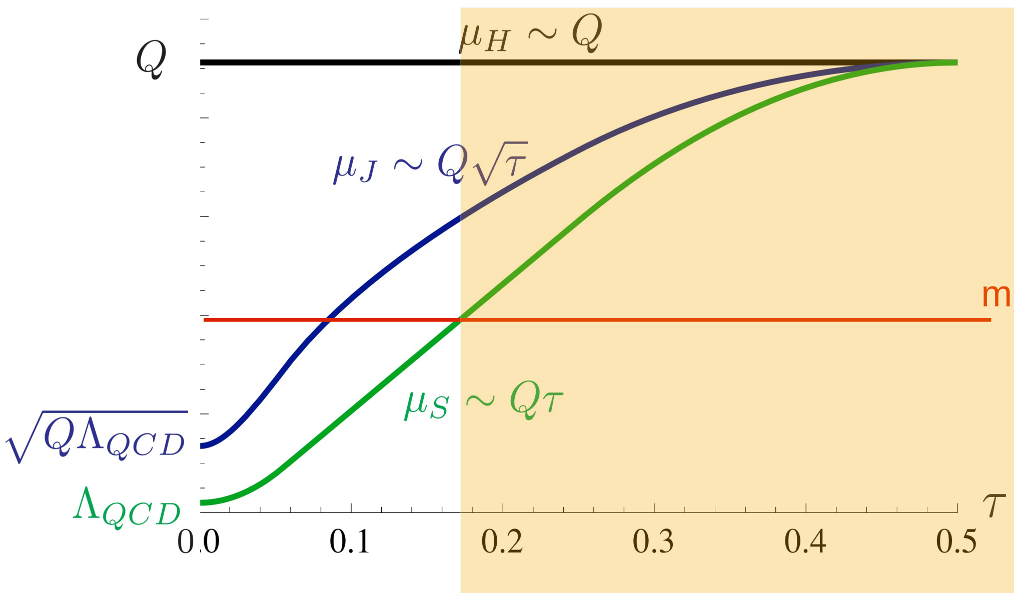


Scenario (IV) - Far Tail

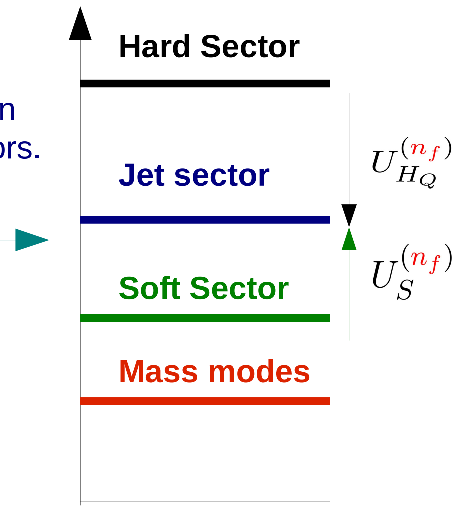
At NNLL

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-IV}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J) \int \mathrm{d}s \int \mathrm{d}k\, \boxed{J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))}$$
$$U_S^{(n_f)}(k,\mu_J,\mu_S)\, S_{\mathrm{part}}^{(n_f)}\Big(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k, \mu_S\Big)$$

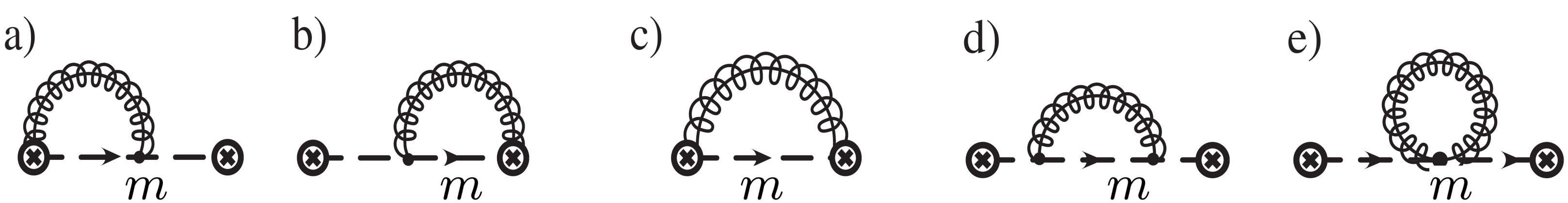
$$n_f = n_l + 1$$



The mass modes enter in all the higher scale sectors.



- **Explicit** modification of jet function at one loop due to **primary massive** quark production.

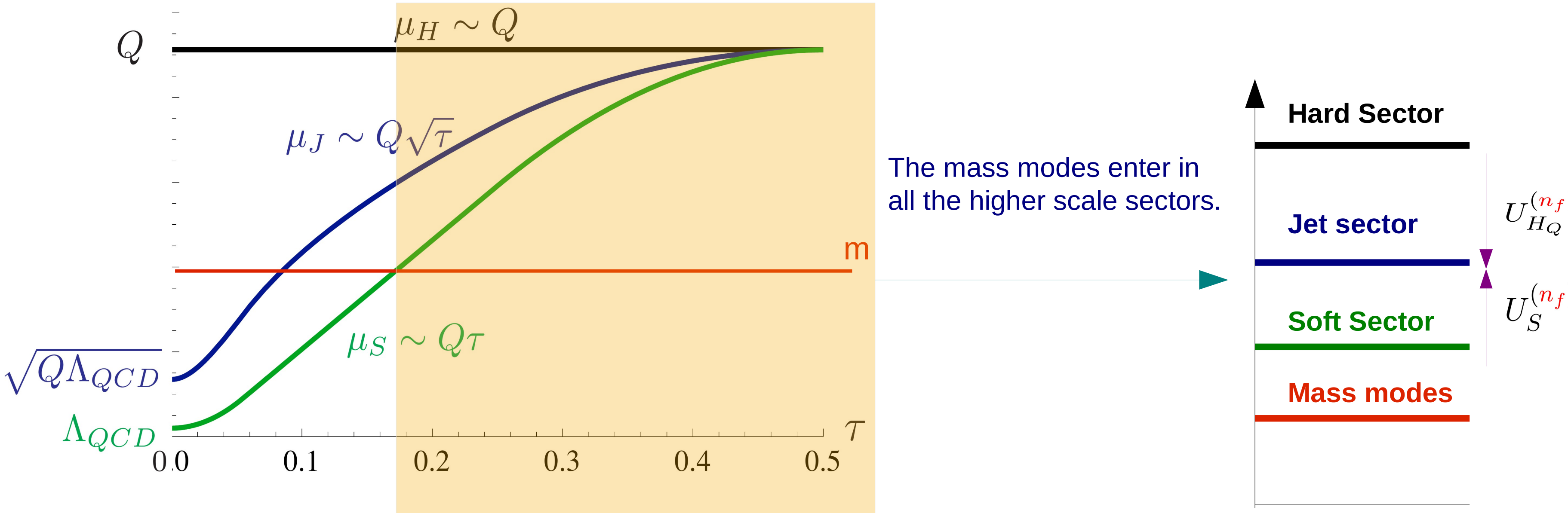


Scenario (IV) - Far Tail

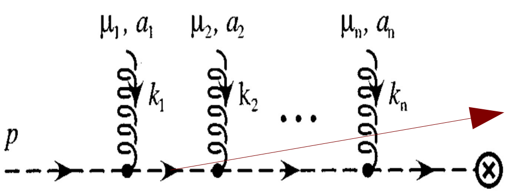
NNLL

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-IV}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int \mathrm{d}s\int \mathrm{d}k\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_S)\, S_{\mathrm{part}}^{(n_f)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k,\mu_S\right)$$

$$n_f = n_l + 1$$



- **Soft** sector remains the same as mass-less (at NNLL) with **nl+1** number of flavors. (universality of Wilson lines for boosted massive particles and mass-less)



$$\frac{\not p + \not k + m}{(k+p)^2 - m^2 + i\epsilon} \rightarrow$$

Leading order expansion for soft gluon emission.

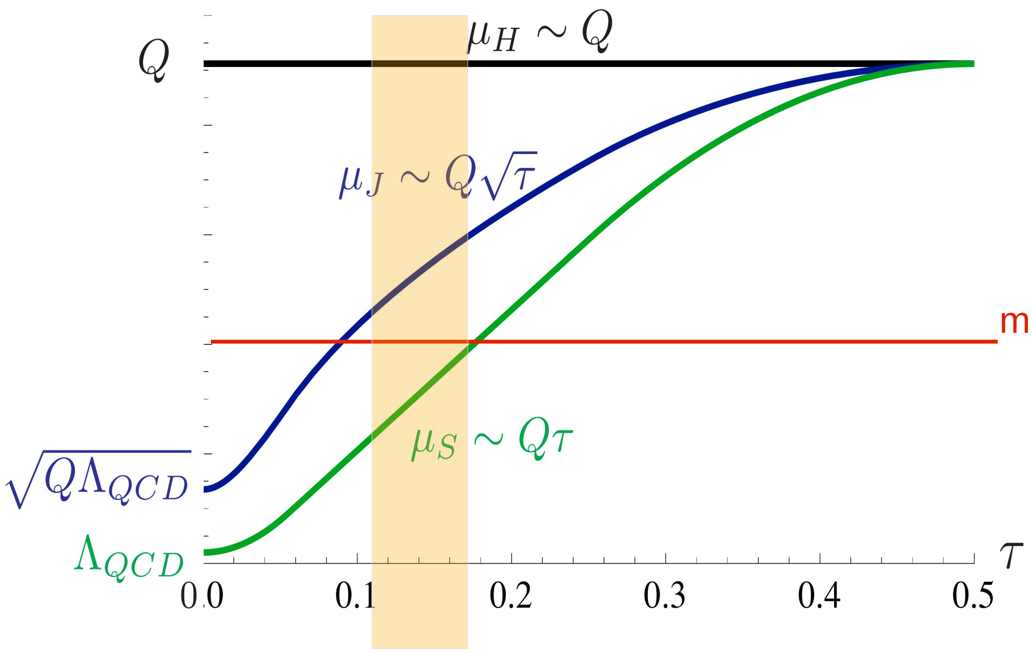
$$\rightarrow \frac{\not n/2}{n\cdot k + i\epsilon}$$

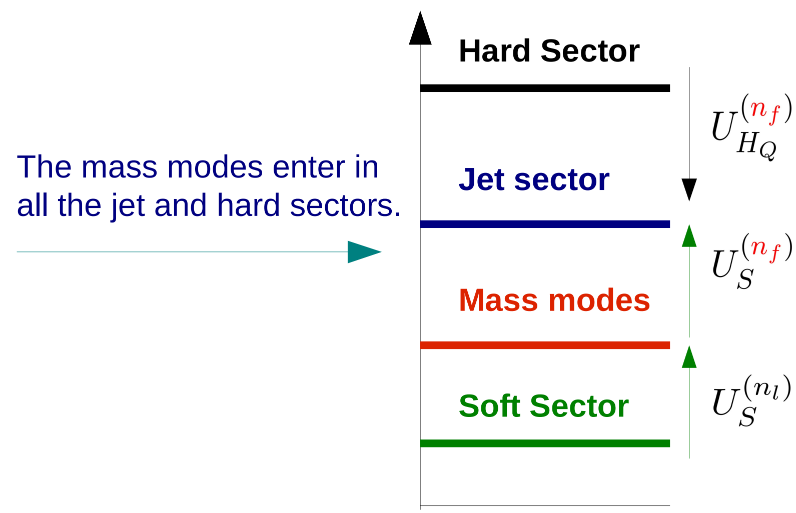
Scenario (III) - Tail

At NNLL

$$\left.\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-III}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J) \int \mathrm{d}s \int \mathrm{d}k\,\mathrm{d}k'\,\mathrm{d}k''\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_m)$$
$$\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s)\, U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\, S_{\mathrm{part}}^{(n_l)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k'',\mu_S\right)$$

$$n_f = n_l + 1$$





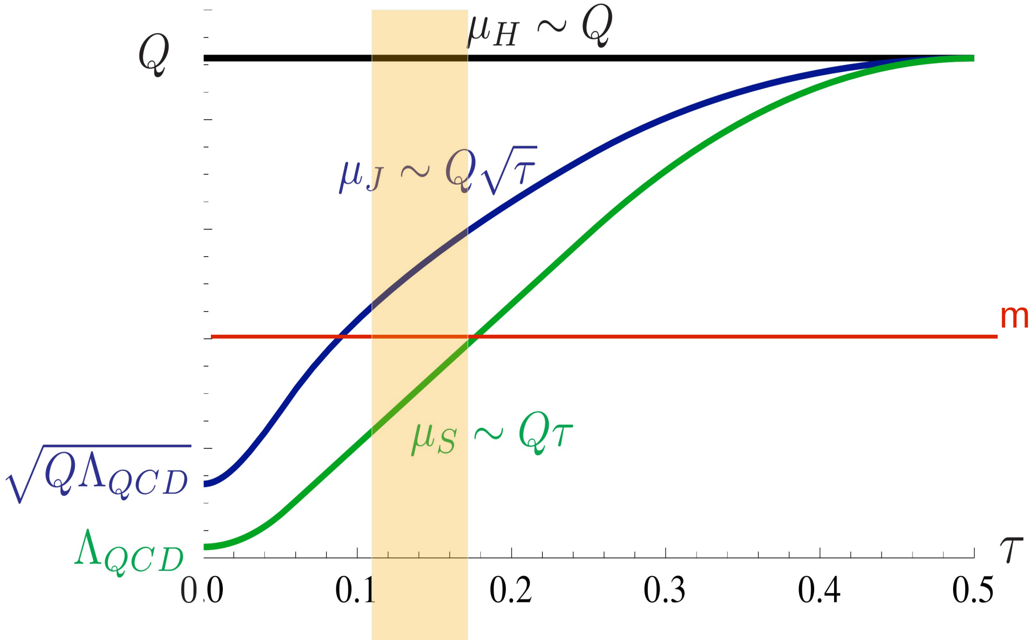
- **Hard and jet** sectors remain the same as scenario (IV) with **nl+1** number of flavors.
- **Soft** sector remains the same as mass-less soft function with **nl** number of flavors.

Scenario (III) - Tail

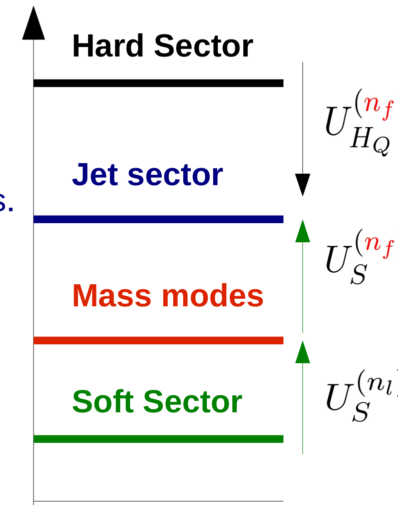
At NNLL

$$\left|\frac{1}{\sigma_0}\frac{d\hat{\sigma}(\tau)}{d\tau}\right|^{\mathrm{SCET-III}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int ds \int dk\, dk'\, dk''\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_m)$$
$$\boxed{\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s)}\, U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\, S_{\mathrm{part}}^{(n_l)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k'',\mu_S\right)$$

$$n_f = n_l + 1$$



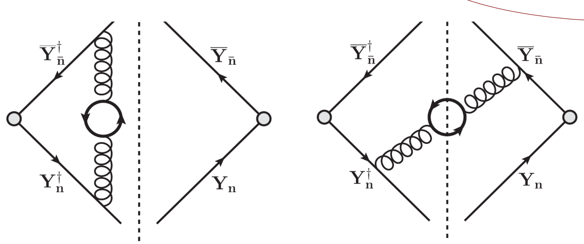
The mass modes enter in all the jet and hard sectors.



- Soft mass mode matching: **integrating in** the **mass mode** effects in the **evolution of soft function.**

$$\mathcal{M}_S^{(n_f)}(k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s) = \delta(k) + \delta(k) C_F \left(\frac{\alpha_s^{(nf)}(\mu_m)}{4\pi}\right)^2 \ln\left(\frac{\mu_s^2}{\mu_m^2}\right)\left(\frac{4}{3}L_m^2 + \frac{40}{9}L_m + \frac{112}{27}\right)$$

$$L_m = \ln\left(\frac{\overline{m}^2}{\mu_m^2}\right)$$



Gritschacher, Hoang, Jemos, Pietrulewicz (2013)

Large logs which should be consider at NNLL. $\alpha_s \log\left(\frac{\mu_s}{\mu_m}\right) \sim \mathcal{O}(1)$

bHQET

At NNLL

bHQET region of thrust(heavy quark in mesons)

- Additional physical scale appears.

$$\hat{s}_{t,\bar{t}} \equiv \frac{s_{t,\bar{t}}}{m} \equiv \frac{M^2_{t,\bar{t}} - m^2}{m} \sim \Gamma \ll m$$

Invariant jet mass

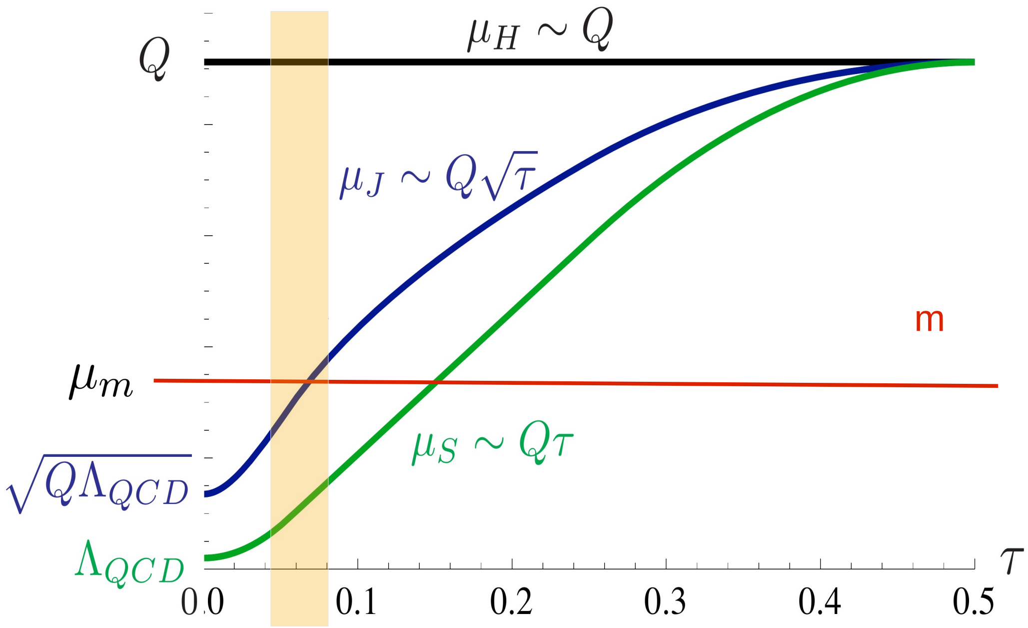
Top width

- Relevant scales at bHQET region $\boxed{Q \gg m \gg \hat{s}, \Gamma, \Lambda_{\mathrm{QCD}}}$

- Matching SCET to a pair of boosted heavy quark effective field theories with stable/ unstable heavy quarks.

$$p^\mu = m v^\mu + k^\mu$$

$$\mathcal{L}_+ = \bar{h}_{v_+}\Big(i v_+ \cdot D_+ - \delta m + \frac{i}{2}\Gamma\Big)h_{v_+} \qquad v_+^\mu = \Big(\frac{m}{Q}, \frac{Q}{m}, \mathbf{0}_\perp\Big), \qquad k_+^\mu \sim \Gamma\Big(\frac{m}{Q}, \frac{Q}{m}, 1\Big)$$



- The collinear fluctuations of the order of mass are integrated out.
- Describes the QCD dynamics and decays of top and anti-top quarks near their mass shell within the jets.
- The soft cross talk of jets is universal (Soft Wilson lines)

$$i\mathcal{D}_+^\mu = i\tilde{\partial}_+^\mu + gA_+^\mu + \frac{\bar{n}^\mu}{2} g n \cdot A_s$$

- **Short distance mass** should be implemented.

$$\delta m_{\overline{\mathrm{MS}}} \sim \alpha_s \overline{m} \text{ V.S } \delta m_{\mathrm{Jet}} \sim \alpha_s R \sim \alpha_s \hat{s}$$

bHQET

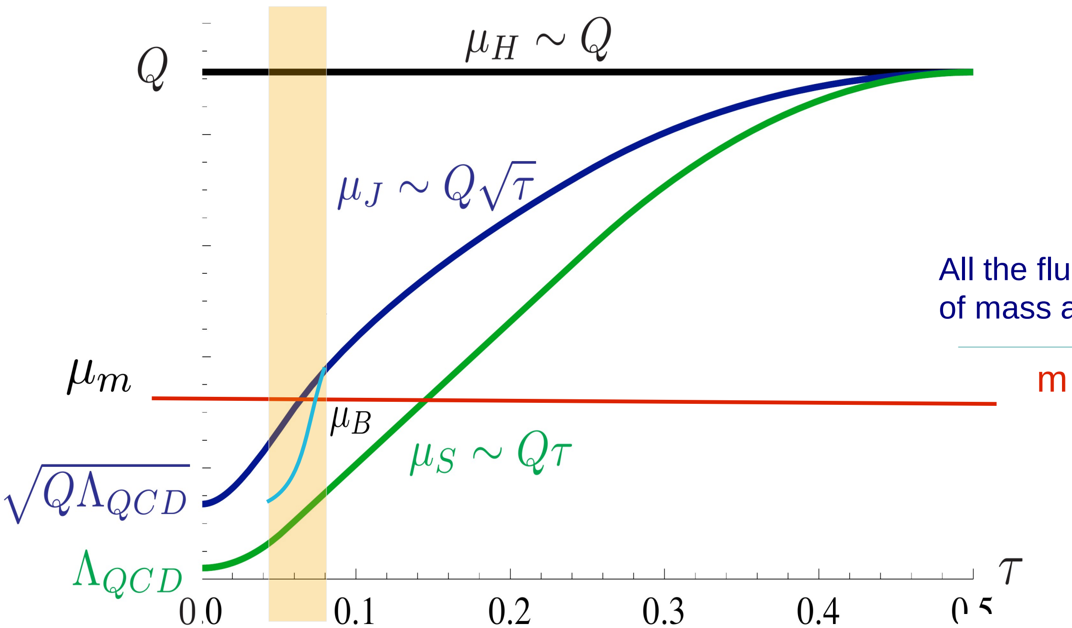
At NNLL

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{bHQET}} = Q\, \boxed{H_Q^{(n_f)}(Q,\mu_Q)}\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\, H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m)\, U_{H_m}^{(n_l)}\Big(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\Big)$$

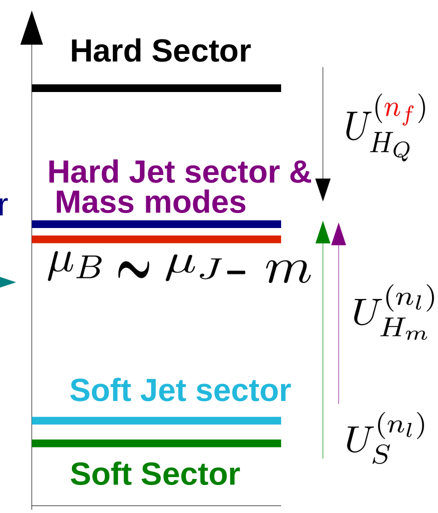
$$\int \mathrm{d}s \int \mathrm{d}k\; B^{(n_l)}\Big(\frac{s}{m_J^{(n_l)}},\mu_B,m_J^{(n_l)}\Big)\, U_S^{(n_l)}(k,\mu_B,\mu_S)\, \boxed{S_{\mathrm{part}}^{(n_l)}\Big(Q\tau - Q\tau_{\mathrm{MIN}} - \frac{s}{Q} - k,\mu_S\Big)}$$

$$n_f = n_l + 1$$

$$\tau_{\mathrm{MIN}} = 1 - \sqrt{1 - 4\big(m_J^{(n_l)}/Q\big)^2}$$



All the fluctuations of the order of mass are integrated out.



- **Hard** function remains the same as scenario (III) and (IV).
- **Soft** function remains the same as scenario (III).

bHQET

At NNLL

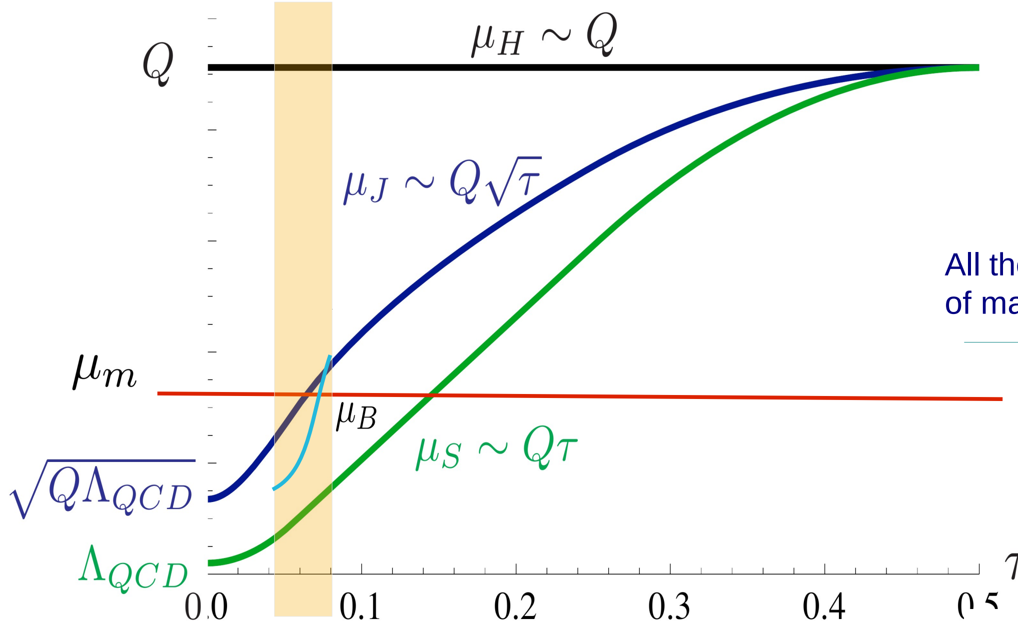
$$\left|\frac{1}{\sigma_0}\frac{d\hat{\sigma}(\tau)}{d\tau}\right|^{\text{bHQET}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\, H_m^{(n_f)}\left(\overline{m}^{(n_f)},\mu_m\right) U_{H_m}^{(n_l)}\left(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\right)$$

$$\int ds \int dk\, B^{(n_l)}\left(\frac{s}{m_J^{(n_l)}},\mu_B, m_J^{(n_l)}\right) U_S^{(n_l)}(k,\mu_B,\mu_S)\, S_{\text{part}}^{(n_l)}\left(Q\tau - Q\tau_{\text{MIN}} - \frac{s}{Q} - k,\mu_S\right)$$

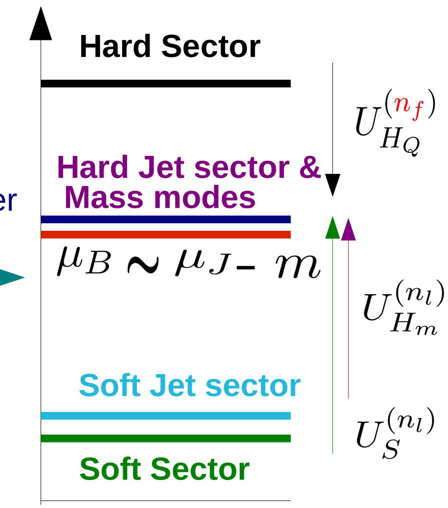
$$n_f = n_l + 1$$

$$\tau_{\text{MIN}} = 1 - \sqrt{1 - 4\left(m_J^{(n_l)}/Q\right)^2}$$

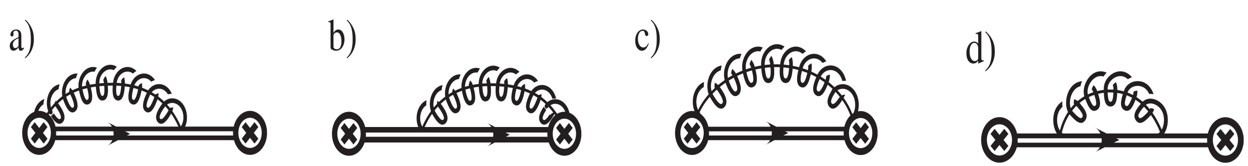
Mass dependence has been explicitly entered the distribution.



All the fluctuations of the order of mass are integrated out.



- **bHQET jet** function and **corresponding matching coefficient (**Large log from secondary corrections**) .**



bHQET

At NNLL

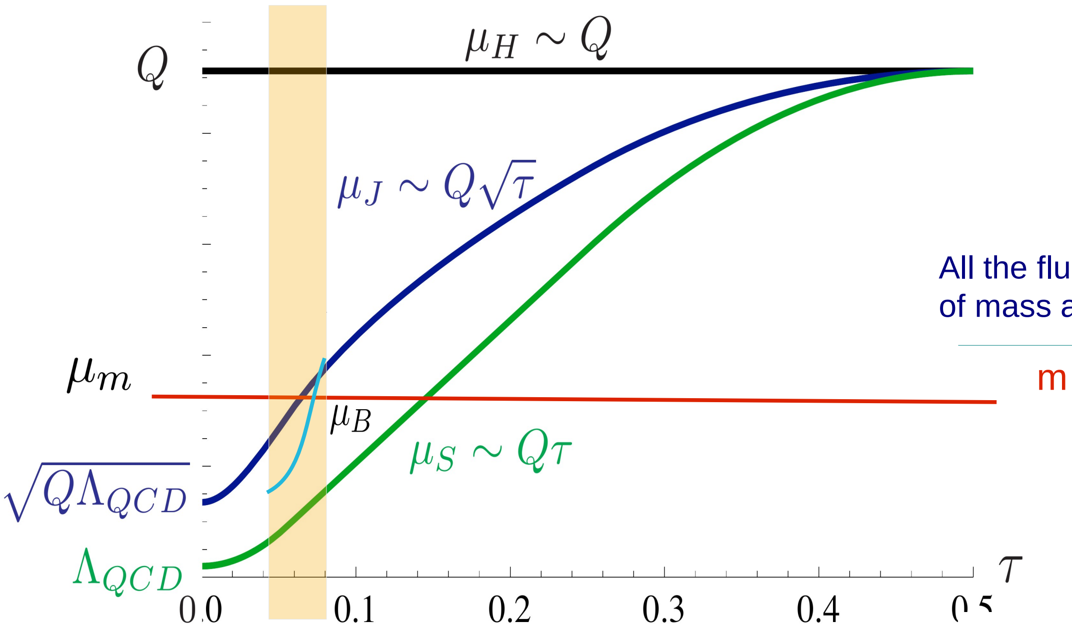
$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat\sigma(\tau)}{\mathrm{d}\tau}\right|^{\rm bHQET} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\, H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m)\, U_{H_m}^{(n_l)}\left(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\right)$$

$$\int \mathrm{d}s \int \mathrm{d}k\, B^{(n_l)}\left(\frac{s}{m_J^{(n_l)}},\mu_B,m_J^{(n_l)}\right) U_S^{(n_l)}(k,\mu_B,\mu_S)\, S_{\rm part}^{(n_l)}\left(Q\tau - Q\tau_{\rm MIN} - \frac{s}{Q} - k,\mu_S\right)$$

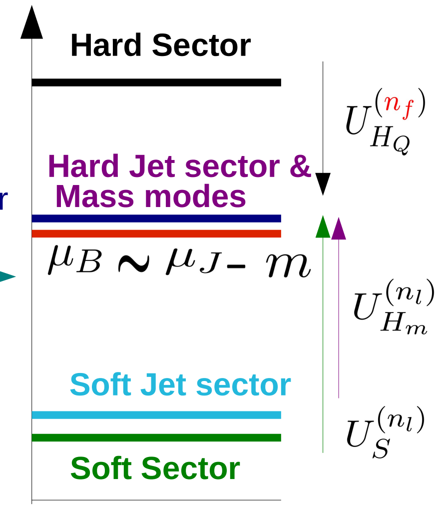
$$n_f = n_l + 1$$

$$\tau_{\rm MIN} = 1 - \sqrt{1-4\left(m_J^{(n_l)}/Q\right)^2}$$

Mass dependence has been explicitly entered the distribution.



All the fluctuations of the order of mass are integrated out.



- **bHQET jet** function and **corresponding matching coefficient (**Large log from secondary corrections**) .**

$$m_J B^{(n_l)}\left(\hat s,\mu,m_J\right) = \delta(\hat s) - 4\delta m_J \delta'(\hat s) + \frac{C_F\alpha_s^{(n_l)}(\mu)}{\pi}\left\{2\delta(\hat s)\left(1-\frac{\pi^2}{8}\right) + \frac{4}{\mu}\left[\frac{\theta(\hat s/\mu)\ln(\hat s/\mu)}{\hat s/\mu}\right]_+ - \frac{2}{\mu}\left[\frac{\theta(\hat s/\mu)}{\hat s/\mu}\right]_+\right\}$$

$$H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m) = 1 + C_F\left(\frac{\alpha_s^{(n_f)}(\mu_m)}{4\pi}\right)\left\{L_m^2 - L_m + 4 + \frac{\pi^2}{6}\right\} + C_F\left(\frac{\alpha_s^{(n_f)}(\mu_m)}{4\pi}\right)^2 \ln\left(\frac{\overline{m}^2(\mu_m)}{Q^2}\right)\left\{\frac{4}{3}L_m^2 + \frac{40}{9}L_m + \frac{112}{27}\right\}$$

Further Theoretical Remarks

- Inclusion of **non-singular** terms vary for vector and axial-vector channels.

$$\frac{d\sigma^{\text{nonsing.}}_{\text{part.}}(\tau)}{d\tau} = \frac{d\sigma^{\text{FO}}(\tau)}{d\tau} - \frac{d\sigma^{\text{SCET}}_{\text{part.}}(\tau)}{d\tau}$$

- Convolution with the **soft model** function (Hadronization effects)

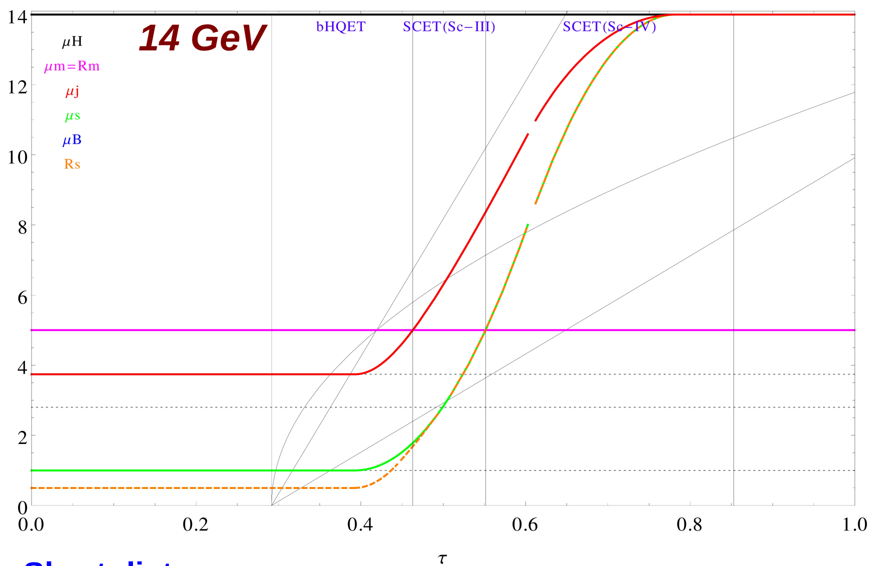
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \Big(\frac{\mathrm{d}\sigma^{\text{SCET}}_{\text{part}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma^{\text{nonsing}}_{\text{part}}}{\mathrm{d}\tau}\Big)\Big(\tau - \frac{k}{Q}\Big) \times S^{\text{model}}_{\tau}(k - 2\Delta(R,\mu)) + \mathcal{O}(\tau, \frac{\Lambda_{\text{QCD}}}{Q})$$

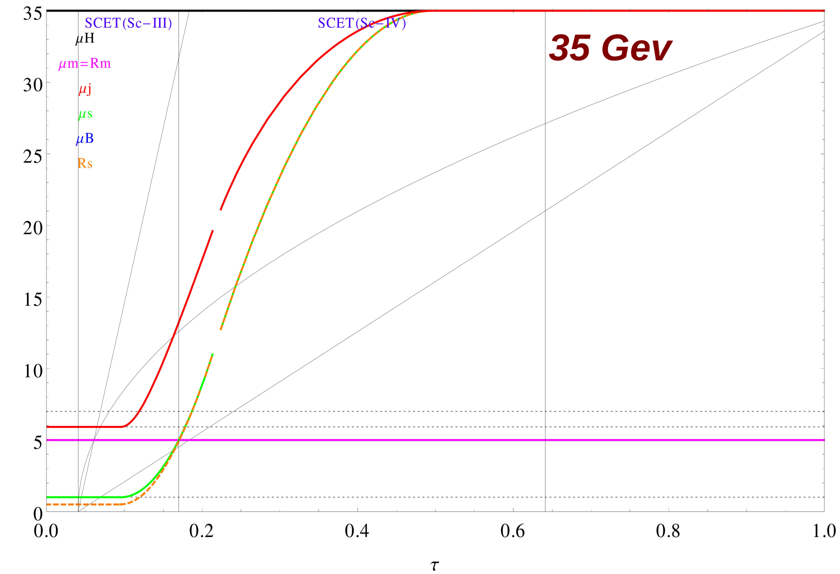
- **Heavy quark Masses From Jets**
    - Peak properties are sensitive observables to mass.
    - Renormalon ambiguity of pole mass (IR sensitive since it is not physical observable)
    - Short distance schemes to preserve the power counting. (Jet mass, renormalization group improved scheme like MSR)

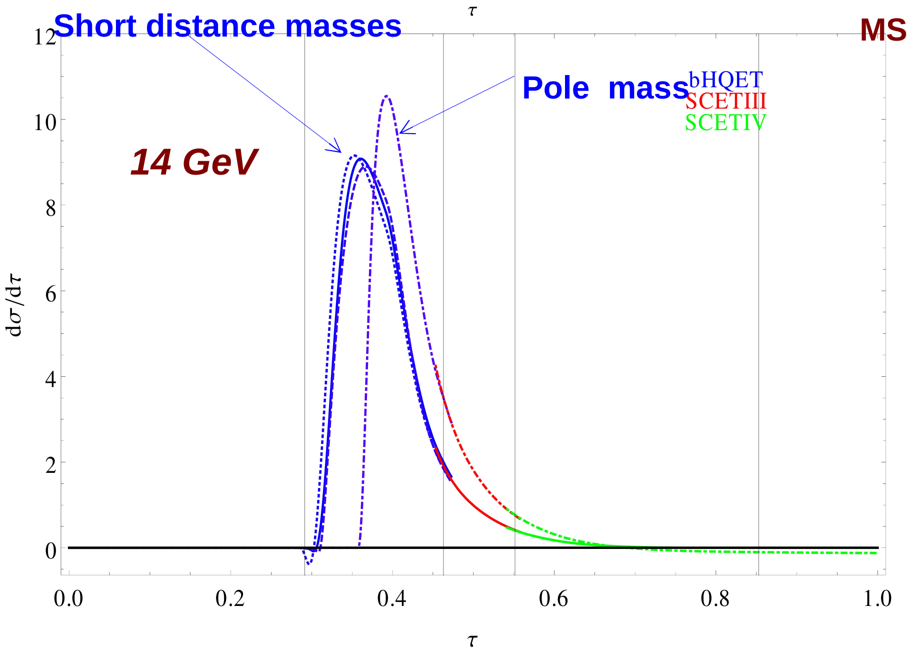
Preliminary Results

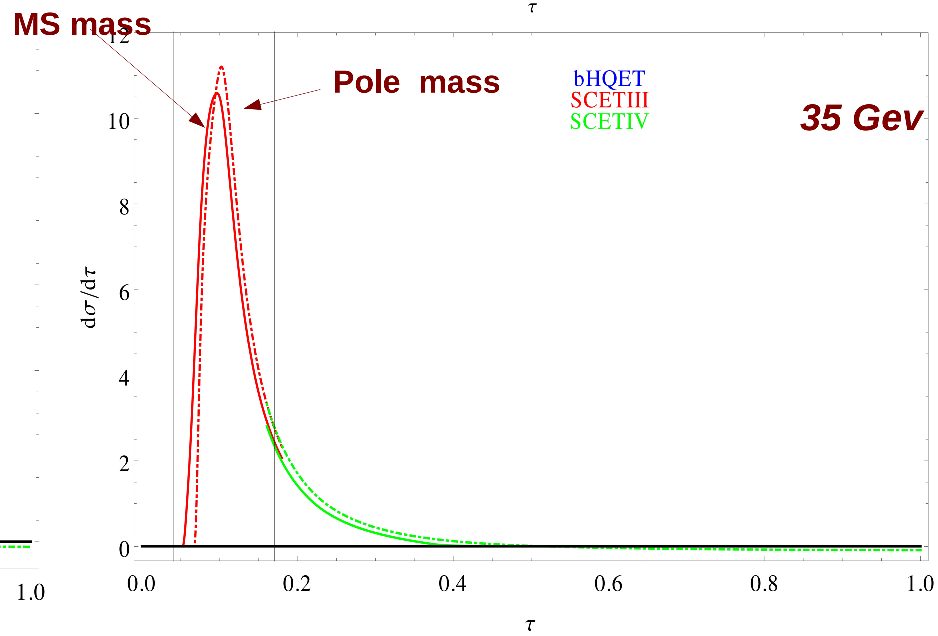
At NNLL

Current status: Theoretical calculations up to **NNLL** (singular) for thrust distribution.
Bottom mass effects at low c.m. energy.









Conclusion and outlooks

Conclusions

- ***Event shapes*** are proper jet observables to study.
- **SCET** is an appropriate framework to study jet physics.
- ***Three different effective field theoretic*** setups to describe the ***entire massive thrust distribution****.*
- Sequence of EFTs to describe the ***jets close their mass-shell***(SCET+**bHQET**)
- ***Peak*** properties are sensitive enough to ***study the heavy quark masses***.
- ***Short distance mass*** vs ***MC mass parameters***

Out looks

- ***Bottom mass*** using data from low $E_{c.m.}$ (Q = 14 , 22 GeV).
- Study the mass parameters of MC generators, which is essential for precise measurement of ***top mass***.
- Improve the analysis to ***NNNLL*** (***Two loop jet function*** should be computed).

Thank you!