Analysis Strategies

for α_s from τ decays

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Tau Moments

Define general τ moments (without factor $|V_{ud}|^2 S_{\rm EW}$):

$$R^w_{V/A}(s_0) \equiv 6\pi i \oint\limits_{|s|=s_0} \frac{ds}{s_0}\, w(s) \left[\Pi^{(1+0)}_{V/A}(s) + \frac{2s}{(s_0+2s)}\, \Pi^{(0)}_{V/A}(s) \right].$$

For $R_{\tau,V/A}$, the kinematic weight reads: $(x \equiv s/s_0)$

$$w_\tau(x) = (1-x)^2 (1+2x) = 1 - 3x^2 + 2x^3 .$$

And the general decomposition of $R^w_{\tau,V/A}(s_0)$:

$$R^w_{V/A}(s_0) = \frac{N_c}{2} \left[\delta^{\rm tree}_w + \delta^{(0)}_w(s_0) + \sum_{D\geq 2} \delta^{(D)}_{w,V/A}(s_0) + \delta^{\rm DV}_{w,V/A}(s_0) \right].$$

Weight Functions

Several types of weight functions are available:

- With pinch-suppression and a "1":

  $w_\tau = (1-x)^2(1+2x)$, $w_7 = 1-x^2$, $w_8 = 1-x^3$.

- With pinch-suppression and without a "1":

  $w_{16} = (1-x)^3x^2(1+2x)$, $w_{16} = (1-x)^3x^3(1+2x)$.

- With or without a linear term in "x":

  $w_6 = 1-x$, $w_{13} = (1-x)^3(1-2x)$.

- Without pinch-suppression;

  $w_1 = 1$, $w_2 = x$, $w_3 = x^2$.

Naming according to: (Boito, Beneke, MJ 2013)

Adler Function Model

(Beneke, MJ 2008)

To incorporate known renormalon structure, use Ansatz:

$$B[\widehat{D}](u) = B[\widehat{D}_1^{\rm UV}](u) + B[\widehat{D}_2^{\rm IR}](u) + B[\widehat{D}_3^{\rm IR}](u) + d_0^{\rm PO} + d_1^{\rm PO}\, u\,.$$

with

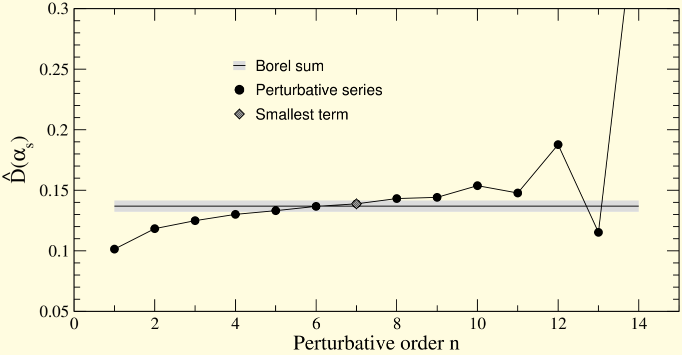
$$B[\widehat{D}_p^{\rm IR}](u) \equiv \frac{d_p^{\rm IR}}{(p-u)^{1+\tilde{\gamma}}}\Big[\,1 + \tilde{b}_1(p-u) + \tilde{b}_2(p-u)^2 + \ldots\,\Big]\,.$$

Fitting $c_{1,1}$ to $c_{5,1}$, the parameters are found to be:

$$d_1^{\rm UV} = -1.56\cdot 10^{-2}\,, \qquad d_2^{\rm IR} = 3.16\,, \qquad d_3^{\rm IR} = -13.5\,,$$

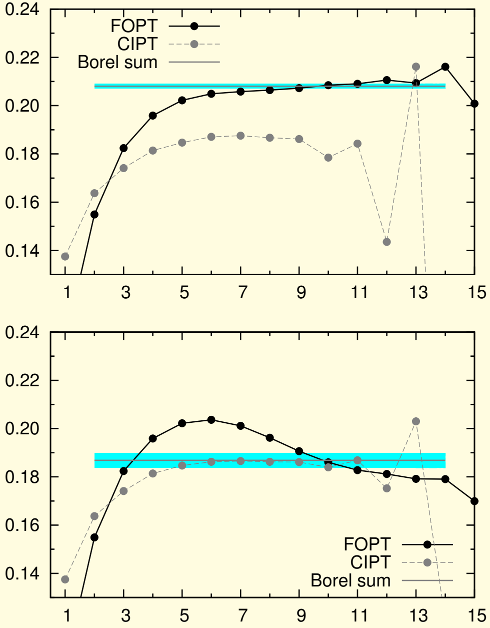
$$d_0^{\rm PO} = 0.781\,, \qquad d_1^{\rm PO} = 7.66\cdot 10^{-3}\,.$$

Adler Function

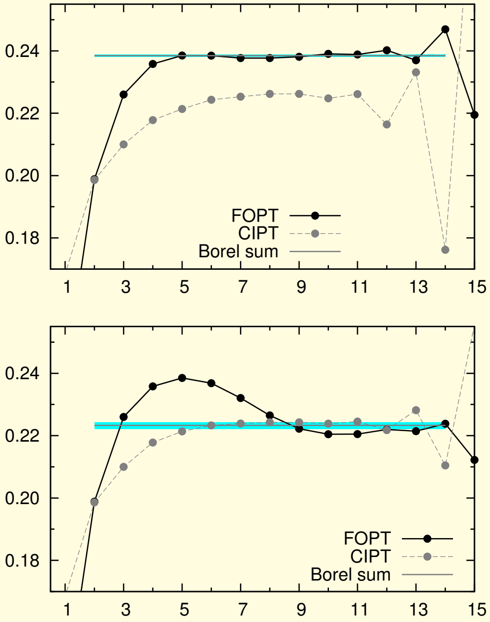


$$\alpha_s(M_\tau) = 0.3186\,, \quad c_{5,1} = 283\,.$$

$$w_\tau(x) = (1-x)^2(1+2x)$$



$$w_7(x) = 1 - x^2$$



0.24
0.22
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FOPT
CIPT
Borel sum
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0.22
0.20
0.18
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3
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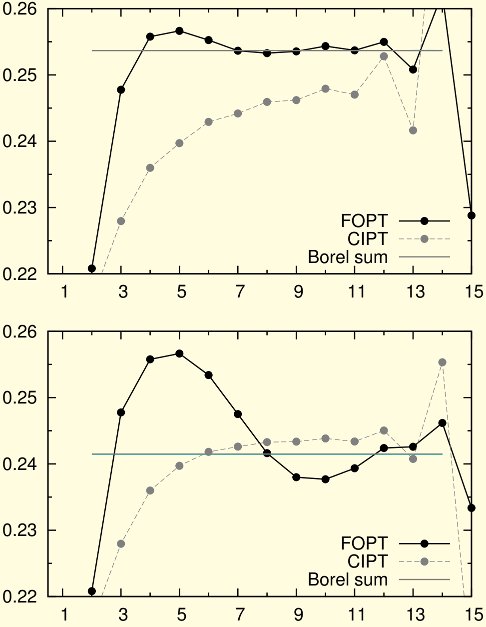
FOPT
CIPT
Borel sum

$$w_8(x) = 1 - x^3$$



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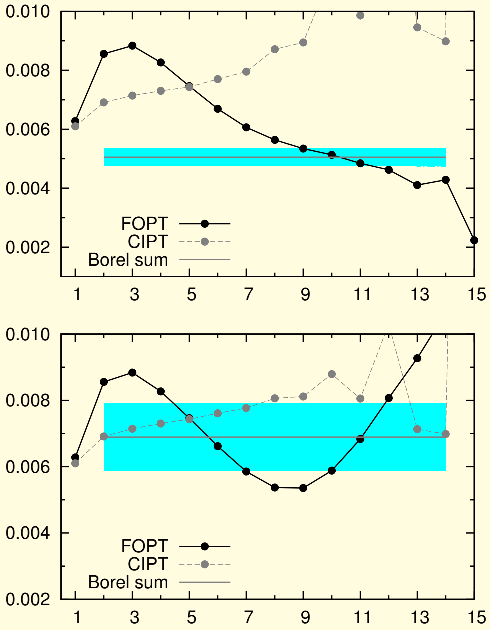
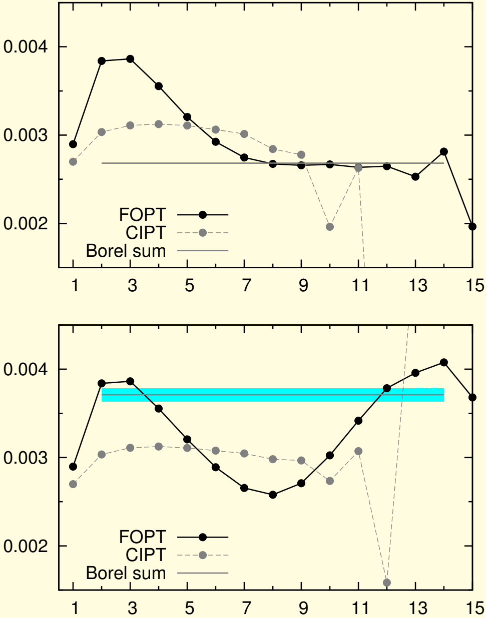
FOPT
CIPT
Borel sum

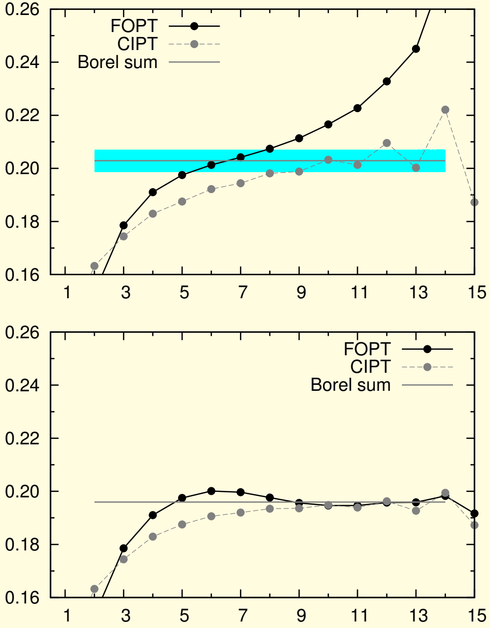
$$w_{16}(x) = (1-x)^3 x^2 (1+2x)$$



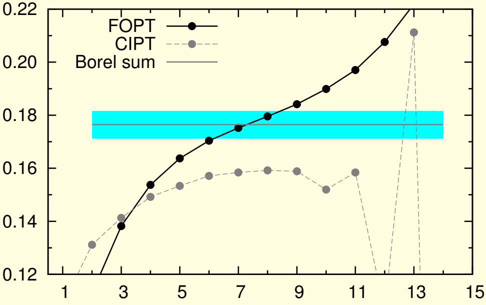
$$w_{17}(x) = (1-x)^3 x^3 (1+2x)$$



$w_6(x) = 1 - x$



$$w_{13}(x) = (1-x)^3(1+2x)$$



0.22
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FOPT
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Borel sum
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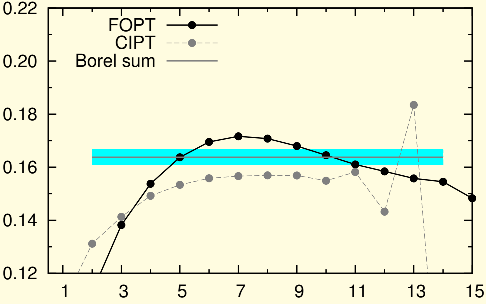

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0.12
FOPT
CIPT
Borel sum
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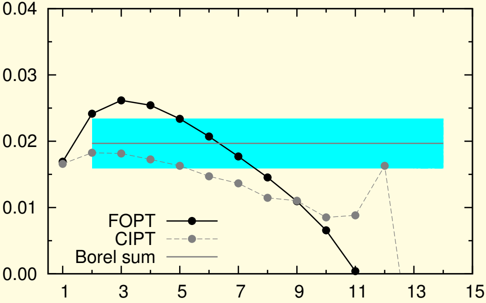
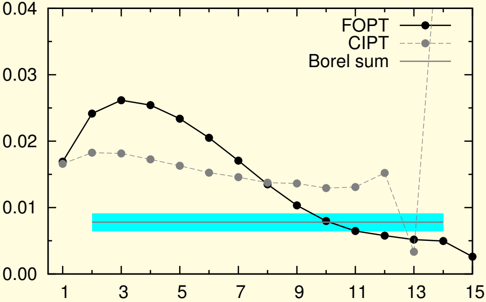
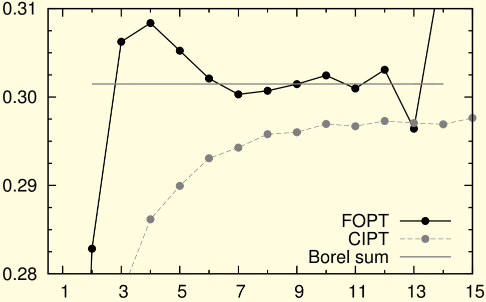
$$w_{15}(x) = (1-x)^3 x(1+2x)$$





$$w_1(x) = 1$$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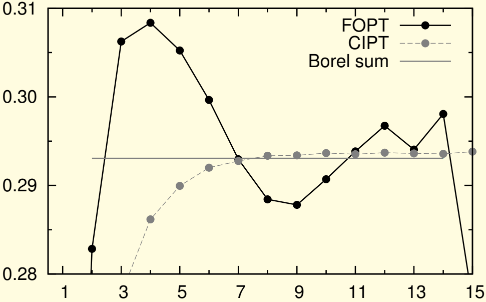
CIPT
Borel sum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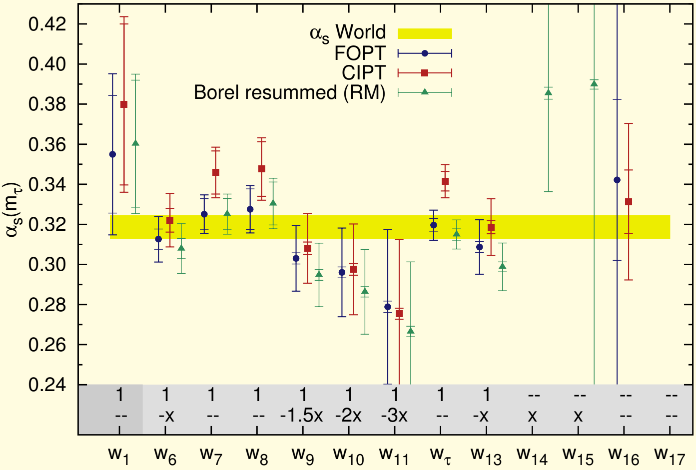
Borel sum



$\alpha_s(m_\tau)$
0.42
0.40
0.38
0.36
0.34
0.32
0.30
0.28
0.26
0.24
α_s World
FOPT
CIPT
Borel resummed (RM)
1 1 1 1 1 1 1 1 1 -- -- -- --
-- -x -- -- -1.5x -2x -3x -- -x x x -- --
w_1 w_6 w_7 w_8 w_9 w_{10} w_{11} w_τ w_{13} w_{14} w_{15} w_{16} w_{17}

Duality Violations

In the OPE, close to the Minkowskian axis ($s > 0$), so-called Duality Violations (DV's) can appear.

They can be studied on the basis of a toy-model:

(Shifman et al. 1998-2000)

(Catà, Golterman, Peris 2005/2008)

$$\Pi_V(s) = -\psi\left(\frac{M_V^2 + u(s)}{\Lambda^2}\right) + \text{const.}\,.$$

where

$$u(s) = \Lambda^2 \left(\frac{-s}{\Lambda^2}\right)^{\zeta} \quad \text{and} \quad \zeta = 1 - \frac{a}{\pi N_c}\,.$$

The model is based on large-N_c QCD and Regge-theory.

$$M_V = 770\,\mathrm{MeV}, \quad \Lambda = 1.2\,\mathrm{GeV}, \quad a = 0.4\,.$$

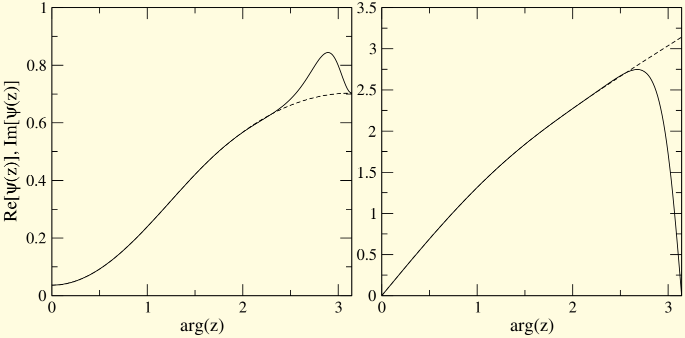
The OPE corresponds to the asymptotic expansion of the ψ-function for large s (large u).

$$\psi(z) \sim \ln z - \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nz^{2n}}\,, \quad \mathrm{Re}z > 0\,.$$

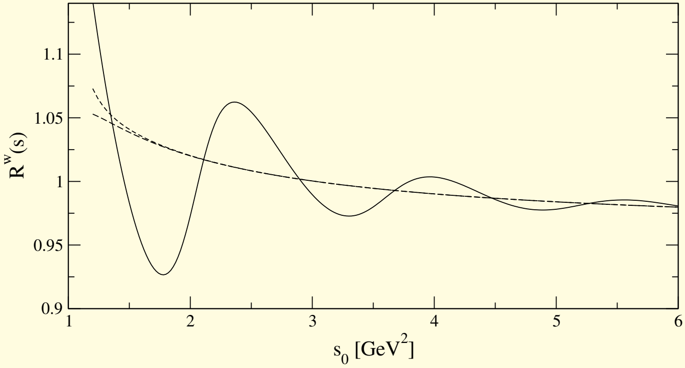
In the Minkowskian region, an additional term arises:

$$-\pi\,[\cot(\pi z) \pm i\,]\,, \quad \mathrm{Re}z < 0\,,\ \mathrm{Im}z \gtrless 0\,.$$

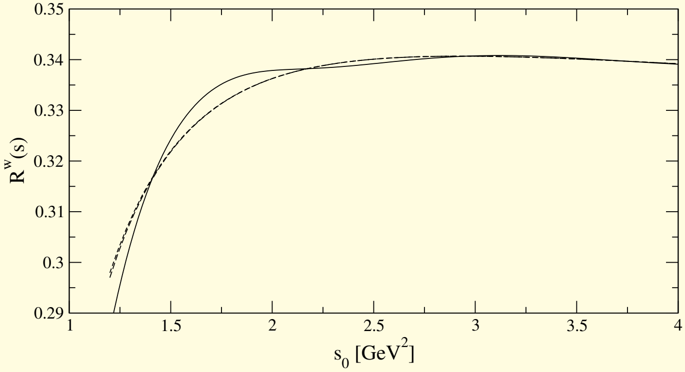
Formally, this term is exponentially suppressed, but it is enhanced by the poles of the ψ-function.



$$z = 1.5 \cdot \exp(i\varphi)$$



ψ-function moment for $w(z) = 1$.



ψ-function moment for $w(z) = (1 - z)^2$.

In fits to data, a model for DV's should be included.

The ψ-function model suggests an oscillating, decaying exponential, which can be chosen of the form:

$$\rho^{\mathrm{DV}}_{V/A}(s) = \kappa_{V/A}\, \mathrm{e}^{\gamma_{V/A} s} \sin(\alpha_{V/A} + \beta_{V/A} s) \,.$$

The fit quantities are the w-moments of the exp spectra.

$$R^{w}_{\tau,V/A}(s_0) \equiv \int\limits_0^{s_0} ds\, w(s)\, \rho_{V/A}(s) \,.$$

The cleanest moment turns out to be $w(s) = 1$.

Fitting combinations of several moments (e.g. w_τ, $1 - x^2$, "1") is complicated by very strong correlations.

Outlook

- Lecture 4: Description of exclusive τ decay distributions.

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Thank You!