Facets of Chiral Perturbation Theory

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Contents

Motivation and overview

Goal

systematic and quantitative treatment of the Standard Model at low energies ($E < 1$ GeV)

- Effective Field Theory (EFT)
- Lattice Field Theory

main objectives

- understand physics of the SM at low energies
- look for evidence of new physics

$E < 1$ GeV:

- strong-coupling regime of QCD $\longrightarrow$ not accessible in standard perturbation theory

key concept for EFT: approximate chiral symmetry of QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm tr}(G_{\mu\nu}G^{\mu\nu}) + \sum_{f=1}^{6} \overline{q}_f \left(i\gamma^\mu D_\mu - m_f \mathbb{1}_c\right) q_f$$

for $m_f = 0$: chiral components can be rotated separately

$$q_{fL} = \frac{1}{2}(1-\gamma_5)q_f\,, \qquad q_{fR} = \frac{1}{2}(1+\gamma_5)q_f$$

$\longrightarrow$ chiral symmetry $SU(n_F)_L \times SU(n_F)_R \times U(1)_V$

$m_f = 0$:

- very good approximation for $n_F = 2 \quad (u, d)$
- reasonable approximation for $n_F = 3 \quad (u, d, s)$

in contrast to isospin $SU(2)$ or flavour $SU(3)$:

no sign of chiral symmetry in hadron spectrum

many other arguments in favour of

spontaneous breaking of chiral symmetry

$$SU(n_F)_L \times SU(n_F)_R \times U(1)_V \quad \longrightarrow \quad SU(n_F)_V \times U(1)_V$$

Goldstone theorem:

$\exists\ n_F^2 - 1$ massless (for $m_f = 0$) Goldstone bosons

Goldstone fields parametrize $SU(n_F)_L \times SU(n_F)_R \ / \ SU(n_F)_V$

n_F	$n_F^2 - 1$	Goldstone bosons
2	3	π
3	8	π, K, η

even in the real world ($m_q \neq 0$):

pseudo-scalar meson exchange dominates amplitudes at low energies

$\longrightarrow$ construct EFT for pseudo-Goldstone bosons

nonlinear realization of chiral symmetry $\longrightarrow$

effective Lagrangian necessarily nonpolynomial consequence:

EFT nonrenormalizable QFT: Chiral Perturbation Theory (CHPT)

Weinberg, Gasser, Leutwyler,. . .

nevertheless: CHPT fully renormalized QFT
(to next-to-next-to-leading order)

basis for systematic low-energy expansion:

pseudo-Goldstone bosons decouple for vanishing momenta and masses

systematic approach for low-energy hadron physics

- most advanced in meson sector (up to 2 loops)
- also single-baryon sector and few-nucleon systems
- electroweak interactions can be included

Effective chiral Lagrangian (meson sector)

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}^{\text{odd}}(0) + \mathcal{L}_{G_F p^2}^{\Delta S=1}(2) + \mathcal{L}_{G_8 e^2 p^0}^{\text{emweak}}(1) + \mathcal{L}_{e^2 p^0}^{\text{em}}(1) + \mathcal{L}_{\text{kin}}^{\text{leptons}}(0)$	$L = 0$
$+ \mathcal{L}_{p^4}(10) + \mathcal{L}_{p^6}^{\text{odd}}(23) + \mathcal{L}_{G_8 p^4}^{\Delta S=1}(22) + \mathcal{L}_{G_{27} p^4}^{\Delta S=1}(28) + \mathcal{L}_{G_8 e^2 p^2}^{\text{emweak}}(14) + \mathcal{L}_{e^2 p^2}^{\text{em}}(13) + \mathcal{L}_{e^2 p^2}^{\text{leptons}}(5)$	$L \leq 1$
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

LECs: low-energy constants $\equiv$ coupling constants of CHPT

in red: Lagrangians relevant for nonleptonic K decays

Nonleptonic kaon decays

dominant decays: $K \to 2\pi, 3\pi$

LO — Cronin

NLO — Kambor, Missimer, Wyler

NLO + isospin violation + rad. corrs. — Cirigliano, E., Neufeld, Pich; Bijnens, Borg

$\longrightarrow$ LO couplings G_8, G_{27} well known

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$+ \mathcal{L}_{p^4}(10) + \mathcal{L}^{\text{odd}}_{p^6}(23) + \mathcal{L}^{\Delta S=1}_{G_8 p^4}(22) + \mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28) + \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14) + \mathcal{L}^{\text{em}}_{e^2 p^2}(13) + \mathcal{L}^{\text{leptons}}_{e^2 p^2}(5)$	$L \leq 1$
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

in red: LO Lagrangian for nonleptonic K decays

in blue: NLO Lagrangian — " —

Nonleptonic kaon decays

dominant decays: $K \longrightarrow 2\pi, 3\pi$

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$\longrightarrow$ LO couplings G_8, G_{27} well known

N.B.: all other nonleptonic transitions start at NLO $= O(G_F p^4)$

problem: 22 (octet) + 28 (27-plet) LECs

Theorists' favourite nonleptonic decays

$K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$ [, $K_S \to \pi^0\pi^0\gamma\gamma$]

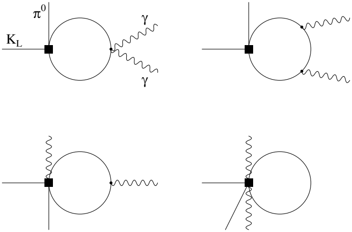
no LECs at all at NLO !

Status at $O(G_F p^4)$

$K_S \to \gamma\gamma$	D'Ambrosio, Espriu; Goity
$K_L \to \pi^0\gamma\gamma$	E., Pich, de Rafael; Cappiello, D'Ambrosio
$K_S \to \pi^0\pi^0\gamma\gamma$	Funck, Kambor

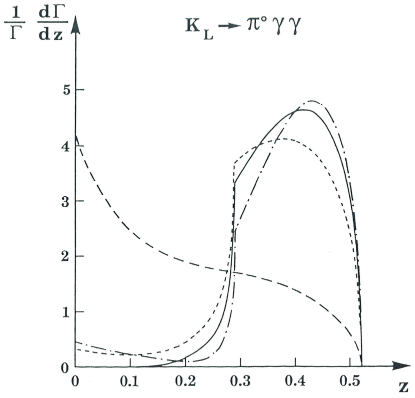
$O(G_F p^2)$ no contribution

$O(G_F p^4)$ LECs do not contribute $\to$ finite loop amplitude



pre-CHPT: $K_L \to \pi^0\gamma\gamma$ vector-meson dominated

compare 2-photon spectra

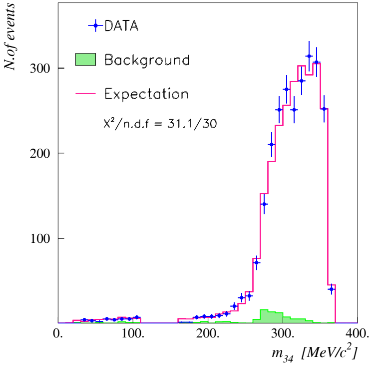


Normalized decay distribution in $z = M_{\gamma\gamma}^2/M_K^2$

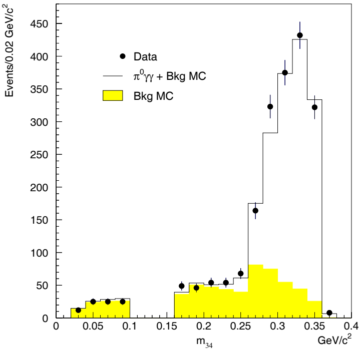
E., Pich, de Rafael

leading order $[O(p^4)]$	**full curve**
pure vector resonance exchange	**dashed curve**
[$a_V = -.32$	**dotted curve]**

$K_L \to \pi^0\gamma\gamma$ decay distribution in $m_{34} = M_{\gamma\gamma}$



NA48 (2002)



KTeV (2008)

higher-order corrections (starting at NNLO $= O(G_F p^6)$)

- rescattering (unitarity) corrections largely model independent

  Cappiello, D'Ambrosio, Miragliuolo; Cohen, E., Pich; Kambor, Holstein

  $K_S \to \gamma\gamma$ "trivial" in terms of $K \to 2\pi$ rate

  $K_L \to \pi^0\gamma\gamma$ more involved but straightforward

- resonance contributions

  Cohen, E., Pich; D'Ambrosio, Portolés; Buchalla, D'Ambrosio, Isidori

  $K_S \to \gamma\gamma$ small (vector mesons cannot contribute)

  $K_L \to \pi^0\gamma\gamma$ vector meson contribution model dependent

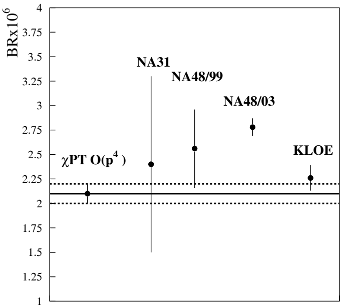
  good approximation: single parameter a_V

$K_S \to \gamma\gamma$

puzzling result of NA48 (2003):
rate substantially bigger than
$O(p^4)$ result

KLOE (2008):
$B(K_S \to \gamma\gamma) = 2.26(12)(06) \times 10^{-6}$

$\longrightarrow$ perfect agreement



not a good idea:

PDG averages NA48/03 and KLOE $\longrightarrow$
$B(K_S \to \gamma\gamma) = 2.63(17) \times 10^{-6}$
??

$K_L \to \pi^0 \gamma\gamma$

for reasonable values of a_V:

pion loop dominates 2γ-spectrum

rate more affected both by rescattering corrections and by a_V

$\Longrightarrow$ excellent agreement between theory and experiment

$$B(K_L \to \pi^0\gamma\gamma)\cdot 10^6 = \begin{cases} 1.27 \pm 0.04 \pm 0.01 & \text{NA48 (2002)} \\ 1.28 \pm 0.06 \pm 0.01 & \text{KTeV (2008)} \\ 1.273 \pm 0.033 & \text{PDG (2012)} \end{cases}$$

$$a_V = -0.43 \pm 0.06 \qquad \text{PDG (2012)}$$

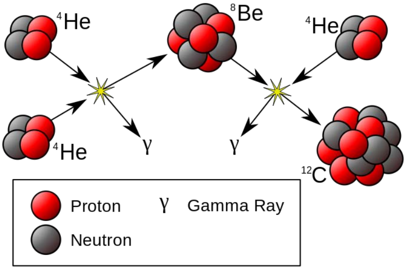
important consequence

CP-conserving contribution $K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+ e^-$ negligible in comparison with CP-violating amplitudes

Carbogenesis: the Hoyle state

almost all carbon produced in stellar nucleosynthesis via

triple-α process



Hoyle (1954): to explain observed carbon abundance

$\longrightarrow$ $\exists$ excited 0^+ state of ^{12}C near ^{8}Be-α threshold

observed soon afterwards

properties of Hoyle state

$\epsilon = 379.47(18)$ keV (above 3α threshold)

$\Gamma_{\text{tot}} = 8.3(1.0)$ eV, $\Gamma_\gamma = 3.7(5)$ meV

triple-α rate $\sim$ $\Gamma_\gamma \exp -\epsilon/kT$ $\longrightarrow$ mainly sensitive to ϵ

example of anthropic principle?

Livio et al. (1989), Oberhummer et al. (2004)

$\Delta\epsilon \lesssim 100$ keV tolerable to explain abundance of ^{12}C, ^{16}O

$\longrightarrow$ not exactly severe fine-tuning

however:

more interesting issue:

dependence of ϵ on fundamental parameters of strong and electromagnetic interactions one-parameter (p) nuclear cluster model — Oberhummer et al.

tolerances

$\Delta p/p \lesssim 0.5\%$ $\Delta F_{\mathrm{Coulomb}}/F_{\mathrm{Coulomb}} \lesssim 4\%$

open question:

relation to fundamental parameters of QCD and QED?

chiral EFT of nuclear forces — Weinberg (1990), . . .

expansion of nuclear potential (2-,3-,4-nucleon forces)

successful approach for small nuclei ($A \lesssim 3$)

more recent development

nuclear lattice simulations (Muller, Lee, Borasoy, . . .)

lattice dofs: nucleons (not quarks!) and pions

Monte-Carlo techniques

$\longrightarrow$ energies of low-lying states of ^{12}C (in MeV)

	0_1^+	$2_1^+(E^+)$	0_2^+
LO	$-96(2)$	$-94(2)$	$-89(2)$
NLO	$-77(3)$	$-74(3)$	$-72(3)$
NNLO	$-92(3)$	$-89(3)$	$-85(3)$
Exp	-92.16	-87.72	-84.51

Epelbaum et al.

0_2^+: Hoyle state

method allows to study dependence on

quark masses (via $M_\pi^2 \sim (m_u + m_d)$)

fine-structure constant $\alpha_{\rm em}$ (not $\alpha_{\rm QCD}$)

final conclusion for tolerances

$\Delta m_q/m_q \lesssim 3\%$ $\Delta\alpha_{\rm em}/\alpha_{\rm em} \lesssim 2.5\%$

$\longrightarrow$ fine-tuning in $m_q, \alpha_{\rm em}$ much more severe than in ϵ

Low-energy constants and lattice QCD

Motto (Laurent Lellouch, John F. Kennedy)

Ask not what CHPT can do for the lattice,
but ask what the lattice can do for CHPT

- CHPT $\longrightarrow$ lattice
  (chiral) extrapolation to physical quark (meson) masses
  still useful, but less needed than 5 years ago
- lattice $\longrightarrow$ CHPT
  determination of LECs (FLAG, ...)
  especially welcome for LECs multiplying quark mass terms
  advantage of lattice simulations compared to phenomenology:
  quark (and therefore meson) masses can be tuned

illustrative example:

chiral $SU(3)$ Lagrangian (strong interactions)

$$\mathcal{L}_{p^2}(2) = \frac{F_0^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle$$

$$\mathcal{L}_{p^4}(10) = \cdots + L_4\langle D_\mu U D^\mu U^\dagger\rangle\langle \chi U^\dagger + \chi^\dagger U\rangle + \ldots$$

$\langle \ldots \rangle$ flavour trace

$F_0 = \lim_{m_u,m_d,m_s\to 0} F_\pi$, $\chi = 2B_0\mathcal{M}_q$ ($B_0 \sim$ quark condensate)

$U = \mathbb{1} +$ meson fields

gauge-covariant derivative $D_\mu U$ (contains $A_\mu, W_\mu^\pm$)

$$\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}(10) = \frac{1}{4}\langle D_\mu U D^\mu U^\dagger \rangle \left[F_0^2 + 8 L_4 \left(2 \mathring{M}_K^2 + \mathring{M}_\pi^2 \right) \right] + \ldots$$

$\mathring{M}_P$ lowest-order meson mass

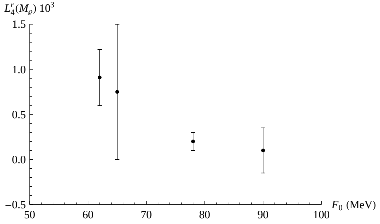
$F_\pi^2/(16 M_K^2) = 2 \times 10^{-3}$ $\sim$ typical size of NLO LEC

consequences

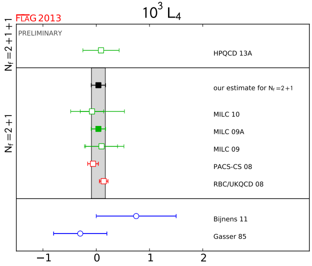
- strong anticorrelation between F_0 and L_4 in global fits

  Bijnens, Jemos

J. Bijnens, I. Jemos : Global fit for SU(3) LECs
Nucl. Phys. B854 (2012) 631



- "convergence" of SU(3) CHPT depends (also) on value of F_0
  chiral expansion: powers of $p^2/(4\pi F_0)^2$
  but : F_0 less known than many higher-order LECs
- rather wide spread in F_0 also from lattice studies $\longrightarrow$ FLAG does not perform an average
- L_4 is large-N_c suppressed $\longrightarrow$ in Gasser, Leutwyler (1985) set to zero (more precisely: $L_4^r(M_\eta) = 0 \pm 0.5 \times 10^{-3}$)
  FLAG (2011): published lattice determinations (for $L_4^r(M_\rho)$)



comparison between $SU(2)$ and $SU(3)$

$F = \lim_{m_u,m_d \to 0} F_\pi$

$$F_0 = F - F^{-1} \left\{ \left(2M_K^2 - M_\pi^2\right) \left(4L_4^r(\mu) + \frac{1}{64\pi^2} \log \mu^2/M_K^2\right) + \frac{M_\pi^2}{64\pi^2} \right\} + O(p^6)$$

"paramagnetic" inequality (Descotes-Genon, Girlanda, Stern)

$$F_0 < F \quad \longrightarrow \quad L_4^r(M_\rho) > -0.4 \times 10^{-3}$$

FLAG (2013): $F = (85.9 \pm 0.6)$ MeV

$\longrightarrow$ linear relation between F_0 and L_4

suggestion:

determine F_0, L_4 from $SU(3)$ lattice data for F_π

advantage:

anti-correlation can be softened by tuning quark masses

$\longrightarrow$ smaller errors for F_0, L_4

essential: CHPT to NNLO $= O(p^6)$ (Amoros,Bijnens, Talavera)

drawbacks:

chiral $SU(3)$ expression at NNLO (2 loops) quite involved

only available in numerical form

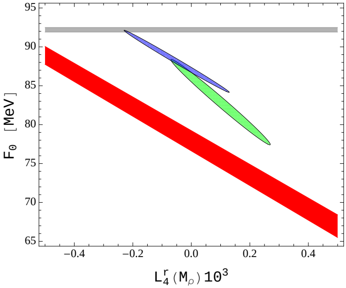
$\Longrightarrow$ most lattice groups ignore CHPT amplitudes

proposal: E., Masjuan, Neufeld

employ large-N_c motivated approximation for 2-loop calculation

requires tree- and 1-loop amplitudes only in addition: need some knowledge of L_5 and LECs of $O(p^6)$

(e.g.: from analysis of F_K/F_π)



RBC/UKQCD data (2013)

$F_0 = (88.1 \pm 4.0)$ MeV

$L_4^r(M_\rho) = (-0.05 \pm 0.18) \cdot 10^{-3}$

green ellipse: lattice data only

blue ellipse: include $F_\pi^{\text{exp}} = (92.2 \pm 0.3)$ MeV

most precise value for F_0

discrepancy with $SU(2)$ constraint (red band) ?

persisting anti-correlation between F_0, L_4 despite lattice data ?

Conclusions

main objectives

- understand physics of the SM at low energies
- look for evidence of new physics

objectives accomplished?

- we have gone some way in understanding the SM at low energies
- on the other hand: we have not found evidence for new physics
- but neither has the LHC !