On the Higgs triplet extension of the Standard Model

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Outline

- Introduction
- The Higgs triplet model
- Neutrino mass terms
- Cross sections
- Decay widths
- Conclusions

Introduction

Introduction: Unsolved problems of the SM

- The Standard Model of particle physics (SM) has to be extended due to unsolved problems like
 - Quantum gravity
 - baryon asymmetry
 - dark matter
 - hierarchy problem
 - strong CP-problem
 - neutrino masses
 - etc.

Introduction: Extension of the SM

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- Fundamental extensions:
 - Introduction of a larger gauge group than SU(3)_C × SU(2)_L × U(1)_Y like in Grand Unifying Theories
 - Assumtion of extra dimensions like in string theories
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 - etc.
- Particular extensions:
 - Augmentation of a single sector of the SM like the scalar sector, etc.

- The scalar sector can be extended by introducing additional Higgs multiplets such as in
 - Two Higgs doublet models (SM + 2ϕ)
 - Zee model (SM + $2\phi + \eta^+$)
 - Zee-Babu model (SM + η^+ + k^{++})
 - Higgs triplet model (SM + $\overrightarrow{\Phi}$ = $(\phi^{++}, \phi^{+}, \phi^{0})$)
 - etc.

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- In the SM neutrinos stay massless due to the absence of right-handed neutrino fields
- But recent experiments have discovered the phenomenon of neutrino flavour oscillations, which is only possible if neutrinos have non-zero and different masses
- One possibility to introduce neutrino masses is the socalled type-II see-saw mechanism, in which a complex scalar triplet with Y = 2, the Higgs triplet, is added to the scalar sector

- The Lagrangian of the Yukawa sector is enhanced with a gauge invariant coupling \mathcal{L}_Δ between the Higgs triplet and the lepton doublets
- \mathcal{L}_{Δ} automatically leads to a Majorana Mass term for neutrinos at tree level proportional to v_T (the vacuum expectation value of the neutral component of the Higss triplet)

The Higgs triplet model

The Higgs triplet model: The idea

 The idea of adding a scalar triplet to the SM was first mentioned in a work of W. Konetschny and W. Kummer in 1977 [1]

^[1] W. Konetschny and W. Kummer, Nonconservation of total lepton number with scalar bosons Phys. Lett. **70B** (1977) 433.

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- It was shown that additional scalar singlets S⁺, S⁺⁺ and a scalar triplet $\Phi = (\phi^{++}, \phi^{+}, \phi^{0})$ permit Yukawa couplings, which allow lepton flavour violating transitions like $\mu \to e \gamma$ and $\mu \to 3e$

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- This idea of an additional a scalar triplet was also used by G.B Gelmini and M. Roncadelli in 1981 in order to introduce neutrino masses [2]

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The Higgs triplet model: The Lagrangian

The Yukawa Lagrangian in the lepton sector is given by [3]

$$\mathcal{L}_{Y} = \sum_{\alpha,\beta} \left\{ -c_{\alpha\beta} \overline{\ell}_{\alpha R} \phi^{\dagger} L_{\beta L} + \frac{1}{2} f_{\alpha\beta} L^{T}_{\alpha L} C^{-1} i \tau_{2} \Delta L_{\beta L} \right\} + \text{H.c.}$$

• α, β

•
$$L_{\alpha L} = (\nu_{\alpha}, \ell_{\alpha L})$$

• $\ell_{\alpha R}$

φ

\(\Delta \)

• C

τ₂

• $c_{\alpha\beta}$, $f_{\alpha\beta}$

flavour indices

left-handed lepton doublets

right-handed lepton singlets

Higgs doublet

2×2 representation of the Higgs triplet

charge conjugation matrix

second pauli matrix

coupling matrices, f symmentric, i.e $f_{\alpha\beta}=f_{\beta\alpha}$

[3] W.Grimus, R.Pfeiffer and T.Schwetz, A 4-neutrino model with a Higgs triplet, Eur. Phys. J. C **13** (2000) 125 [arXiv:hep-ph/9905320]

The Higgs triplet model: The multiplets

• The multiplets transform under $U \in SU(2)$ as

$$L_{\alpha L} \rightarrow UL_{\alpha L}, \; \ell_{\alpha R} \rightarrow \ell_{\alpha R}, \; \phi \rightarrow U\phi, \; \Delta \rightarrow U\Delta U^{\dagger}$$

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• The U(1) transformation properties are determined by their hypercharges:

	$L_{lpha L}$	$\ell_{lpha R}$	ϕ	Δ
Υ	-1	-2	1	2

The Higgs triplet model: The 2×2 representation

• The relation between the triplet and the 2×2 representation is given by

$$\Delta = \vec{\Phi} \cdot \vec{\tau} = \begin{pmatrix} H^{+} & \sqrt{2}H^{++} \\ \sqrt{2}H^{0} & -H^{+} \end{pmatrix} \text{ with } \vec{\Phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}(H^{0} + H^{++}) \\ \frac{1}{\sqrt{2}}(H^{0} - H^{++}) \\ H^{+} \end{pmatrix}$$

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The charge eigenfields are given by

$$H^{++} = \frac{1}{\sqrt{2}}(H_1 - iH_2), H^+ = H_3, H^0 = \frac{1}{\sqrt{2}}(H_1 + iH_2)$$

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The Higgs triplet model: VEVs

 The VEVs of the Higgs multiplets consistent with electric charge conservation are given by

$$\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$
 and $\langle \Delta \rangle_0 = \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix}$

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- We have set $\langle H^0 \rangle_0 = \frac{v_T}{\sqrt{2}}$
- We expect $|v_T| \ll v$, since a larger triplet VEV would destroy the tree-level relation $M_W = M_Z \cos(\theta_W)$ between the gauge boson masses and the Weinberg angle and precision measurements place a stringent bound on $v_T[4]$

[4] J.Erler and P.Langacker, Constraints on extended neutral gauge structures, Phys.Lett. B **456** (1999) 68 [arXiv:hep-ph/9903476]

• The most general Higgs potential involving ϕ and Δ is given by

$$V(\phi, \Delta) = a\phi^{\dagger}\phi + \frac{b}{2}\text{Tr}(\Delta\Delta^{\dagger}) + c(\phi^{\dagger}\phi)^{2} + \frac{d}{4}(\text{Tr}(\Delta\Delta^{\dagger}))^{2}$$
$$+ \frac{e-h}{2}\phi^{\dagger}\phi\text{Tr}(\Delta\Delta^{\dagger}) + \frac{f}{4}\text{Tr}(\Delta^{\dagger}\Delta^{\dagger})\text{Tr}(\Delta\Delta)$$
$$+ h\phi^{\dagger}\Delta^{\dagger}\Delta\phi + (t\phi^{\dagger}\Delta\tilde{\phi} + \text{H.c.})$$

with
$$\tilde{\phi} = i \tau_2 \, \phi^*$$

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- The doublet VEV \boldsymbol{v} can be chosen real, by a performing global U(1) transformation
- Because of the t-term we do not have a second global symmetry, the lepton number to make v_T real therefore we can write $v_T=w\,e^{i\gamma}$ with $w=|v_T|$

 Following orders of magnitude for the parameters of the potential are assumend:

$$a, b \sim v^2$$
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The potential as function of the VEVs is given by

$$V(\langle \phi \rangle_0, \langle \Delta \rangle_0) = \frac{1}{2} a v^2 + \frac{1}{2} b w^2 + \frac{1}{4} c v^4 + \frac{1}{2} d w^4 + \frac{e-h}{4} v^2 w^2 + v^2 w |t| \cos(\omega + \gamma)$$

• It has to be minimized with respect to v, w and γ in order to obtain relations between parameters of the potential

The Higgs triplet model: Minimum conditions

• Minimization with respect to γ , the phase of v_T , gives $\omega + \gamma = \pi$ or

$$v_T = -we^{-i\omega}$$
 and $v_T t = -w|t|$

With this relation the other two minimum conditions are

$$a + cv^{2} + \frac{e - h}{2}w^{2} + 2|t|w = 0$$
$$b + dw^{2} + \frac{e - h}{2}v^{2} + \frac{|t|}{w}v^{2} = 0$$

We find the approximate soloution

$$v^2 \cong \frac{a^2}{c}$$
 and $w \cong |t| \frac{v^2}{b + (e-h)v^2/2}$

The Higgs triplet model: Minimum conditions

- We see that $w \sim |t|$, the triplet VEV is of the order of the parameter |t| in the potential
- The fine-tuning to get a small triplet is therefore simply given by $|t| \ll v$
- Alternatively, one could use $b\gg v^2$ to get a small triplet VEV

The Higgs triplet model: Mass terms

• Mass terms for charged leptons and neutrinos are induced by \mathcal{L}_{V} and the VEVs $\langle \phi \rangle_{0}$, $\langle \Delta \rangle_{0}$:

$$-(\overline{\ell}_R \mathcal{M}_\ell \ell_L + \text{H.c.}) \quad \text{with} \quad \mathcal{M}_\ell = \frac{v}{\sqrt{2}} (c_{\alpha\beta})$$

$$\frac{1}{2} v_L^T C^{-1} \mathcal{M}_\nu v_L + \text{H.c. with} \quad \mathcal{M}_\nu = v_T (f_{\alpha\beta})$$

Neutrino mass terms

Neutrino mass terms: Dirac vs. Majorana neutrinos

- Majorana Fermions are their own anti-particles
- The equation for Majorana field is the same as for Dirac fields:

$$(i\gamma^{\mu}\partial_{\mu}-m)\psi=0$$

The Majorana nature is hidden in the Majorana condition:

$$\psi = \psi^C = C\gamma_0^T \psi^*$$

- Most of the SM extensions suggest that neutrinos have Majorana Nature
- In consequence of the smallness of the neutrino masses, it is difficult to distinguish between Dirac and Majorana neutrinos
- Neutrinoless $\beta\beta$ -decay to be the only prospective road so far

Neutrino mass terms: Dirac mass term

• With two independent chiral 4-spinor fields $\nu_{L,R}$ one can construct a Dirac mass term by writing a *Lorentz-invariant* bilinear for Dirac fields [5]:

$$\bar{\nu}_R \mathcal{M} \nu_L + \text{H.c.} = \bar{\nu}' \widehat{m} \nu'$$

- \mathcal{M} is an arbitrary $n \times n$ mass matrix
- \widehat{m} is a diagonal and positive mass matrix with the bidiagonalization ${U_R}^\dagger \mathcal{M} U_L = \widehat{m}$
- The physical Dirac fields are given by

$$\nu' = \nu'_L + \nu'_R$$
 with $\nu_{L,R} = U_{L,R} \nu'_{L,R}$

[5] W.Grimus, Neutrino physics: Theory, Lect. Notes Phys. 629 (2004) 169 [arXiv:hep-ph/0307149].

Neutrino mass terms: Majorana mass term

• With only one chiral 4-spinor field v_L a Lorentz-invariant bilinear can still be constructed with the help of the charge conjugation matrix C. This bilinear is the so called Majorana mass term [5]:

$$\frac{1}{2}v_L^T C^{-1} \mathcal{M} v_L + \text{H.c.} = -\frac{1}{2} \bar{v}' \hat{m} v'$$

- \mathcal{M} is now a complex and symmetric $n \times n$ mass matrix
- \widehat{m} is a diagonal and positive mass matrix with the diagonalization $U_L^T \mathcal{M} U_L = \widehat{m}$
- The physical Majorana fields are given by

$$\nu' = \nu'_L + (\nu'_L)^C$$
 with $\nu_L = U_L \nu'_L$

• The Majorana mass term violates not only the individual lepton family numbers just as the Dirac mass term, but it also violates the total lepton number $L=\sum_{\alpha}L_{\alpha}$

[5] W.Grimus, Neutrino physics: Theory, Lect. Notes Phys. 629 (2004) 169 [arXiv:hep-ph/0307149].

Neutrino mass terms: The mixing matrix U_{PMNS}

- Situation in the lepton sector analogue to the quark sector:
 Flavour eigenfields ≠ mass eigenfields
- Neutrino mixing given by

$$\nu_{\alpha L} = \sum_{j} U_{\alpha j} \nu_{jL}$$

• The lepton mixing matrix U_{PMNS} (Pontecorvo-Maki-Nakagawa-Sakata-matrix) is given as

$$U = U_L^{(\ell)^{\dagger}} U_L^{(\nu)}$$

where $U_L^{(\ell)^\dagger}$ and $U_L^{(\nu)}$ are the matrices which (bi)diagonalize the charged lepton/neutrino mass matrices

Neutrino mass terms: The mixing matrix U_{PMNS}

• The lepton mixing matrix is usually parametrized as

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

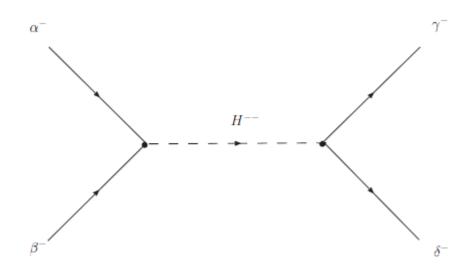
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ -s_{12} & c_{23} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} e^{i\alpha_{1}/2} & 0 & 0 \\ 0 & e^{i\alpha_{2}/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$, δ is non-zero only if neutrino oscillations violate CP symmetry. α_1 , α_2 are physically meaningfull if neutrinos have Majorana nature.

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• The Lagrangian \mathcal{L}_{Y} permits lepton flavour violating processes $\alpha^{-}\beta^{-} \to \gamma^{-}\delta^{-}$ (for $\alpha, \beta, \gamma, \delta = e, \mu, \tau$)



The cross section was calculated as [6]

$$\sigma(\alpha^{-}\beta^{-} \to \gamma^{-}\delta^{-}) = \frac{|f_{\alpha\beta}|^{2}|f_{\gamma\delta}|^{2}}{16\pi(1+\delta\gamma\delta)} \frac{s}{(s-m^{2})^{2}+m^{2}\Gamma^{2}}$$

- $\delta_{\gamma\delta}$
- $s = (p_1 + p_2)^2$ Mandelstam variable
- m

Yukawa coupling matrix

Kronecker delta

 $\mathsf{mass}\ \mathsf{of}\ H$

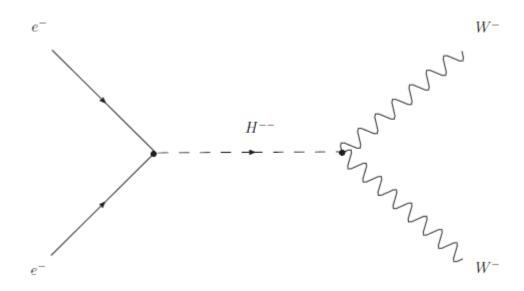
total width of H^{-}

[6] W. Rodejohann and H. Zhang, Higgs triplets at like-sign linear colliders and neutrino mixing, Phys. Rev. D 83 (2011) 073005 [arXiv:1011.3606 [hep-ph]].

The gauge coupling

$$\mathcal{L}_{\Delta gauge} = \frac{1}{2} \text{Tr} \{ (D_{\mu} \Delta)^{\dagger} (D^{\mu} \Delta) \}$$

permits lepton nuber violating processes like $e^-e^- \rightarrow W^-W^-$



The cross section was calculated as

$$\sigma(e^{-}e^{-} \to W^{-}W^{-})$$

$$= \frac{G_F |f_{ee}|^2 |v_T|^2}{4\pi} \frac{(s - 2m_W^2)^2 + 8m_W^4}{(s - m)^2 + m^2\Gamma^2} \sqrt{1 - \frac{m_W^2}{m^2}}$$

- G_F
- *f*
- ullet v_T
- m
- [
- ullet m_W

Fermi constant

Yukawa coupling matrix

triplet VEV

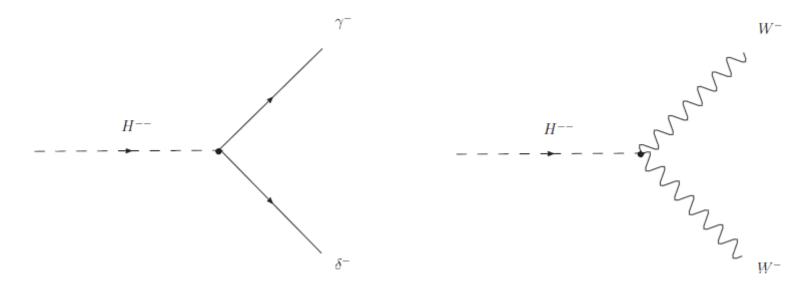
mass of H^{--}

total width of H

mass of the W-boson

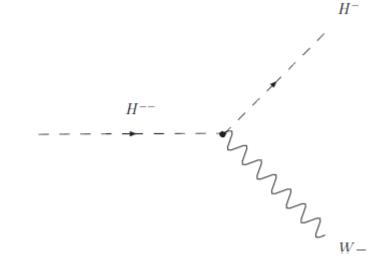
- Lets take a look at some decays of H^{--}
- $\Gamma(H^{--} \to \gamma^- \delta^-) = \frac{|f_{\gamma\delta}|^2}{4\pi(1+\delta_{\gamma\delta})} m$

•
$$\Gamma (H^{--} \to W^- W^-) = \frac{g^4 |v_T|^2}{32\pi m} \frac{(s - 2m_W^2)^2 + 8m_W^4}{m_W^4} \sqrt{1 - \frac{m_W^2}{m^2}}$$

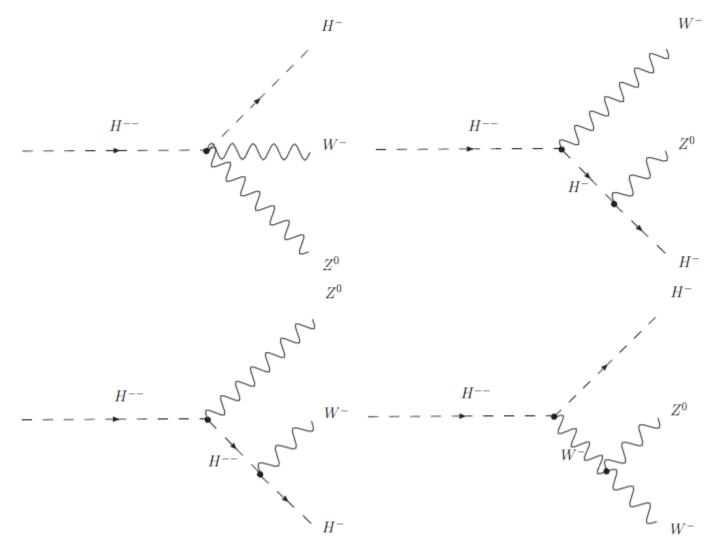


•
$$\Gamma(H^{--} \to H^- W^-) = \frac{g^2}{16\pi m^3 m_w^2} [\lambda(m^2, m_-^2, m_W^2)]^{\frac{3}{2}}$$

- m mass of H^{--}
- m_{-} mass of H^{-}
- m_W mass of the W-boson
- g coulpling constant
- $\lambda(x, y, z) = x^2 + y^2 + z^2$ -2(xy + xz + yz)

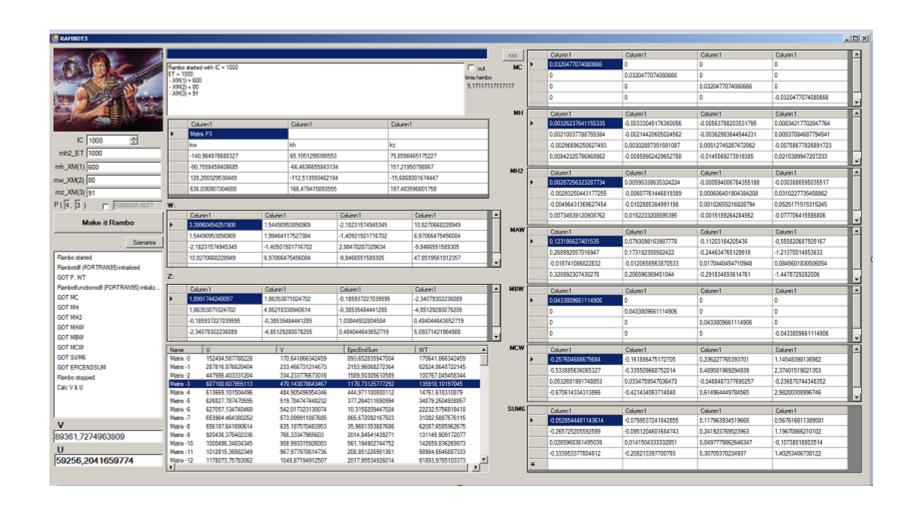


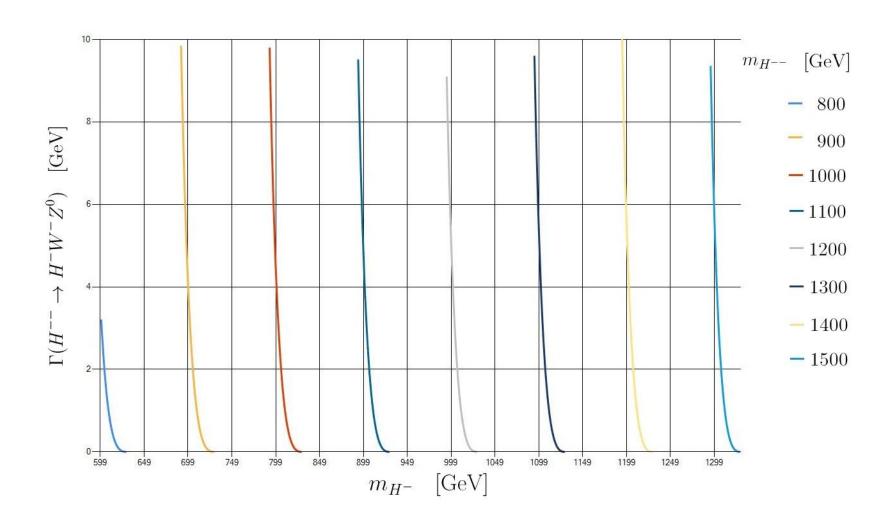
• The 3-body decay $H^{--} \to H^-W^-Z^0$ has four contributions:



- The squared amplitude $|\mathcal{M}|^2$ of this process consists of over a hundred terms
- Most of the terms contain H^- , H^{--} or W^- propagators
- The 3-body phase-space integral over the rational functions was not solvable analytically
- Therefore $\Gamma(H^{--} \to H^-W^-Z^0)$ was solved numerically with the FORTRAN program RAMBOC, based on "RAMBO`` (random momenta beautifully organized)[7]
- "RAMBO" is based on the Monte Carlo algorithm. The integration over phase space is replaced by a number of random choices over the integration variable.

[7] R. Kleiss, W. J. Stirling and S. D. Ellis, A new Monte Carlo treatment of multiparticle phase space at high-energies, Comput. Phys. Commun. **40** (1986) 359.





Conclusions

Conclusions

- The additional term in the Yukawa Lagrangian \mathcal{L}_Y induced a lepton number violating Majorana mass term for neutrinos at tree level, which was found as $\frac{1}{2}v_L^TC^{-1}\mathcal{M}_v v_L$ + H.c.
- The mass matrix was given by $\mathcal{M}_{v}=v_{T}\left(f_{\alpha\beta}\right)$
- The triplet VEV had to be very small, i.e. $|v_T| \ll v$, which was achieved by the fine-tuning $|t| \ll v$; t was the parameter of the lepton number violating term in the potential $(t\phi^\dagger\Delta\tilde{\phi} + \text{H.c.})$
- Neutrinos can be Dirac or Majorana fermions, most SM extension suggest Majorana nature of the neutrino
- \mathcal{L}_Y permits interesting lepton flavour and lepton number violating processes like $\alpha^-\beta^- \to \gamma^-\delta^-$, $e^-e^- \to W^-W^-$, $H^{--} \to H^-W^-$, $H^{--} \to H^-W^-$ Z⁰

Thank you for your attention!