Masses in perturbative QCD

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based on work in collaboration with

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Outline

The pQCD formalism: Bird's eye view

QCD: A QFT for the strong interactions

- Statement: Hadronic matter is made of spin-1/2 quarks [$\leftrightarrow SU(3)_{\mathrm{fl}}$]
- Baryons like $\Delta^{++} = |u^\uparrow u^\uparrow u^\uparrow\rangle$ forbidden by Pauli exclusion/Fermi-Dirac stat.
  Need additional colour degree of freedom!
- Local SU(3)-color gauge symmetry:

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{q=u,d,s,c,b,t} \bar{q}(i\not{\partial} - m_q)q - g\bar{q}\not{G}q - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{ghost}$$

- Fundamental d.o.f.: quark and gluon fields
- Free parameters:
  - gauge coupling: g
  - quark masses: $m_u, m_d, m_s, m_c, m_b, m_t$

QCD

Properties:

- **Confinement and Hadronization:**
  - Free quarks and gluons have not been observed:
    A) They are confined in color-neutral hadrons of size $\sim$ 1 fm.
    B) They hadronize into the observed hadrons.

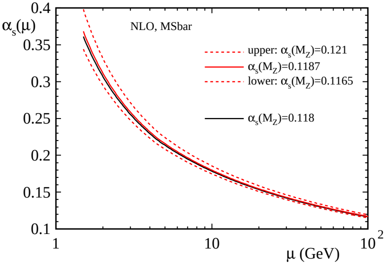
  - Hadronic energy scale: a few hundred MeV [1 fm $\leftrightarrow$ 200 MeV]
  - Strong coupling large at long distances ($\gtrsim$ 1 fm): 'IR-slavery'
  - Hadrons and hadron masses enter the game

- **Asymptotic freedom:**
  - Strong couling small at short distances: perturbation theory
  - Quarks and gluons behave as free particles at asymptotically large energies

Asymptotic freedom

Renormalization of UV-divergences:
Running coupling constant $a_s := \alpha_s/(4\pi)$

$$a_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}$$



- Gross, Wilczek ('73); Politzer ('73)



Non-abelian gauge theories:
negative beta-functions

$$\frac{da_s}{d\ln\mu^2} = -\beta_0 a_s^2 + \ldots$$

where $\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$

$\Rightarrow$ asympt. freedom: $a_s \searrow$ for $\mu \nearrow$

- Nobel Prize 2004

Asymptotic freedom:

- pQCD directly applicable if **all** energy scales large (hard scales)
- However, usually long-distance contributions to the amplitudes present:
  - emission of soft gluons
  - emission of collinear gluons and quarks

pQCD still useful for two classes of observables:

- IR– and collinear–save observables (insensitive to soft or collinear branching)
- Factorizable observables (separate physics from different scales)

Factorization:

- Separate amplitudes into (quantum mechanically) independent factors:
  - Soft parts (long distances/small energies): universal
  - hard parts (short distances/large energies): perturbatively calculable
- Soft parts non-perturbative but universal → **Predictive framework**

QCD factorization theorems:

$$d\sigma \quad = \quad \text{PDF} \otimes d\hat{\sigma} + \text{remainder}$$

- PDF:
  - Proton composed of partons = quarks, gluons
  - Structure of proton described by parton distribution functions (PDF)
  - Factorization theorems provide field theoretic definition of PDFs
  - PDFs universal $\rightarrow$ **PREDICTIVE POWER**
- Hard part $d\hat{\sigma}$:
  - depends on the process
  - calculable order by order in perturbation theory
  - Factorization theorems prescribe how to calculate $d\hat{\sigma}$:
    "$d\hat{\sigma} =$ partonic cross section - mass factorization"
- Statement about error: remainder suppressed by hard scale, $\mathcal{O}((\Lambda/Q)^p)$

Original factorization proofs considered massless partons

Mass factorization $\rightarrow$ Scale-dependence of PDFs

DGLAP evolution:

$$\begin{aligned}\frac{\mathrm{d}f_i(x,Q^2)}{\mathrm{d}\ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}\, P_{ij}(y)\, f_j(x/y,Q^2)\\ &= \frac{\alpha_s(Q^2)}{2\pi}[P_{ij}\otimes f_j](x,Q^2)\,.\end{aligned}$$

Sum rules:

$$\int_0^1 \mathrm{d}x\; u_v(x,Q^2) = 2\,,\qquad \int_0^1 \mathrm{d}x\; d_v(x,Q^2) = 1$$

and

$$\int_0^1 \mathrm{d}x\; x\, \left[\Sigma(x,Q^2) + g(x,Q^2)\right] = 1\,,$$

PDFs determined in global χ^2-analyses from data

Traditionally partons considered to be massless

- In factorization ansatz:
  - relate 4-momenta of partons to 4-momenta of hadrons

$$\mathrm{d}\sigma[P] = \int_0^1 dx\, f(x)\mathrm{d}\hat{\sigma}[\hat{p} = xP] \quad \text{with} \quad P^2 = \hat{p}^2 = 0$$

  - Use massless evolution kernels (in $\overline{\mathrm{MS}}$ scheme)
- In dynamics: massless parton propagators
- In kinematics: massless partons in phase space
- At higher orders: calculate in *n* dimensions, renormalization and mass factorization in $\overline{\mathrm{MS}}$ $\Rightarrow$ short distance $\mathrm{d}\hat{\sigma}$ massless.
  Unavoidable for practical reasons!

A quark h is heavy $:\Leftrightarrow m_h \gg \Lambda_{\rm QCD} \sim 250\ {\rm MeV}$

- $m_h \gg \Lambda_{\rm QCD} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}(\frac{m_h^2}{\Lambda_{\rm QCD}^2}) \ll 1$ (asymptotic freedom)
- m_h sets hard scale; acts as long distance cut-off
- Perturbtation theory (pQCD) applicable

charm:	$m_c \sim 1.5\ {\rm GeV}$	$\Lambda_{\rm QCD}/m_c \sim 0.17$	$\alpha_s(m_c^2) \sim 0.34$
bottom:	$m_b \sim 5\ {\rm GeV}$	$\Lambda_{\rm QCD}/m_b \sim 0.05$	$\alpha_s(m_b^2) \sim 0.21$
top:	$m_t \sim 175\ {\rm GeV}$	$\Lambda_{\rm QCD}/m_t \sim 0.001$	$\alpha_s(m_t^2) \sim 0.11$

- The smaller the ratio $\Lambda_{\rm QCD}/m_h$, the smaller effects of non-perturbative QCD (such as hadronization)
- Top quark decays before it could hadronize due to its large mass ($\Gamma \propto m_t^3$):
  $\Gamma \simeq \Gamma(t \to bW) \simeq \frac{G_F m_t^3}{8\pi\sqrt{2}}|V_{tb}|^2 \simeq 1.76\ {\rm GeV}\ (\frac{m_t}{175\ {\rm GeV}})^3$

How to incorporate heavy quark masses into the pQCD formalism?

The pQCD formalism with quark masses

Hard-scattering factorization with heavy quarks: A general treatment

J. Collins, PRD58(1998)094002

See also:
R. Thorne, W. K. Tung, arXiv:0809.0714

F. Olness, I. Schienbein, Proceedings of Ringberg workshop 2008, arXiv: 08mm.nnnn

Requirements:

(1) $\mu \ll m$: Decoupling of heavy degrees of freedom

(2) $\mu \gg m$: IR-safety

(3) $\mu \sim m$: Correct threshold behavior

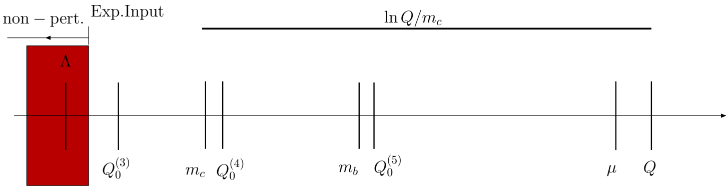
Problems:

- Multiple hard scales: m_c, m_b, m_t, μ
- Mass-independent factorization/renormalization schemes like $\overline{\text{MS}}$
- A single $\overline{\text{MS}}$ scheme cannot meet requirements (1) and (3) (is unphysical).

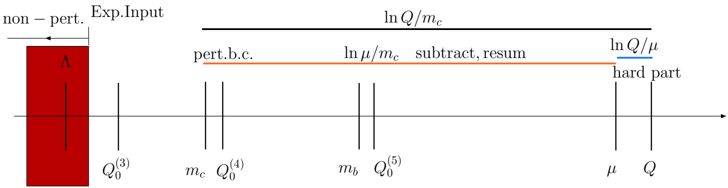
Way out: Patchwork of $\overline{\text{MS}}$ schemes S^{n_f, n_R}

- Variable Flavor-Number Scheme (VFNS): $S^{3,3} \xrightarrow{m_c} S^{4,4} \xrightarrow{m_b} S^{5,5} \xrightarrow{m_t} S^{6,6}$
- **Masses reintroduced** by backdoor: matching $S^{3,3} \leftrightarrow S^{4,4}$ at m_c:
  $f_i^{n_f=4}(x, m_c) = [A_{ij} \otimes f_j^{n_f=3}](x, m_c)$, $\alpha_s^{n_f=4}(m_c) = B\, \alpha_s^{n_f=3}(m_c)$, etc.
- Fixed Flavor-Number Scheme (3-FFNS): $S^{3,3}$ or $S^{3,3} \to S^{3,4} \to S^{3,5} \to S^{3,6}$

- No charm PDF! Of course need exp. Input for u, d, s, g PDFs at scale $Q_0^{(3)}$
- finite collinear logs $\ln Q/m_c$ arise → are kept in hard part (unresummed, in fixed order)
- Requirement (3) naturally satisfied
- Not IR-safe, does not meet requirement (2):
  - Not valid for $Q >> m_c$
  - Can we quantify? Valid for $Q < m_c, 3m_c, 5m_c$?



- often large ratios of scales involved: multi-scale problems
  For $Q \gg m_c$: write $\ln Q/m_c = \ln \mu/m_c + \ln Q/\mu$, subtract $\ln \mu/m_c$ and resum $\ln \mu/m_c$ by introducing charm PDF at $Q_0^{(4)} \simeq m_c$ using a perturbative boundary condition
- $Q < Q_0 : n_f = 3$ no charm PDF, $Q \geq Q_0 : n_f \to n_f + 1$, charm PDF without fi t parameters
- IR-safe, satisfi es requirement (2); resums colliner logarithms
- Problem: original ZM-VFNS (=massless parton model) only valid for $Q >> m$ (Quantify?)
- GM-VFNS: need extra work to satisfy requirement (3) but then valid for all scales Q!
  approaches FFNS for $Q \sim m$, approaches ZM-VFNS for $Q >> m$



Variable Flavor-Number Scheme (VFNS):

$$S^{3,3} \to S^{4,4} \to S^{5,5} \to S^{6,6}$$

Matching conditions for PDFs at matching point $\mu_M \simeq m$: BMNS'96,'98

$$f_i^{(4)}(x, m_c) = [A_{ij}^{(4)} \otimes f_j^{(3)}](x, m_c)$$

with the 4×3 matrix $A_{ij}^{(4)}$:

$$A = \delta + \frac{\alpha_s}{2\pi}[P_1 L + c_1] + (\frac{\alpha_s}{2\pi})^2[P_1^2 L^2 + P_2 L + c_2] + \dots$$

where $L = \ln(\mu^2/m^2)$.

Matching conditions for α_s: Chetyrkin,Kniehl,Steinhauser '97,'98

$$\alpha_s^{(4)}(m_c) = B^{(4)} \alpha_s^{(3)}(m_c)$$

At $\mu_M \simeq m_c$: $f_i^{(4)} = A_{ij}^{(4)} \otimes f_j^{(3)}, A = \delta + \frac{\alpha_s}{2\pi}[P_1 L + c_1] + (\frac{\alpha_s}{2\pi})^2 [P_1^2 L^2 + P_2 L + c_2] + \ldots$

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1. A has exact structure of DGLAP. Only 'unknowns': coefficients c_1, c_2
2. $c_1^{\overline{\text{MS}}} = 0 \Rightarrow$ At NLO, $\mu_M = m$ is special matching point:

$$L = \ln(\mu^2/m^2) = 0 \Rightarrow f_{1,2,3}^{(4)}(x,m) = f_{1,2,3}^{(3)}(x,m)\,,\; c^{(4)}(x,m) = 0$$

3. $c_2^{\overline{\text{MS}}} \neq 0$. At NNLO $\mu_M = m$ not special.
   In fact, in the matching conditions for fragmentation functions already $c_1^{\overline{\text{MS}}} \neq 0$.
   Also at NNLO: $f_c(\mu = m) < 0$. Worrisome? PDFs are not observables!
   Also α_s theoretical construct: $\alpha_s^{n_f+1} = \alpha_s^{n_f} + \mathcal{O}(\alpha_s^3)$
4. <u>Conclusion</u>: At higher orders matching discontinuous!
   - PDFs discontinuous at $\mathcal{O}(\alpha_s^2)$
   - α_s discontinuous at $\mathcal{O}(\alpha_s^3)$
5. **Observables continuous (up to higher orders)**: $\sigma^{n_f} = \sigma^{n_f+1} + \mathcal{O}(\alpha_s^{K+1})$

- Matching conditions valid for any VFNS (ZM-VFNS, GM-VFNS)
- GM-VFNS: Quark masses are retained in the coefficient functions/hard scattering cross sections
- Factorization proof with massive quarks for inclusive DIS: Collins '98
  Remainder $\sim \mathcal{O}(\Lambda^2/Q^2)$ not $\sim \mathcal{O}(m^2/Q^2)$

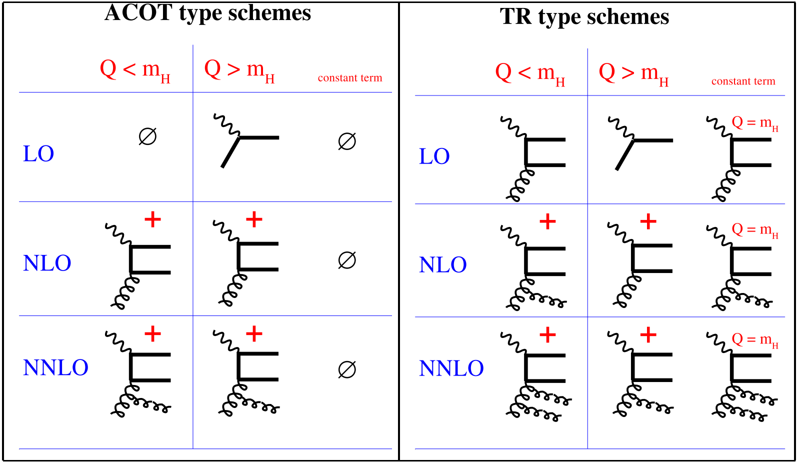
Deep-inelastic scattering and PDFs

Most important application of GM-VFNS: Deep-inelastic scattering (DIS)
Backbone of any PDF global analysis!

- GM-VFNS essential for W, Z cross sections at the LHC
- Most of the most recent global analyses of proton PDFs use a version of a GM-VFNS
  - MSTW08: TR scheme
  - CTEQ6.6/CT10: S-ACOT$_\chi$
  - NNPDF2.1: FONLL
  - HERANPDF1.0: same as MSTW08
  - GJR08, JR09: as GRV in a FFNS
  - CTEQ5,CTEQ6.1,NNPDF2.0,... and older: ZM-VFNS
- The various GM-VFN schemes are 'tuned to' the DIS structure functions F_2^c, F_L^c

If these scheme are not just cooking recipes but pQCD formalisms with heavy quarks, they should be applicable to other processes as well

- Factorization proof with massive quarks for inclusive DIS: Collins '98
  Remainder $\sim \mathcal{O}(\Lambda^2/Q^2)$ not $\sim \mathcal{O}(m^2/Q^2)$
- Many incarnations of VFNS (ACOT, ACOT-χ, TR): Freedom to shift finite *m*-terms between PDFs and hard part without spoiling IR-safety
- S-ACOT scheme: Krämer,Olness,Soper '2000
  incoming and cut heavy quark lines with $m = 0$ ($\leftrightarrow$ scheme choice)
  more complex at NNLO
- Massive quarks can be described by massless evolution kernels ($\leftrightarrow$ scheme choice)


ACOT type schemes
$Q < m_H$
$Q > m_H$
constant term
LO
$\varnothing$
$\varnothing$
NLO
+
+
$\varnothing$
NNLO
+
+
$\varnothing$
TR type schemes
$Q < m_H$
$Q > m_H$
constant term
LO
$Q = m_H$
NLO
+
+
$Q = m_H$
NNLO
+
+
$Q = m_H$

- ACOT scheme: Aivazis, Collins, Olness, Tung, PRD50(1994)3102
  Implementation of mass effects based on simple principles. Directly based on Collin's factorization theorem.
- NLO corrections to ACOT: Kretzer, Schienbein '98
  Massive kinematics, Factorization ansatz:

$$W_{\mu\nu} = \int \frac{\mathrm{d}\xi}{\xi} f(\xi)\, \hat{w}_{\mu\nu\,|\hat{p}^+=\xi P^+}$$

- S-ACOT scheme Krämer,Olness,Soper '2000
- ACOT$_\chi$, S-ACOT$_\chi$: Kretzer, Tung et al., >2000:
  Mimic physical charm-production threshold by slow-rescaling prescription

$$c(x) \to c(\chi)\,, \quad \chi = x\left[1 + (\frac{n m_c}{Q})^2\right]\,, \quad n = 2$$

- Work in progress: extension to NNLO and N3LO Olness, Schienbein, Stavreva et al.

Twist-2 TMC and quark masses can be easily treated together!

$\exists$ simple master formula see Review Schienbein et al, 2008

Rescaling variable factorises:

$$\xi = \eta \chi$$

with the Nachtmann variable η:

$$\eta = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2/Q^2}}$$

One-particle inclusive production in a GM-VFNS

$A+B\to H+X$: $\quad d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\sigma(ij\to kX) \otimes D_k^H(z)$

sum over all possible subprocesses $i+j\to k+X$

Parton distribution functions:
$f_i^A(x_1,\mu_F)$, $f_j^B(x_2,\mu_F)$
non-perturbative input
long distance
universal

Hard scattering
cross section:
$d\sigma(\mu_F,\mu_F',\alpha_s(\mu_R),[\frac{m_h}{p_T}])$
perturbatively computable
short distance
(coefficient functions)

Fragmentation functions:
$D_k^H(z,[\mu_F'])$
non-perturbative input
long distance
universal

Accuracy:

light hadrons: $\mathcal{O}((\Lambda/p_T)^p)$ with p_T hard scale, Λ hadronic scale, $p=1,2$
heavy hadrons: if m_h is neglected in $d\sigma$: $\mathcal{O}((m_h/p_T)^p)$

Details (subprocesses, PDFs, FFs; mass terms) depend on
the Heavy Flavour Scheme

FFNS/Fixed Order

Start with FFNS = Fixed Order:

- NLO calculation more than 20 years old, very well tested
- Allows to predict the total cross section
- Allows to compute p_T spectrum if $!(p_T >> m)$
  (up to inclusion of a non-perturbative FF which is very hard)

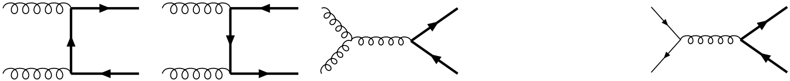
Compare data with best FFNS prediction!

- Find p_T-range where FFNS applicable. Guess: $p_T = 5m$ still ok.
- When need for resummation of $\ln m$ terms visible?
  (Apart from smaller uncertainty band in resummed theory)

As described before: GM-VFNS $\rightarrow$ FFNS for $p_T \sim m$

Leading order subprocesses:

1. $gg \to Q\bar{Q}$
2. $q\bar{q} \to Q\bar{Q} \quad (q = u, d, s)$



- The gg-channel is dominant at the LHC ($\sim$ 85% at $\sqrt{S} = 14$ TeV).
- The total production cross section for heavy quarks is finite.
  The minimum virtuality of the t-channel propagator is m^2. Sets the scale in α_s.
  Perturbation theory should be reliable.
- Note: For $m^2 \to 0$ total cross section would diverge.

[See M. Mangano, hep-ph/9711337; Textbook by Ellis, Stirling and Webber]

Next-to-leading order (NLO) subprocesses:

1. $gg \to Q\bar{Q}g$
2. $q\bar{q} \to Q\bar{Q}g \quad (q = u, d, s)$
3. $gq \to Q\bar{Q}q$, $g\bar{q} \to Q\bar{Q}\bar{q}$ [new at NLO]
4. Virtual corrections to $gg \to Q\bar{Q}$ and $q\bar{q} \to Q\bar{Q}$

NLO corrections for $\sigma_{\rm tot}$ and differential cross sections $d\sigma/dp_T dy$ known since long:

- Nason, Dawson, Ellis, NPB303(1988)607; Beenakker, Kuif, van Neerven, Smith, PRD40(1989)54 [$\sigma_{\rm tot}$]
- NDE, NPB327(1989)49; (E)B335(1990)260; Beenakker *et al.*,NPB351(1991)507 [$d\sigma/dp_T dy$]

Well tested by recalculations and zero-mass limit:

- Bojak, Stratmann, PRD67(2003)034010 [$d\sigma/dp_T dy$ (un)polarized]
- Kniehl, Kramer, Spiesberger, IS, PRD71(2005)014018 [$m \to 0$ limit of diff. x-sec]
- Czakon, Mitov, NPB824(2010)111 [$\sigma_{\rm tot}$, fully analytic]

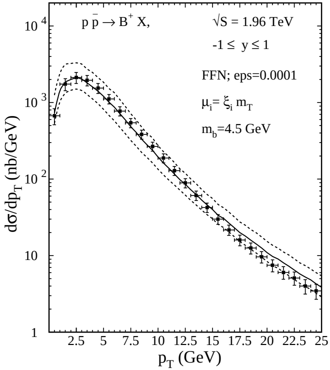
NNLO?

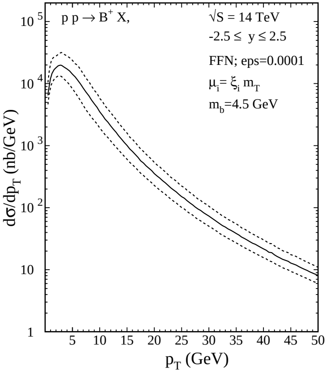
ToDo:

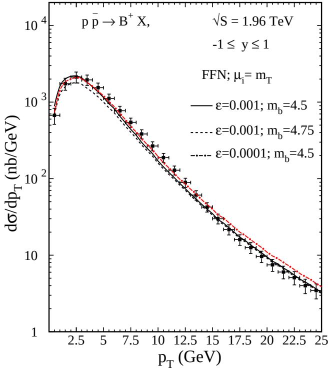
- Calculate for $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$
  - $M^{(0)}(2 \to 4)$ (tree level)
  - $M^{(1)}(2 \to 3)$ (one-loop)
  - $M^{(2)}(2 \to 2)$ (two-loop)
  - NNLO subtraction of IR singularities!
- In addition gq channel
- Goal: total cross section and differential spectra

- Several groups are working on it (Bonciani, Gehrmann, Ferroglia, ...; Czakon, ...)
- $q\bar{q}$ complete (easier); $M^{(2)}(2 \to 4)$ for gg channel difficult
- Complete calculation still a couple of years away
- See, e.g., arXiv:1012.0258 for a recent talk

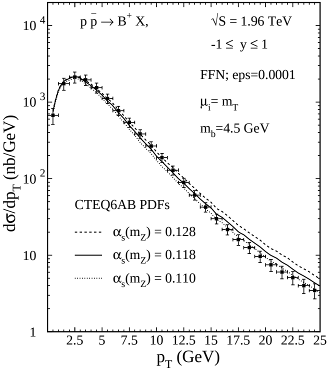
- $d\sigma/dp_T$ for the process $pp \to B^+X$; Fragmentation $b \to B$ via Peterson-FF
- CTEQ6.1 PDFs
- Prediction in NLO perturbation theory







$p\bar{p} \to B^+ X,$
$\sqrt{S} = 1.96$ TeV
$-1 \leq y \leq 1$
FFN; $\mu_i = m_T$
$\varepsilon=0.001;\ m_b=4.5$
$\varepsilon=0.001;\ m_b=4.75$
$\varepsilon=0.0001;\ m_b=4.5$
$d\sigma/dp_T$ (nb/GeV)
p_T (GeV)



$p\bar{p} \to B^+ X,$
$\sqrt{S} = 1.96$ TeV
$-1 \leq y \leq 1$
FFN; eps=0.0001
$\mu_i = m_T$
$m_b=4.5$ GeV
CTEQ6AB PDFs
$\alpha_s(m_Z) = 0.128$
$\alpha_s(m_Z) = 0.118$
$\alpha_s(m_Z) = 0.110$
$d\sigma/dp_T$ (nb/GeV)
p_T (GeV)

Remarks:

- Fixed order theory in reasonable agreement with Tevatron data up to $p_T \simeq 5m_b$
- At $p_T \lesssim m_b$ factorization less obvious. Depends on definition of convolution variable z: $p_B = zp_b$ or $p_T^B = zp_T^b$ or $p_B^+ = zp_b^+$ or $\vec{p}_B = z\vec{p}_b$
- Less hadronization effects than originally believed:
  ϵ-parameter small corresponding to a hard fragmentation function.
  Harder FF $\rightarrow$ harder p_T-spectrum
- Larger $\alpha_s(M_Z) \rightarrow$ harder p_T-spectrum
- Mass dependence imortant for $p_T \lesssim m$ (peak) $\rightarrow \sigma_{tot}$
- Only the 4th or 5th Mellin-moment of the FF is relevant for large p_T [M. Mangano]:
  $d\sigma^b/dp_T(b) \simeq A/p_T(b)^n$ with $n \simeq 4, \ldots, 5$

  $d\sigma^B/dp_T(B) = \int dz/z\, D(z)\, d\sigma^b/dp_T(b)[p_T(b) = p_T(B)/z] =$
  $A/p_T(B)^n \times \int dz\, z^{n-1}\, D(z)$

ZM-VFNS/RS (RS: Resummed)

Next ZM-VFNS/RS which is the baseline for $p_T >> m$

- Again NLO calculation more than 20 years old, very well tested
- Allows to compute p_T spectrum if $p_T >> m$
- Needs scale-dependent FFs for quarks and gluons $D_q^H(z, \mu_F')$, $D_g^H(z, \mu_F')$
- Same theory used for the computation of inclusive π or K production.

Compare data with best ZM-VFNS prediction!

- Find smallest p_T where ZM-VFNS applicable.
- m/p_T terms neglected.
- Is there an overlapping region where both, FFNS and ZM-VFNS are valid?

As said before: GM-VFNS $\rightarrow$ ZM-VFNS for $p_T >> m$

Massless NLO calculation: [Aversa,Chiappetta,Greco,Guillet,NPB327(1989)105]

1. $gg \to qX$
2. $gg \to gX$
3. $qg \to gX$
4. $qg \to qX$
5. $q\bar{q} \to gX$
6. $q\bar{q} \to qX$
7. $qg \to \bar{q}X$
8. $qg \to \bar{q}'X$
9. $qg \to q'X$
10. $qq \to gX$
11. $qq \to qX$
12. $q\bar{q} \to q'X$
13. $q\bar{q}' \to gX$
14. $q\bar{q}' \to qX$
15. $qq' \to gX$
16. $qq' \to qX$

$\oplus$ charge conjugated processes

GM-VFNS/FONLL

$$\boxed{\mathrm{FONLL} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\mathrm{G}(\mathrm{m}, \mathrm{p_T})}$$

FO: Fixed Order; FOM0: Massless limit of FO; RS: Resummed

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2} \simeq \begin{cases} 0.04 & : \quad p_T = m \\ 0.25 & : \quad p_T = 3m \\ 0.50 & : \quad p_T = 5m \\ 0.66 & : \quad p_T = 7m \\ 0.80 & : \quad p_T = 10m \end{cases}$$

$$\Rightarrow \mathrm{FONLL} = \begin{cases} \mathrm{FO} & : \quad \mathrm{p_T} \lesssim 3\mathrm{m} \\ \mathrm{RS} & : \quad \mathrm{p_T} \gtrsim 10\mathrm{m} \end{cases}$$

[1] Cacciari, Greco, Nason, JHEP05(1998)007

Fragmentation functions (FFs):

- $D_i^H(z, \mu_F') = D_i^Q \otimes D_Q^H$ where:
  - $D_i^Q(z, \mu_F')$: perturbative FFs of $i = q, g, Q$ into an on-shell heavy quark Q
  - $D_Q^H(z)$: scale-independent, non-perturbative FF describing transition of heavy quark to heavy hadron
- Non-perturbative FF fitted to $e^+e^- \to DX, BX$ data

Applications available for:

- $\gamma^* + p \to D^{*,0,+} + X$
  photoproduction [JHEP0103(2001)006]
- $p + \bar{p} \to (D^0, D^{*\pm}, D^\pm, D_s^\pm, \Lambda_c^\pm) + X$
  good description of Tevatron data [JHEP05(1998)007]
- $p + \bar{p} \to B + X$
  good description of Tevatron data [PRL89(2002)122003,JHEP07(2004)033]
- $p + p \to D, B + X$
  good description of RHIC data [PRL95(2005)122001]

Factorization Formula: [1]

$$d\sigma(p\bar{p} \to D^\star X) = \sum_{i,j,k} \int dx_1\, dx_2\, dz\, f_i^p(x_1)\, f_j^{\bar{p}}(x_2) \times \mathrm{d}\hat{\sigma}(ij \to kX)\, D_k^{D^\star}(z) + \mathcal{O}(\alpha_s^{n+1}, (\tfrac{\Lambda}{Q})^p)$$

Q: hard scale, $p = 1, 2$

- $\mathrm{d}\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$: hard scattering cross sections free of long-distance physics $\longrightarrow$ m_h kept
- PDFs $f_i^p(x_1, \mu_F)$, $f_j^{\bar{p}}(x_2, \mu_F)$: $i, j = g, q, c$ $[q = u, d, s]$
- FFs $D_k^{D^\star}(z, \mu'_F)$: $k = g, q, c$

$\Rightarrow$ need short distance coeffi cients including heavy quark masses

---

[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment', PRD58(1998)094002

	Only light lines	Heavy quark initiated ($m_Q = 0$)	Mass effects: $m_Q \neq 0$
1	$gg \to qX$	-	$gg \to QX$
2	$gg \to gX$	-	-
3	$qg \to gX$	$Qg \to gX$	-
4	$qg \to qX$	$Qg \to QX$	-
5	$q\bar{q} \to gX$	$Q\bar{Q} \to gX$	-
6	$q\bar{q} \to qX$	$Q\bar{Q} \to QX$	-
7	$qg \to \bar{q}X$	$Qg \to \bar{Q}X$	-
8	$qg \to \bar{q}'X$	$Qg \to \bar{q}X$	$qg \to \bar{Q}X$
9	$qg \to q'X$	$Qg \to qX$	$qg \to QX$
10	$qq \to gX$	$QQ \to gX$	-
11	$qq \to qX$	$QQ \to QX$	-
12	$q\bar{q} \to q'X$	$Q\bar{Q} \to qX$	$q\bar{q} \to QX$
13	$q\bar{q}' \to gX$	$Q\bar{q} \to gX$, $q\bar{Q} \to gX$	-
14	$q\bar{q}' \to qX$	$Q\bar{q} \to QX$, $q\bar{Q} \to qX$	-
15	$qq' \to gX$	$Qq \to gX$, $qQ \to gX$	-
16	$qq' \to qX$	$Qq \to QX$, $qQ \to qX$	-

$\oplus$ charge conjugated processes

Mass terms contained in the hard scattering coefficients:

$$\mathrm{d}\hat{\sigma}(\mu_F, \mu_{F'}, \alpha_s(\mu_R), \frac{m}{p_T})$$

Two ways to derive them:

(1) Compare massless limit of a massive fixed-order calculation
with a massless $\overline{\mathrm{MS}}$ calculation
to determine subtraction terms

[Kniehl,Kramer,IS,Spiesberger,PRD71(2005)014018]

OR

(2) Perform mass factorization using partonic PDFs and FFs

[Kniehl,Kramer,IS,Spiesberger,EPJC41(2005)199]

skip details

- Compare limit $m \to 0$ of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...)
  with massless $\overline{\mathrm{MS}}$ calculation (Aurenche et al., Aversa et al., ...)

$$\lim_{m\to 0} \mathrm{d}\tilde{\sigma}(m) = \mathrm{d}\hat{\sigma}_{\overline{\mathrm{MS}}} + \Delta \mathrm{d}\sigma$$

  $\Rightarrow$ Subtraction terms

$$\mathrm{d}\sigma_{\mathrm{sub}} \equiv \Delta \mathrm{d}\sigma = \lim_{m\to 0} \mathrm{d}\tilde{\sigma}(m) - \mathrm{d}\hat{\sigma}_{\overline{\mathrm{MS}}}$$

- Subtract $\mathrm{d}\sigma_{\mathrm{sub}}$ from massive partonic cross section while keeping mass terms

$$\mathrm{d}\hat{\sigma}(m) = \mathrm{d}\tilde{\sigma}(m) - \mathrm{d}\sigma_{\mathrm{sub}}$$

  $\to$ $\boxed{\mathrm{d}\hat{\sigma}(m) \text{ short distance coefficient including } m \text{ dependence}}$

  $\to$ allows to use PDFs and FFs with $\overline{\mathrm{MS}}$ factorization $\otimes$ massive short distance cross sections

- Treat contributions with <u>charm in the initial state</u> with $m = 0$
- Massless limit: technically non-trivial, map from phase-space slicing to subtraction method

Mass factorization

Subtraction terms are associated to mass singularities:
can be described by
partonic PDFs and FFs for collinear splittings $a \to b + X$

- initial state:

$$f^{(1)}_{g\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(x) \ln\frac{\mu^2}{m^2}$$

$$f^{(1)}_{Q\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+$$

$$f^{(1)}_{g\to g}(x,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\frac{1}{3}\ln\frac{\mu^2}{m^2}\delta(1-x)$$

- final state:

$$d^{(1)}_{g\to Q}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(z) \ln\frac{\mu^2}{m^2}$$

$$d^{(1)}_{Q\to Q}(z,\mu^2) = C_F\frac{\alpha_s(\mu)}{2\pi} \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+$$

- Other partonic distribution functions are zero to order α_s

[Mele, Nason; Kretzer, Schienbein; Melnikov, Mitov]

Sketch of kinematics:

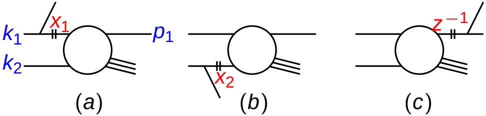


Fig. (a):

$$\mathrm{d}\sigma^{\rm sub}(ab \to QX) = \int_0^1 \mathrm{d}x_1\, f^{(1)}_{a\to i}(x_1, \mu_F^2)\, \mathrm{d}\hat{\sigma}^{(0)}(ib \to QX)[x_1 k_1, k_2, p_1]$$
$$\equiv f^{(1)}_{a\to i}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(ib \to QX)$$

Fig. (b):

$$\mathrm{d}\sigma^{\rm sub}(ab \to QX) = \int_0^1 \mathrm{d}x_2\, f^{(1)}_{b\to j}(x_2, \mu_F^2)\, \mathrm{d}\hat{\sigma}^{(0)}(aj \to QX)[k_1, x_2 k_2, p_1]$$
$$\equiv f^{(1)}_{b\to j}(x_2) \otimes \mathrm{d}\hat{\sigma}^{(0)}(aj \to QX)$$

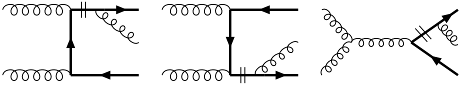
Fig. (c):

$$\mathrm{d}\sigma^{\rm sub}(ab \to QX) = \int_0^1 \mathrm{d}z\, \mathrm{d}\hat{\sigma}^{(0)}(ab \to kX)[k_1, k_2, z^{-1} p_1]\, d^{(1)}_{k\to Q}(z, {\mu'_F}^2)$$
$$\equiv \mathrm{d}\hat{\sigma}^{(0)}(ab \to kX) \otimes d^{(1)}_{k\to Q}(z)$$

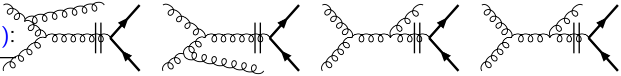
---

[1] Kniehl, Kramer, I.S., Spiesberger, EPJC41(2005)199

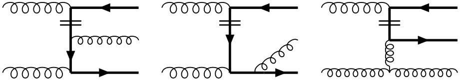
$\mathrm{d}\hat{\sigma}^{(0)}(gg \to Q\bar{Q}) \otimes d^{(1)}_{Q\to Q}(z)$:



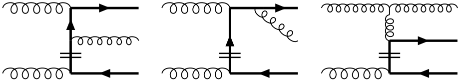
$\mathrm{d}\hat{\sigma}^{(0)}(gg \to gg) \otimes d^{(1)}_{g\to Q}(z)$:



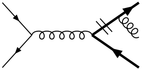
$f^{(1)}_{g\to Q}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(Qg \to Qg)$:



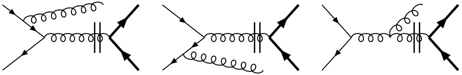
$f^{(1)}_{g\to Q}(x_2) \otimes \mathrm{d}\hat{\sigma}^{(0)}(gQ \to Qg)$:



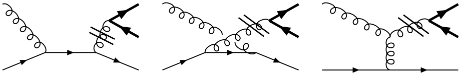
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to Q\bar{Q}) \otimes d^{(1)}_{Q\to Q}(z)$:



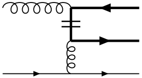
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to gg) \otimes d^{(1)}_{g\to Q}(z)$:



$\mathrm{d}\hat{\sigma}^{(0)}(gq \to gq) \otimes d^{(1)}_{g\to Q}(z)$:



$f^{(1)}_{g\to Q}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(Qq \to Qq)$:



$$\boxed{\mathrm{S-ACOT_\chi = FO + (RS - FOM0) H(m, p_T)}}$$

FO: Fixed Order; $\mathrm{FOM0}$: Massless limit of FO; RS: Resummed

$$H(m, p_T) = \begin{cases} 1 & : \quad \mathrm{S-ACOT} \\ f(p_T/m) & : \quad \mathrm{S-ACOT_\chi} \end{cases}$$

Goal: $H(m, p_T)$ not ad hoc! Calculated from a general principle.

$$\Rightarrow \mathrm{S-ACOT_\chi} \rightarrow \begin{cases} \mathrm{FO} & : \quad \mathrm{p_T \sim m} \\ \mathrm{RS} & : \quad \mathrm{p_T \gg m} \end{cases}$$

Present implementation: $\mathrm{S-ACOT}$ (transition to FFNS at small p_T problematic)

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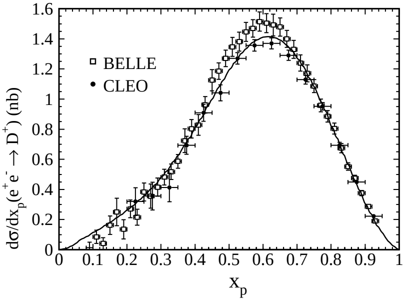
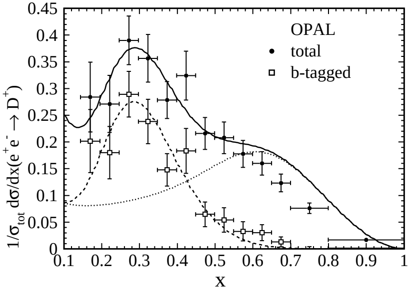
[1] Cacciari, Greco, Nason, JHEP05(1998)007

Applications available for

- $\gamma + \gamma \rightarrow D^{*\pm} + X$
  direct and resolved contributions — EPJC22, EPJC28
- $\gamma^* + p \rightarrow D^{*\pm} + X$
  photoproduction — EPJC38, EPJC62
- $p + \bar{p} \rightarrow (D^0, D^{*\pm}, D^\pm, D_s^\pm, \Lambda_c^\pm) + X$
  good description of Tevatron data — PRD71, PRL96, PRD79
- $p + \bar{p} \rightarrow B + X$
  works for Tevatron data at large p_T — PRD77
- work in progress for $e + p \rightarrow D + X$

Non-perturbative input





FF for $c \to D^*$
from fitting to e^+e^- data

2008 analysis based on GM-VFNS
$\mu_0 = m$

global fit: data from
ALEPH, OPAL, BELLE, CLEO

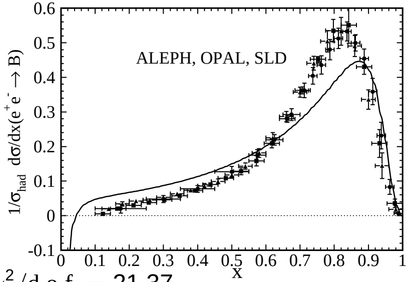
BELLE/CLEO fit

[KKKS: Kneesch, Kramer, Kniehl, IS NPB799 (2008)]

tension between low and high energy data sets $\to$ speculations about non-perturbative (power-suppressed) terms

Petersen

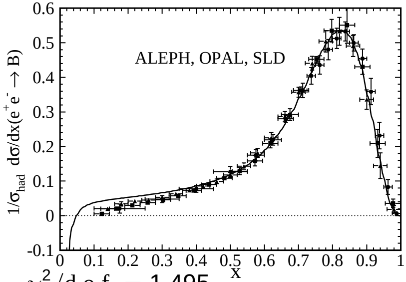
$$D(x,\mu_0^2) = N \frac{x(1-x)^2}{[(1-x)^2+\epsilon x]^2}$$



$\chi^2/\mathrm{d.o.f.} = 21.37$

Kartvelishvili-Likhoded

$$D(x,\mu_0^2) = N x^\alpha (1-x)^\beta$$



$\chi^2/\mathrm{d.o.f.} = 1.495$

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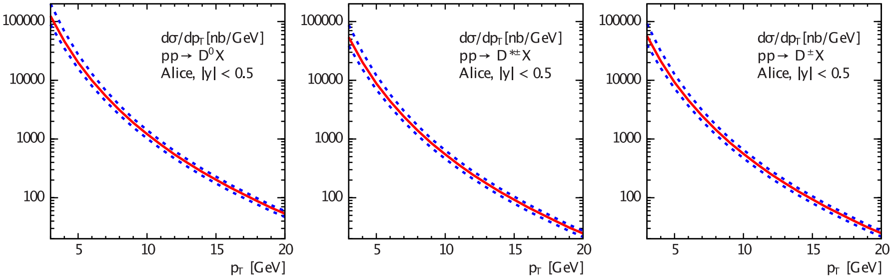
[1] Kniehl,Kramer,IS,Spiesberger,PRD77(2008)014011

[2] ALEPH, PLB512(2001)30; OPAL, EPJC29(2003)463; SLD, PRL84(2000)4300; PRD65(2002)092006

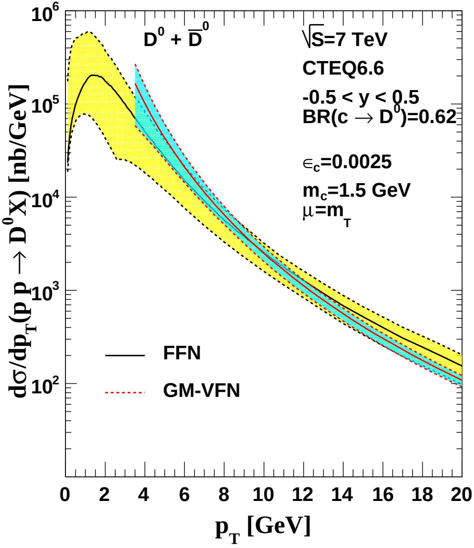
Numerical Results

- Predictions for the LHC
  - ALICE: $pp \to DX$ at 7 TeV
  - CMS: $pp \to BX$ at 7 TeV
  - ATLAS: $pp \to DX$ at 7 TeV
  - LHCb: $pp \to DX$ at 7 TeV
- Predictions for the Tevatron
  - CDF: $p\bar{p} \to DX$ at 1.96 TeV
  - CDF: $p\bar{p} \to BX$ at 1.96 TeV
- Predictions for RHIC
  - RHIC: $pp \to DX$ at 200 GeV and 500 GeV

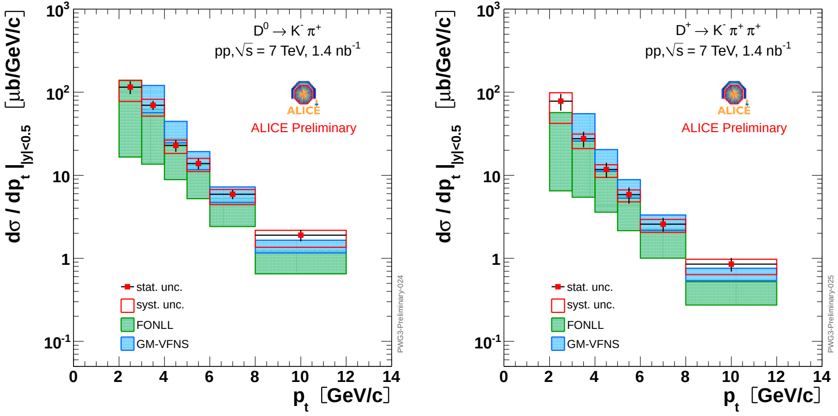


- *pp* collisions, $\sqrt{S} = 7$ TeV
- CTEQ6.5 PDF, KKKSc FF, $m_c = 1.5$ GeV
- Results for $(D^0 + \bar{D}^0)/2$, $(D^{\star +} + D^{\star -})/2$, $(D^+ + D^-)/2$ (check)
- Error bands: Varying μ_R by factors 2 up and down
  (Except for very small p_T this gives maximal variation in the cross section)

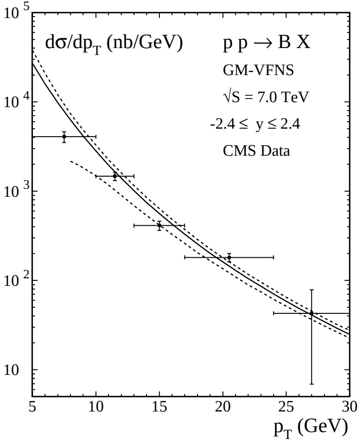
Comparison of FFNS and GM-VFNS predictions



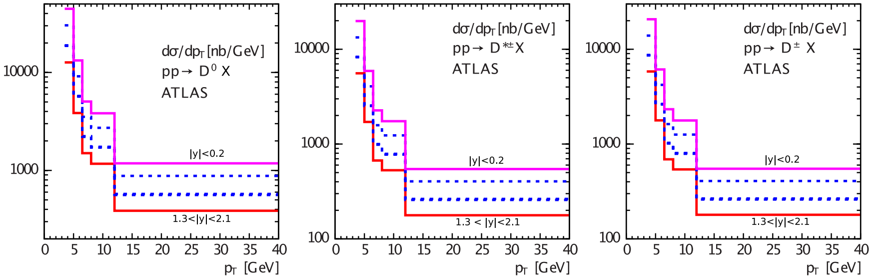
- Presented by A. Dainese at LHC Physics Day, 3. Dec. 2010



- pQCD predictions (FONLL, GM-VFNS) compatible with data

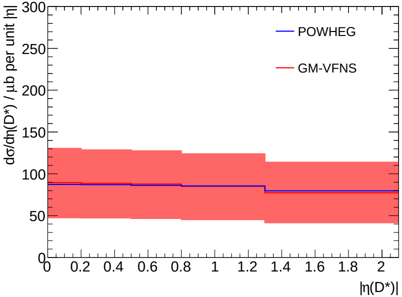


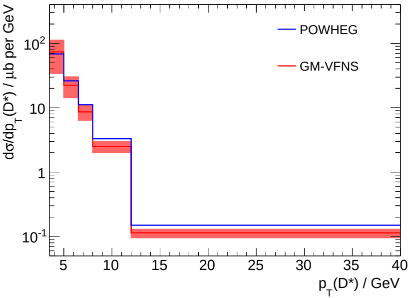
- CMS data: PRL106(2011)112001
- Error band: scale uncertainty



- *pp* collisions, $\sqrt{S} = 7$ TeV
- CTEQ6.6 PDF, KKKS06 FF, $m_c = 1.5$ GeV
- Rapidity bins (top to bottom):
  $|\eta| < 0.2$, $0.2 < |\eta| < 0.5$,$0.5 < |\eta| < 0.8$,$0.8 < |\eta| < 1.3$, $1.3 < |\eta| < 2.1$
- Results for average $(D^0 + \bar{D}^0)/2$, $(D^{\star+} + D^{\star-})/2$, $(D^+ + D^-)/2$

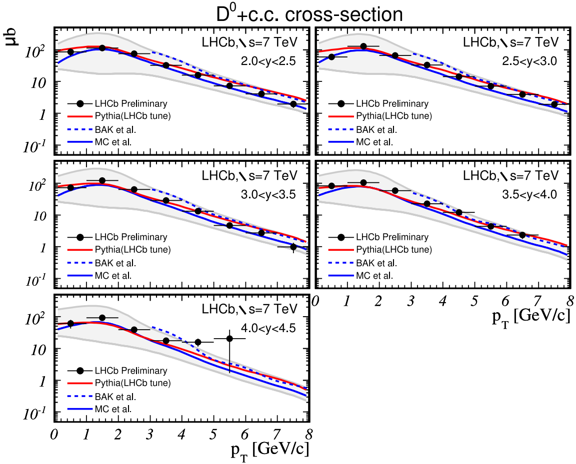
Figures provided by S. Head





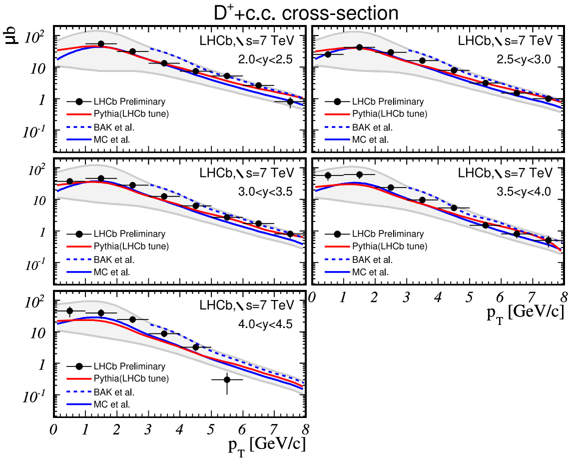
- pp collisions, $\sqrt{S} = 7$ TeV
- CTEQ6.6 PDF, KKKS06 FF, $m_c = 1.5$ GeV
- Left figure: $d\sigma/d\eta$ for $3.5 < p_T < 40$
- Right figure: $d\sigma/dp_T$ for $0 < \eta < 2.1$
- Results for sum $D^0 + \bar{D}^0$, $D^{\star +} + D^{\star -}$, $D^+ + D^-$
- Independent variation of μ_R and μ_F by a factor two up and down

- Prelim. results ($\mathcal{L} = 1.8$ nb^{-1}), $D^0 \to K^-\pi^+$, Data: 12 % correlated error not shown



- BAK et al.= GM-VFNS: B. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger
- MC et al.= FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi

- Prelim. results ($\mathcal{L} = 1.8\ \mathrm{nb}^{-1}$), $D^+ \to K^-\pi^+\pi^+$, Data: 14 % correlated error not shown



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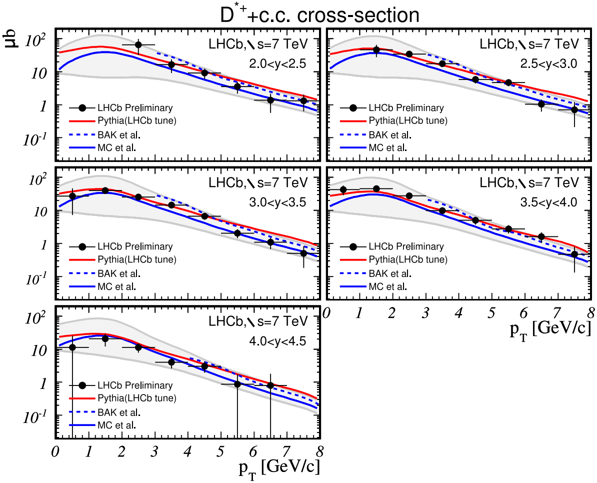
- MC et al.= FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi

- Preliminary ($\mathcal{L} = 1.8\ \text{nb}^{-1}$), $D^{\star+} \to (D^0 \to K^-\pi^+)\pi^+$, Data: 14 % corr. error not shown



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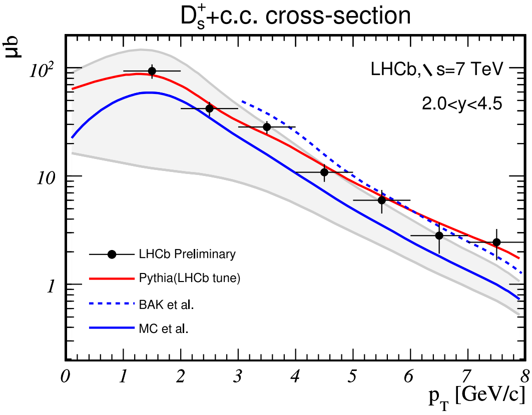
- MC et al.= FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi

- Preliminary ($\mathcal{L} = 1.8\ \text{nb}^{-1}$), $D^s \to K^- K^+ \pi^+$, Data: 16 % corr. error not shown



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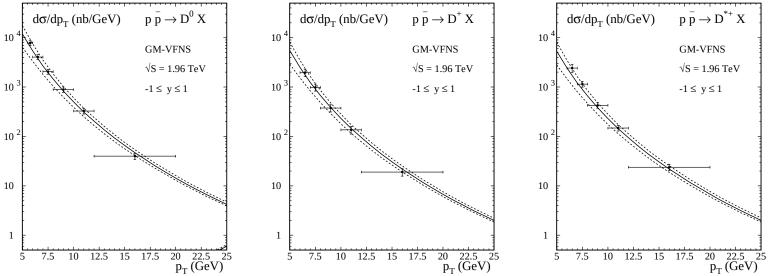
- MC et al.= FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi

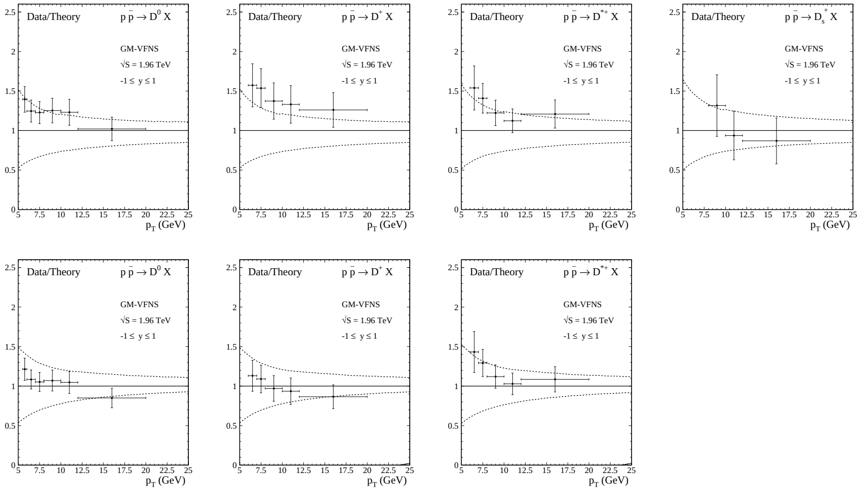
HADROPRODUCTION OF D^0, D^+, D^{*+}, D_s^+
GM-VFNS RESULTS W/ KKKSC FFS [1]



- $d\sigma/dp_T$ [nb/GeV] $|y| \leq 1$ prompt charm
- Uncertainty band: $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2$ $\quad (m_T = \sqrt{p_T^2 + m_c^2})$
- CDF data from run II [2]
- GM-VFNS describes data within errors

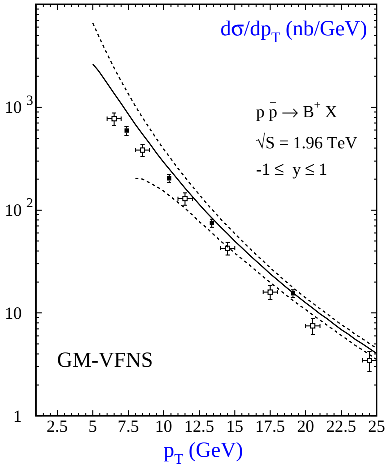
[1] Kniehl,Kramer,IS,Spiesberger, arXiv:0901.4130[hep-ph], PRD(to appear)

[2] Acosta et al., PRL91(2003)241804



- New KKKSc FFs improve agreement w/ CDF data.

[1] Kniehl,Kramer, PRD74(2006)037502

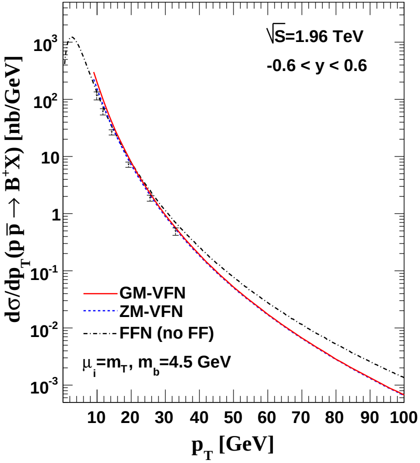


- CDF (1.96 TeV):
  - open squares $J/\psi X$ [1]
  - solid squares $J/\psi K^+$ [2]
- CTEQ6.1M PDFs
- $m_b = 4.5$ GeV
- $\Lambda_{\overline{\mathrm{MS}}}^{(5)} = 227\ \mathrm{MeV} \rightsquigarrow \alpha_s^{(5)} = 0.1181$
- $1/2 \leq \mu_R/m_T, \mu_F/m_T, \mu_R/\mu_F \leq 2$
  $(m_T = \sqrt{p_T^2 + m_b^2})$

[1] CDF, PRD71(2005)032001

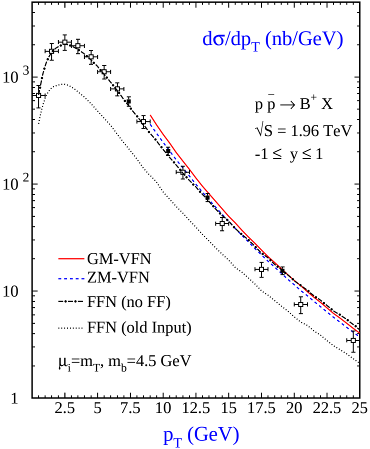
[2] CDF, PRD75(2007)012010

GM-VFNS PREDICTION VS. CDF II [1]



- CDF II (preliminary) [1]
- $\mu_R = \mu_F = m_T$
- for $p_T \gg m_b$:
  - GM-VFN merges w/ ZM-VFN
  - FFN breaks down
- data point in bin [29,40] favors GM-VFN

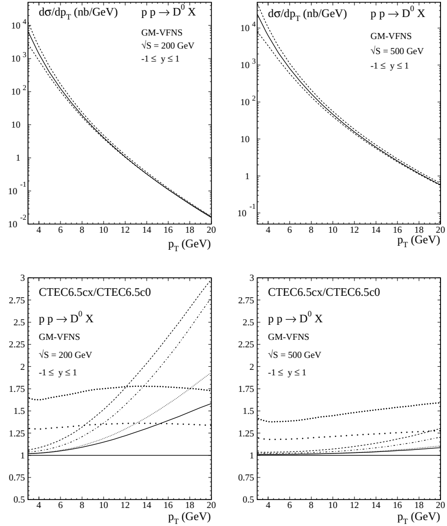
[1] Kraus, FERMILAB-THESIS-2006-47; Annovi, FERMILAB-CONF-07-509-E



- obsolete FFN as above
- up-to-date FFN evaluated with
  - CTEQ6.1M PDFs
  - $m_b = 4.5$ GeV
  - $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 227\ \mathrm{MeV} \rightsquigarrow \alpha_s^{(5)} = 0.1181$
  - $D(x) = B(b \to B)\delta(1-x)$ with $B(b \to B) = 39.8\%$

[1] Kniehl,Kramer,IS,Spiesberger,PRD77(2008)014011

INTRINSIC CHARM IN THE PROTON
D-MESONS AT RHIC



$(\mathrm{d}\sigma/\mathrm{d}p_T)(pp \to D^0 + X)$
$\sqrt{s} = 200, 500$ GeV
$|y| < 1$

IC Model	moderate	marginal
BHPS	solid	dashed
Meson-cloud	densely dotted	dot-dashed
Sealike	scarsely dotted	dotted