Soft and Hard Mesons in Chiral Perturbation Theory

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Outline

1. Chiral Perturbation Theory

2. Relations at order p^6 in Chiral Perturbation Theory

3. A new global fit of the L_i^r at next-to-next-to-leading order

4. Hard pion Chiral Perturbation Theory

Outline

$$\mathcal{L} = \sum_{q=1}^{n_f} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q(\bar{q}_R q_L + \bar{q}_L q_R)]$$

(n_f =number of flavours)

If $m_q = 0$ then $SU(n_f)_L \times SU(n_f)_R$ (chiral symmetry)$\Rightarrow$ parity doublets in the spectrum.

They do not exist! $\Rightarrow SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_V$

- $n_f = 2 \to 3$ Goldstone bosons
- $n_f = 3 \to 8$ Goldstone bosons

$m_q \neq 0$ (but small) $\Rightarrow$ chiral symmetry is also explicitly broken, Goldstone bosons are not massless

Construction as Effective Field Theory

Degrees of freedom pseudo-Goldstone bosons (lightest mesons in the spectrum)

- $n_f = 2 \rightarrow \pi^+, \pi^-, \pi^0$
- $n_f = 3 \rightarrow \pi, K, \eta$

$$U = e^{i\frac{\sqrt{2}\phi}{F_0}} \qquad \phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Expected breakdown scale Resonances (m_ρ)

Lagrangian All operators allowed by QCD symmetries

$$\mathcal{L}_2 = \frac{F_0{}^2}{4}(\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + U^\dagger \chi\rangle),$$

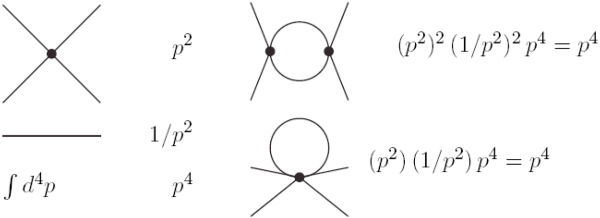
$$\chi = 2B_0(s + ip) \qquad D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu,$$

s, p, r_μ, l_μ =external fields, $s = \mathcal{M} + \ldots$ (quark masses)

F_0, B_0 = Low Energy Constants (LECs)

Power counting

- ChPT at low energies → small momenta (p) and masses
- In $\mathcal{L}_2$ operators with either two derivatives (p^2) or masses (m^2)



- observables depend on $p^2(/(4\pi F_0)^2)$ and $m^2(/(4\pi F_0)^2)$ which are small parameters ($(4\pi F_0)^2 \gg p^2, m^2$)→ use them for perturbative expansion
- so power counting is a dimensional counting!
- $O = O_{p^2} + O_{p^4} + \mathcal{O}(p^6)$

Higher Order

- Loop diagrams are divergent $\Rightarrow$ need counterterms to cancel the infinities arising (RENORMALIZATION)
- From $\mathcal{L}_2$ only terms $\sim p^2$, but we need to renormalize diagrams $\sim p^4$
- Operators $\sim p^4$ also allowed by symmetries $\rightarrow \mathcal{L}_4$

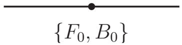
$$
\begin{aligned}
\mathcal{L}_4 = \; & L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\
& + L_3 \langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U \rangle + L_4 \langle D^\mu U^\dagger D_\mu U \rangle \langle \chi^\dagger U + \chi U^\dagger \rangle \\
& + L_5 \langle D^\mu U^\dagger D_\mu U (\chi^\dagger U + U^\dagger \chi) \rangle + L_6 \langle \chi^\dagger U + \chi U^\dagger \rangle^2 \\
& + L_7 \langle \chi^\dagger U - \chi U^\dagger \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \rangle \\
& - i L_9 \langle F^R_{\mu\nu} D^\mu U D^\nu U^\dagger + F^L_{\mu\nu} D^\mu U^\dagger D^\nu U \rangle \\
& + L_{10} \langle U^\dagger F^R_{\mu\nu} U F^{L\mu\nu} \rangle + H_1 \langle F^R_{\mu\nu} F^{R\mu\nu} F^L_{\mu\nu} F^{L\mu\nu} \rangle + H_2 \langle \chi \chi^\dagger \rangle
\end{aligned}
$$

- solve problem of divergencies, but more couplings arising!
- at two loops? same procedure: $\mathcal{L}_6$ couplings (C_i) used to cancel divergencies of loop diagrams of order p^6.

Higher Orders (an example)

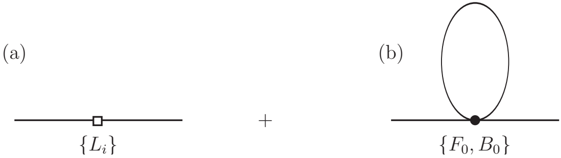
1. Leading Order (LO) = p^2

$O_{p^2} =$



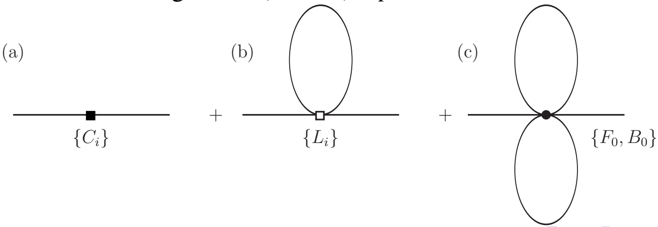
2. Next to Leading Order (NLO) = p^4

$O_{p^4} =$



3. Next to Next to Leading Order (NNLO) = p^6

$O_{p^6} =$



Low Energy Constants

	3 flavour (u,d,s) ChPT	
p^2	F_0, B_0	2
p^4	L_i^r, H_i^r	10+2
p^6	C_i^r	90+4

Determination of LECs is important:

- to have precise predictions of ChPT
- to check convergence of perturbative expansion
- to study the underlying QCD

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PROBLEMS:

1. large number of phenomenological constants
2. strong correlations among them
3. many of the observables calculated in ChPT have not been measured yet. (But dispersion relations and lattice results can be used)

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Why are we looking for relations between observables?

Chiral Perturbation Theory $\rightarrow$ every observable can be written as a sum of terms of decreasing importance in the Chiral expansion.

$$O = O_{p^2} + O_{p^4} + O_{p^6}$$

The p^6 part can be split in as

$$O_{p^6} = O_{C_i(\text{tree level})} + O_{L_i(\text{one loop})} + O_{F_0(\text{two loops})}$$

If we have a relation such that the first contribution cancels out

- we can check how large is the loop contribution and test ChPT convergence in a C_i independent way
- in this way we isolated combinations of the C_i

How did we test ChPT?

If we have a set of observables $\{O_i\}$ calculated at NNLO and such that:

$$-5\,[O_1]_{C_i} + 2\,[O_2]_{C_i} = \text{some combination of the } C_i = 21\,[O_3]_{C_i}$$

where $[A]_{C_i} \equiv C_i$ -part of the observable A

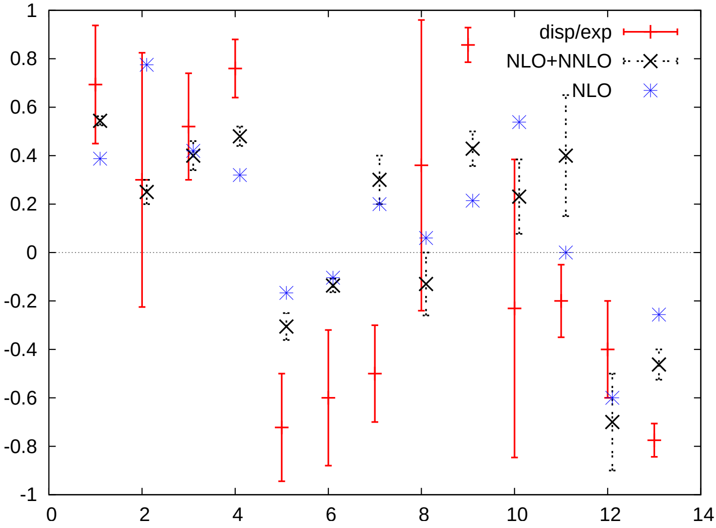
then

- $[LHS]_{C_i} = [RHS]_{C_i}$ as far as regards the C_i part
- but for the other contributions to the $\{O_i\}$ (e.g. loop diagrams) $\Rightarrow LHS_{(rest)} \neq RHS_{(rest)}$(!!!)
- for all measured observables $O_{i(exp)} = O_{i(rest)} + [O_i]_{C_i}$ $\Rightarrow LHS_{(exp)} - RHS_{(exp)} \sim LHS_{(rest)} - RHS_{(rest)}$
- the C_i cancel in the relations $\Rightarrow$this is a C_i-independent way to test ChPT

Overview of the processes considered and relations found

process	# observables	# relations
$\pi\pi$ scattering	11	5
πK scattering	14	5
πK and $\pi\pi$ scattering	no extra observables	2
$K_{\ell 4}$ (with πK scattering)	10	1
$\eta \to 3\pi$ (with πK)	6	2
scalar form factors $F_S^{\pi/K}(t)$	18	6
$F_S^{\pi/K}(t)$, $\pi\pi$ and πK scattering	no extra observables	2
$F_S^{\pi/K}(t)$,$K_{\ell 4}$,$\pi\pi$ and πK scattering	no extra observables	1
$F_S^{\pi/K}(t)$, masses and decay constants	6	4
Vector form factors $F_V^{\pi/K}$	11	7
Total	76	35

Numerical results



[1-5] $\pi\pi$ scatt., [5-10] πK scatt., [11-12] $\pi\pi$ and πK scatt., [13] πK scatt. and $K_{\ell 4}$

Conclusions

- found several relations between observables such that dependence on many couplings drops out
- useful to study validity of ChPT perturbative expansion
- 13 relations studied numerically
- overall picture quite ok, but still a few trouble cases
- probably with the new values of L_i^r (see next) the results will change

Outline

Fit 10: input

- Existing fit of the L_i at NNLO:
  fit 10 Amoros, Bijnens, Talavera, Nucl. Phys. B 602 (2001) 87 [hep-ph/0101127]

- INPUT:
  1. masses: $m^2_{\pi^0}$, m^2_{η}, $m^2_{K^+}$, $m^2_{K^0}$ PDG-00
  2. $F_\pi = 92.4\,\mathrm{MeV}$
  3. $K_{\ell 4}$: f_s, f'_s, g_s, g'_s (linear fit E865) — L^r_1, L^r_2, L^r_3
  4. $F_K/F_\pi = 1.22 \pm 0.01$ — L^r_5
  5. $m_s/\hat{m} = 24$ — L^r_5, L^r_7, L^r_8
  6. $L^r_4 \equiv L^r_6 \equiv 0$ — ($1/N_c$ suppressed)
  7. $L^r_9 \equiv 6.9 \times 10^{-3}$ ($\langle r^2 \rangle^{\pi}_V$) — ($\sim$no contributions in quantities involved)
  8. $L^r_{10} \equiv 0$ — (no contributions in quantities involved)

- C^r_i from resonance saturation: Vector, Axial-Vector, Scalar, η'.
  Scale of saturation $\mu \equiv 0.77\,\mathrm{Gev}$. $\mu = 0.5, 1\,\mathrm{Gev}$ within errors.

New fits: input

- Since 2001 many two-loop calculations+quantities better known phenomenologically ⇒ new global fit of the L_i^r at NNLO: Bijnens,IJ (2011)
- INPUT (fit 10) →(new fits)
  1. masses: $m^2_{\pi^0}, m^2_\eta, m^2_{K^+}, m^2_{K^0}$ PDG-00 →PDG-10
  2. $F_\pi = 92.4\,\text{MeV}$ → $F_\pi = 92.2$ PDG-10
  3. $K_{\ell 4}$: f_s, f_s', g_s, g_s' (linear fit E865) → (quadratic fit NA48/2)
  4. $F_K/F_\pi = 1.22 \pm 0.01$ → $F_K/F_\pi = 1.197 \pm 0.007$
  5. $m_s/\hat{m} = 24$ → $m_s/\hat{m} = 27.8$
  6. $L_4^r \equiv L_6^r \equiv 0$ → constraint released
  7. $L_9^r \equiv 6.9 \times 10^{-3}$ $(\langle r^2 \rangle^\pi_V)$ → $L_9^r \equiv 5.93 \times 10^{-3}$
  8. $L_{10}^r \equiv 0$
  9. $\pi\pi$ scattering threshold parameters
  10. πK scattering threshold parameters
  11. $\langle r^2 \rangle^\pi_S$ pion scalar radius
- C_i^r from resonance saturation: Vector, Scalar, η'.
- C_i^r from Jiang et al.(2010)) and with random values (simulated annealing algorithm) also tested

Fit 10 and new fit: output

fit10 iso: as fit 10 but no isospin breaking corrections

fit All: new best fit

	fit 10 iso	fit All
$10^3 L_1^r$	0.39 ± 0.12	0.88 ± 0.09
$10^3 L_2^r$	0.73 ± 0.12	0.61 ± 0.20
$10^3 L_3^r$	-2.34 ± 0.37	-3.04 ± 0.43
$10^3 L_4^r$	$\equiv 0$	0.75 ± 0.75
$10^3 L_5^r$	0.97 ± 0.11	0.58 ± 0.13
$10^3 L_6^r$	$\equiv 0$	0.29 ± 0.85
$10^3 L_7^r$	-0.30 ± 0.15	-0.11 ± 0.15
$10^3 L_8^r$	0.60 ± 0.20	0.18 ± 0.18
χ^2 (dof)	0.26 (1)	1.28 (4)

- new info on $K_{\ell 4}$ form factors $\Rightarrow$ large N_c relation $2L_1 \sim L_2$ unsatisfied
- L_4^r, L_6^r, L_7^r, L_8^r related to masses (not well constrained)
- change in values of $F_K/F_\pi \Rightarrow$ very different L_5^r
- better convergence for fit All (see after)

Convergence

	fit 10 iso			fit All		
	p^2	p^4	p^6	p^2	p^4	p^6
m_π^2	0.753	0.006	0.241	1.035	-0.084	0.049
m_K^2	0.702	0.007	0.291	1.106	-0.181	0.075
m_η^2	0.747	-0.047	0.291	1.186	-0.224	0.038
F_π	1	0.136	-0.075	1	0.311	0.108
F_K	1	0.308	-0.003	1	0.441	0.216
F_K/F_π	1	0.171	0.049	1	0.129	0.068

- expansion for masses improved for fit All (but a bit suspicious)
- C_i^r appearing in the masses are zero in resonance estimate
- F_K/F_π convergence is quite good for both fits

Extra checks

- **$K_{\ell 4}$ data analysis: two possible fits of F_s formfactor**

$$F_s(q^2) \sim f_s + f_s' q^2 + f_s'' q^4 \rightarrow \text{fit All}$$
$$F_s(q^2) \sim f_s + f_s' q^2 \rightarrow \text{fit 10/fit 10 iso}$$

ChPT does not predict the value for f_s''!
Using f_s' we obtain $L_2^r > L_1^r$ but L_7^r, L_8^r small. Convergence for masses worse.

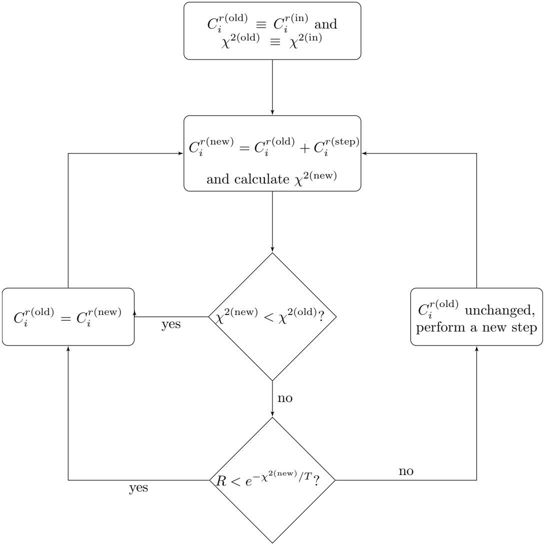
- **Matching between two- and three-flavour ChPT** Gasser *et al.* (2007,2009)
Expand three-flavour results for $m_{u,d} \rightarrow 0$, keeping m_s fixed $\Rightarrow$ $\bar{\ell}_i$ can be written in terms of L_i^r, C_i^r.

	fit 10 iso	All	All linear	lattice/disp
$\bar{\ell}_1$	-0.6	-0.1	-1.9	-0.4 ± 0.6
$\bar{\ell}_2$	5.7	5.3	5.7	4.3 ± 0.1
$\bar{\ell}_3$	1.3	4.2	4.1	3.3 ± 0.7
$\bar{\ell}_4$	4.0	4.8	4.5	4.4 ± 0.4

$\bar{\ell}_2$ always wrong. It depends on L_2^r and on two N_c suppressed C_i^r (see after).

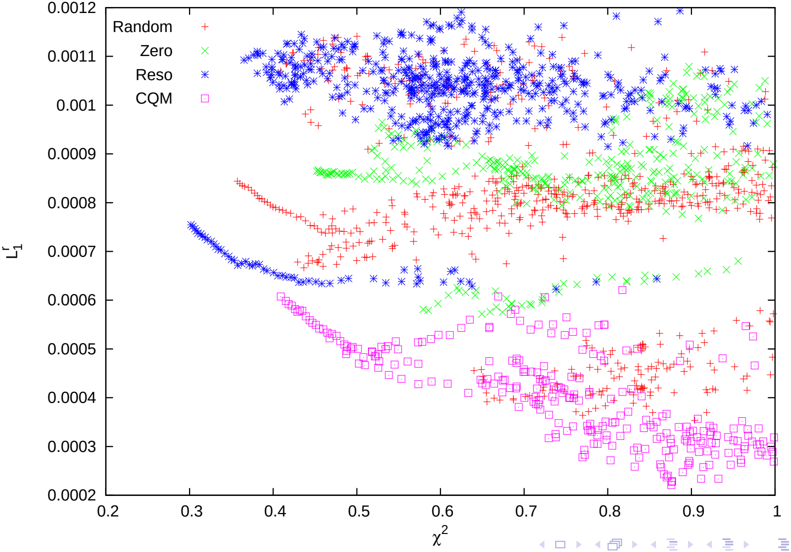
C_i^r with random values

Can we find a set of C_i^r of expected size (i.e. $1/(16\pi^2)^2$) producing good L_i^r fits?



- started with different $C_i^{r(in)}$
- $C_i^{r(step)} \propto 10^{-2}/(16\pi^2)^2$ and chosen so to respect large N_c suppressions
- need to add convergence constraints for masses and decay constants
- $\bar{\ell}_i$ included as input
- obtain several good fits (with good convergence) and ok C_i^r

An example: L_1^r



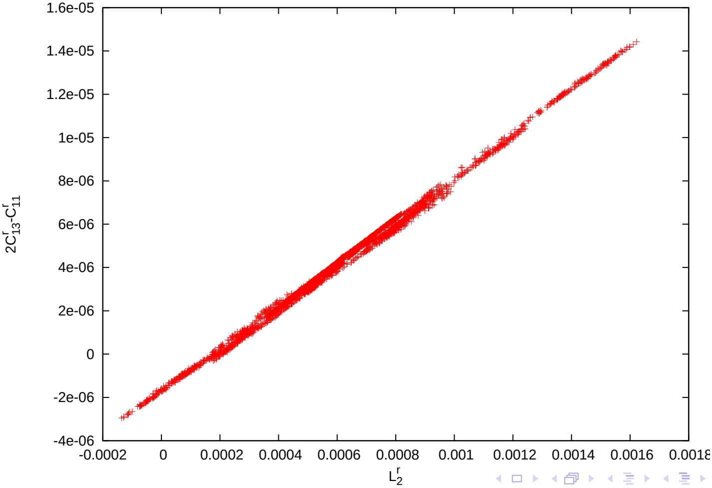
Our best fits for random C_i

C_i^r	best reso	best random
$10^3 L_1^r$	0.75 ± 0.09	0.85 ± 0.09
$10^3 L_2^r$	0.81 ± 0.45	0.54 ± 0.05
$10^3 L_3^r$	-3.91 ± 0.28	-3.51 ± 0.28
$10^3 L_4^r$	0.16 ± 0.10	0.20 ± 0.10
$10^3 L_5^r$	1.40 ± 0.09	1.40 ± 0.09
$10^3 L_6^r$	0.10 ± 0.14	0.12 ± 0.14
$10^3 L_7^r$	-0.32 ± 0.13	-0.32 ± 0.13
$10^3 L_8^r$	0.64 ± 0.16	0.63 ± 0.16
χ^2	0.30	0.36

C_i^r	best reso			best random		
	p^2	p^4	p^6	p^2	p^4	p^6
m_π^2	0.987	0.021	−0.008	0.993	0.021	−0.012
m_K^2	1.057	−0.054	−0.003	1.060	−0.058	−0.002
m_η^2	1.132	−0.133	0.001	1.136	−0.135	−0.001
F_π/F_0	1	0.178	−0.010	1	0.187	−0.010
F_K/F_0	1	0.395	0.009	1	0.404	0.011
F_K/F_π	1	0.217	−0.020	1	0.217	−0.020

A correlation plot

- By varying the C_i^r we can study correlations between different LECs
- Here strong correlation between L_2^r and C_{13}^r, C_{11}^r due to $\bar{\ell}_2$



Comparison with a fit to lattice data PRELIMINARY

- Ecker *et al.*(2010) use symplified NNLO expression to fit lattice data points on F_K/F_π.
- LECs fitted: L_5^r, $C_{14}^r + C_{15}^r$, $C_{15}^r + 2C_{17}^r$
- Other L_i^r as in fit 10

	Ecker *et al.*(2010)	fit All	best reso	best rand
$10^3 L_5^r$	0.76 ± 0.008	0.58 ± 0.13	1.40 ± 0.009	1.40 ± 0.009
$10^5\,(C_{14}^r + C_{15}^r)$	0.31 ± 0.007	0	-0.987	-1.061
$10^5\,(C_{15}^r + 2C_{17}^r)$	1.1 ± 0.14	0	0.22	2.009

- Setting C_{14}^r and C_{17}^r as in Ecker *et al.*(2010), (C_{15}^r=0) we find a fit with $\chi^2 \approx 3$ and $10^3 L_5^r \approx 0.59 \Rightarrow L_7^r$ and L_8^r very small
- However very bad convergence for mass expansions.

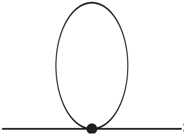
Conclusions

- a new global fit of the $\mathcal{L}_4$ couplings at NNLO from phenomenology has been performed
- the new data/dispersive analysis available give results very different from the previous fit 10
- results show disagreement with large N_c estimates of the couplings
- however many quantities better predicted and convergence of masses improved
- better treatments of some observables ($K_{\ell 4}$, inclusion of lattice data) might improve a lot this fit
- the study with random sets of C_i^r shows that ChPT has a chance to work well once the C_i^r are better estimated

Outline

Chiral Logarithms as main prediction of ChPT

Expansion of ChPT is not Taylor expansion: logarithms of masses (and energies) arise



$$\approx \infty + m^2 \log\left(\frac{m^2}{\mu^2}\right) + \cdots$$

$m =$ mass of meson in loop, $\mu =$arbitrary scale

E.g. for the masses

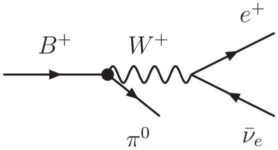
$$M^2 \approx \underbrace{m_0^2}_{\text{LO}} + \underbrace{\frac{m_0^4}{(4\pi F_0)^2} \log \frac{m_0^2}{\mu^2} + \frac{L_i^r}{F_0^2} m_0^4}_{\text{NLO}} + \mathcal{O}(m_0^6)$$

$L_i^r \approx 10^{-2} \Rightarrow$ chiral logarithm $\log \frac{m_0^2}{\mu^2}$ is leading contribution at $\mathcal{O}(p^4)$ (NLO).

The chiral logs encode mass dependence of the observables

Motivation

- Consider decays of a heavy meson into light mesons



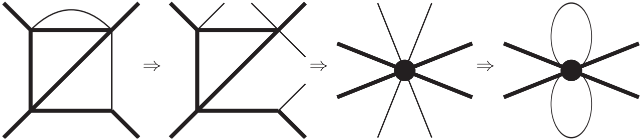
- $m_B \approx 3000$ MeV, $m_D \approx 2000$ MeV, $m_\pi \approx 140$ MeV
- $q^\mu = (p_{B/D} - p_\pi)^\mu =$ momentum transfer to leptons
  $0 \leq q^2 \leq (M - m_\pi)^2 = q^2_{\max}$ with $M = m_B, m_D$
- two different kinematical regimes
  1. $q^2 \approx q^2_{\max} \Rightarrow E_\pi \lesssim 1$ GeV **soft pion** (ChPT ok)
  2. $q^2 \approx 0 \Rightarrow E_\pi > 1$ GeV **hard pion** (ChPT ???)

PROBLEM

- lattice calculates the decay at any q^2 but simulations done with HEAVY pions ($m_\pi > 300$ MeV) $\Rightarrow$ need extrapolation formulas to achieve $m_\pi \sim 140$ MeV

Argument for Hard Pion Chiral Perturbation Theory

Flynn and Sachrajda (2009), Bijnens and Celis (2009), Bijnens and IJ (2010)

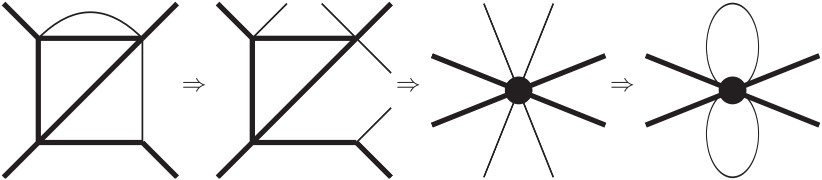


In the Feynman diagrams appear both **hard** and *soft* lines.

The *soft* lines can be separated from the **hard/short-distance** structure of the rest of the diagram.
They are the only responsible of the non analyticities arising for $m_\pi \to 0$ (e.g. the chiral logarithms)

The **hard** part is describable by an effective Lagrangian consistent with all the symmetries and with couplings that depend on hard kinematical quantities.
Assumption: this Lagrangian is sufficiently complete to describe the neighbourhood of the hard process.

Example



1. consider a diagram with *soft (thin)* and **hard (thick)** lines
2. identify the *soft* lines and cut them ⇒ remove the soft singularities
3. the resulting diagram is analytic in the soft part and thus should be describable by a vertex of an effective Lagrangian. The coupling contains information on the hard quantities
4. insert back the loops with the *soft* lines: this last diagram should reproduce the soft singularities of the first one

However only arguments not proof!!!

Results

$$O(q^2, m^2) = L(q^2, 0) \times \left(1 + \alpha m^2 \log\left(\frac{m^2}{\mu^2}\right) + \mathcal{O}(m^2)\right)$$

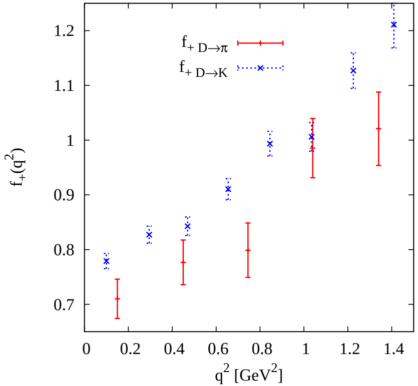
α is what we calculate. $L(q^2, 0)$ depends on the hard quantities (q^2, m_B).
Hard pion ChPT applied so far to

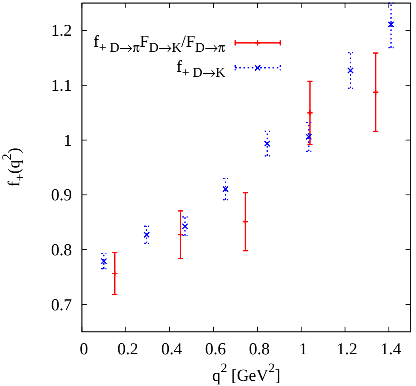
- $K \to \pi \ell \nu_\ell$ (two-flavour) Flynn and Sachrajda (2009)
- $K \to \pi\pi$ (two-flavour) Bijnens and Celis (2009)
- results agree with three-flavour standard ChPT
- $B(D) \to \pi \ell \nu_\ell$ (two-flavour) Bijnens and IJ (2010) $\to$ agree with a relativistic formalism
- Bijnens and IJ (2011)
  1. $B(D) \to M \ell \nu_\ell$ with $(M = \pi, K, \eta)$ (three-flavour),
  2. $B \to D \ell \nu_\ell$ (three-flavour)
  3. π and K scalar and vector formfactors (three-flavour)
  4. checked including two-loop diagrams for scalar and vector formfactors of the pion (two-flavour)

Comparison with data

- use three-flavour results and compare $f^+_{D\to\pi}(q^2)$ with $f^+_{D\to K}(q^2)$ CLEO coll. (2009)
- the chiral logs are responsible of the differences between the two decays

$$\frac{f^+_{D\to\pi}}{1+\log_{D\to\pi}} \approx \frac{f^+_{D\to K}}{1+\log_{D\to K}}$$





Vector and scalar formfactors of the pion

$$\left\langle \pi^+(p_2) \left| j_\mu^{\text{elm}} \right| \pi^+(p_1) \right\rangle = (p_2 + p_1)_\mu F_V^\pi(s)$$

- similar for scalar formfactor
- hard pion ChPT can be applied here. It predicts for $s \gg m_\pi^2$, $m_\pi^2 \to 0$

$$F_V^\pi(s) = F_V^{\pi\chi}(s) \left(1 - \frac{1}{F^2} \frac{m_\pi^2}{16\pi^2} \log\left(\frac{m_\pi^2}{\mu^2} \right) + \mathcal{O}(m_\pi^2) \right)$$

  where $F_V^{\pi\chi}(s)$ unknown (contains only hard quantities, no m_π)
  $\alpha = -1/(16\pi^2 F^2)$
- same result obtained taking ChPT prediction Bijnens *et al.* (1998) and expand it for $s \gg m_\pi^2$, $m_\pi^2 \to 0$
- standard ChPT also predicts

$$F_V^{\pi\chi}(s) = 1 + \frac{s}{16\pi^2 F^2} \left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6} \ln \frac{s}{\mu^2} \right)$$

A two-loop check (two-flavour hard pion ChPT)

- so far including only one-loop diagrams
- what happens if two-loop diagrams are added? If hard pion ChPT is valid α must not change and we must obtain the same chiral logarithm:

$$F_V^\pi(s) = F_V^{\pi\chi}(s)\left(1 - \frac{1}{F^2}\frac{m_\pi^2}{16\pi^2}\log\left(\frac{m_\pi^2}{\mu^2}\right) + \mathcal{O}(m_\pi^2)\right)$$

- take ChPT two-loop result and expand it for $s \gg m_\pi^2$, $m_\pi^2 \to 0$
- terms like $sm_\pi^2 \log^2(m_\pi^2)$ exactly cancel as hard pion ChPT predicts
- furthermore $F_V^\pi(s)$ is the same as before, coefficient α of the chiral log not affected

$$F_V^{\pi\chi}(s) = 1 + \frac{s}{16\pi^2 F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{s}{\mu^2}\right)$$

- this is a test of validity of hard pion ChPT

Conclusions

- ChPT valid only for soft mesons (at low energies), but sometimes needed at higher energies to perfom chiral extrapolations
- chiral logarithms can still be predicted with hard pion ChPT
- we applied (two-flavour) hard pion ChPT to semileptonic decays of B and D mesons
- we extended results to three-flavour hard pion ChPT: more $B(D) \to M$ transitions ($M = \pi, K, \eta$) and first comparison with experiments
- $B \to D\ell\nu_\ell$ decay also better understood
- we checked hard pion ChPT arguments for scalar and vector pion form factors including two-loop diagrams

Numerical analysis explanation

1. Evaluation of each side of the relation using experimental data and/or dispersive analysis:
   [CGL] G. Colangelo, J. Gasser and H. Leutwyler (2001) ($\pi\pi$ scattering)
   [BDM] Büttiker, Descotes-Genon, Moussallam (2004) (πK scattering)
   [NA48/2] NA48/2 coll. (2008) ($K_{\ell 4}$)
   [E865] S. Pislak *et al.* (2003) ($K_{\ell 4}$)
2. Evaluation using NNLO ChPT results; L_i =fit10 Amoros *et al.* (2001). J. Bijnens (2007)
3. We quote the difference of the two evaluations $\Rightarrow$ it contains only the p^6 piece coming from the C_i and higher order terms.
4. Errors obtained adding in quadrature the uncertainties from experiments/dispersive results. No theoretical uncertainty due to higher orders. Theoretical uncertainty due to L_i^r (probably under-)estimated.

An example

Results for 5 relations involving $\pi\pi$ scattering observables

	[CGL]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	**remainder**
LHS (1)	0.009 ± 0.039	0.054	-0.044	-0.041	$-0.002(3)$	0.041 ± 0.039
RHS (1)	-0.102 ± 0.002	-0.009	-0.044	-0.060	$-0.008(6)$	0.018 ± 0.002
10 LHS (2)	0.334 ± 0.019	0.209	0.097	0.103	$0.029(11)$	-0.105 ± 0.019
10 RHS (2)	0.322 ± 0.008	0.177	0.097	0.120	$0.034(13)$	-0.107 ± 0.008
LHS (3)	0.216 ± 0.010	0.166	0.029	0.053	$0.016(6)$	-0.047 ± 0.010
RHS (3)	0.189 ± 0.003	0.145	0.029	0.049	$0.020(7)$	-0.054 ± 0.003
10 LHS (4)	0.213 ± 0.005	0.137	0.032	0.053	$0.035(12)$	-0.043 ± 0.005
10 RHS (4)	0.175 ± 0.003	0.121	0.032	0.050	$0.029(10)$	-0.057 ± 0.003
10^3 LHS (5)	0.92 ± 0.07	0.36	0.00	0.56	$-0.01(13)$	0.00 ± 0.07
10^3 RHS (5)	1.18 ± 0.04	0.42	0.00	0.57	$0.03(13)$	0.15 ± 0.04

Need to include heavy mesons: $m_b \to \infty \Rightarrow$ Heavy Quark Effective Theory combined with ChPT.

$$H^a(v) = \frac{1+\not{v}}{2}\left[B_\mu^{*a}(v)\gamma^\mu - B^a(v)\gamma_5\right]$$

v: four-velocity of the heavy meson
a: light quark flavour index, $B^1 = B^+$ and $B^2 = B^0$ (similarly for B^*_μ)

$$D^\mu_{ab} H_b(v) = \delta_{ab}\partial^\mu H_b(v) + \Gamma^\mu_{ab} H_b(v) \quad \Gamma^\mu_{ab} = \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right]_{ab}$$

$$\mathcal{L}_{\text{heavy}} = -i\,\text{Tr}\left[\overline{H}_a i v \cdot D_{ab} H_b\right] + g\,\text{Tr}\left[\overline{H}_a u^\mu_{ab} H_b \gamma_\mu \gamma_5\right]$$

g: coupling of the heavy meson doublet to the Goldstone boson ($BB^*\pi$, $B^*B^*\pi$)
Tr is over the γ-matrix indices

Extra Check: the relativistic Lagrangian

- $q^2 \neq q^2_{\text{max}} \Rightarrow$ in the loops may appear very off-shell B and B^*
- do different treatments of the off-shell behavior lead to different nonanalyticities?
- according to our argument it should not be the case
- to test this, we calculate also in a relativistic formulation

The relativistic Lagrangian still respects the spin-flavour symmetries:

$$
\begin{aligned}
\mathcal{L}_{\text{kin}} &= \nabla^\mu B^\dagger \nabla_\mu B - m_B^2 B^\dagger B - \frac{1}{2} B^{*\dagger}_{\mu\nu} B^{*\mu\nu} + m_B^2 B^{*\dagger}_\mu B^{*\mu} \\
\mathcal{L}_{\text{int}} &= g M_0 \left(B^\dagger u^\mu B^*_\mu + B^{*\dagger}_\mu u^\mu B \right) + \frac{g}{2} \epsilon^{\mu\nu\alpha\beta} \left(-B^{*\dagger}_\mu u_\alpha \nabla_\mu B^*_\beta + \nabla_\mu B^{*\dagger}_\nu u_\alpha B^*_\beta \right) \\
B^*_{\mu\nu} &= \nabla_\mu B^*_\nu - \nabla_\nu B^*_\mu, \qquad \nabla_\mu = \partial_\mu + \Gamma_\mu
\end{aligned}
$$

B, B^* now in the relativistic form and column-vectors in light flavour space

Extra Check: the relativistic Lagrangian

From $\mathcal{L}_{\text{kin}}$ we find the propagators of the B and B^* respectively:

$$\frac{i}{p^2 - m_B^2} \qquad \frac{-i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_B^2}\right)}{p^2 - m_B^2}$$

in contrast with the propagators in HMChPT formalism

$$\frac{i}{v \cdot p} \qquad \frac{-i\left(g_{\mu\nu} - v_\mu v_\nu\right)}{v \cdot p}$$

This showes the different off-shell behavior.

Correctly we find the same coefficients in the two formalisms!

Sketch of the proof for heavy mesons semileptonic decay

- $q^2 \ll q^2_{\max} \Rightarrow$ operators with an arbitrary numbers of derivatives on the external π are not negligible since its momentum is large
- look at $\langle \pi(p_\pi) |O| B(v) \rangle$ $\quad O$ =operator in J^V_μ with more derivatives
- keep only: $\mathcal{O}(1)$, $\mathcal{O}(m_\pi)$ and $\mathcal{O}(m^2_\pi \log m^2_\pi)$. NO: $\mathcal{O}(m^2_\pi)$ without logarithms
- partial integration and dimensional analysis on $\langle \pi(p_\pi) |O| B(v) \rangle$
- $\langle \pi(p_\pi) |O| B(v) \rangle$ are all proportional to the lowest order ones up to terms $O(m^2_\pi)$ (and without logarithms) which are of higher order
- constants of proportionality can be absorbed in the couplings $\Rightarrow$ they change with the q^2!
- NOTE: kaon loops do not affect the discussion$\Rightarrow$ two- and three- flavour hard pion ChPT both OK